9. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive root, if and only if
   a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

10. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then the value of x is equal to
   a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

11. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is
   a) [1,2] b) [–1,1] c) [0,1] d) [–1,0]

12. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, the value of $(x + y + z)^3$ is
   a) 0 b) –1 c) –3 d) –27

13. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is
   a) $\frac{4}{3}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{3}{2}$

14. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ are
   a) $4(a^2 + b^2)$ b) $2(a^2 + b^2)$ c) $a^2 + b^2$ d) $\frac{1}{2}(a^2 + b^2)$

15. If P(x,y) be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is
   a) 8 b) 6 c) 10 d) 12

16. The parametric equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are
   a) $x = at^2, y = 2at$ b) $x = a\cos\theta, y = b\sin\theta$
   c) $x = a\sec\theta, y = b\tan\theta$ d) $x = a\tan\theta, y = b\sec\theta$

17. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is
   a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) –1

18. If the volume of the parallelepiped with $\vec{a}\times\vec{b}$, $\vec{b}\times\vec{c}$, $\vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$, $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is
   a) 8 cubic units b) 512 cubic units c) 64 cubic units d) 24 cubic units

19. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2} = z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is
   a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

20. The distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0 is
   a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{\sqrt{2}}$

## Part - B

**II. Answer any 7 questions: (Ques.No.30 is compulsory)** 7 x 2 = 14

21. Solve $5x + 2y = 3$, $3x + 2y = 5$ by using inverse matrix method.

22. If $\text{adj}\,A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find $A^{-1}$

23. Write in polar form of the following complex number $1+i\sqrt{3}$

24. Find the square roots of 6-8i

25. If $\alpha$, $\beta$ and $\gamma$ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

26. Solve: $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

27. Find the equation of the ellipse with foci $(\pm 2, 0)$, vertices $(\pm 3, 0)$

28. Find the equation of the circle with centre $(2,-1)$ and passing throught the point $(3,6)$ in standard form.

29. Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x-y+x=5$

30. For what value of x does $\sin x = \sin^{-1} x$ ?

## Part - C

**III. Answer any 7 questions: (Ques.No.40 is compulsory)** 7 x 3 = 21

31. Find the inverse of the matrix $A = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$ by Gauss-Jordan method.

32. Show that $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

33. Suppose $z_1$, $z_2$ are any two complex numbers, prove $|z_1.z_2| = |z_1|\,|z_2|$ and $\arg(z_1.z_2) = \arg(z_1) + \arg(z_2)$

34. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume $a,b,c,d \neq 0$

35. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$

36. Find the equation of the tangent at $t = 2$ to the parabola $y^2 = 8x$

37. A concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long amd maximum height of the arch is 15 m. Write the equation of the parablic arch.

38. If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are coplanar vectors, then show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$

39. Find the points of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y+1}{2} = z$

40. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & ? & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

## Part - D

**IV. Answer all the questions:** $7 \times 5 = 35$

41. a) If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a,b and c, and hence find $A^{-1}$

***(or)***

b) By using Gaussian elimination method, balance the chemical reaction equation $C_2H_6 + O_2 \rightarrow H_2O + CO_2$

42. a) Investigate for what values of $\lambda$ and $\mu$ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$. Has i) no solution ii) a unique solution iii) an infinite number of solutions

***(or)***

b) If $z = x + iy$ is a complex number such that $\left|\frac{z+i}{z-1}\right| = 1$, find the locus of Z.

43. a) Find all the cubic roots of $\sqrt{3} + i$

***(or)***

b) Solve $(x - 4)(x - 7)(x - 2)(x + 1) = 16$

44. a) Find the value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right)$

***(or)***

b) Find the equation of circle passing through the points (1,2), (2,–1) and (3,2)

45. a) Identify the type of conic whose equation is $9x^2 - y^2 - 36x - 6y + 18 = 0$ and find centre, foci, vertices and directrices.

***(or)***

b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

46. a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

***(or)***

b) If $\vec{a} = 2\vec{i} - 3\vec{j} + 2\vec{k}$, $\vec{b} = 3\vec{i} - \vec{j} + 3\vec{k}$, $\vec{c} = 2\vec{i} - 5\vec{j} + \vec{k}$, verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$

47. a) Find the parametric form of vector equation, and the cartesian equations of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$

***(or)***

b) Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines.

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