Reg.No. | | | | | | |

COMMON FIRST REVISION TEST - 2020

STANDARD - XII

Time : 3.00 hrs **Mathematics** Marks: 90

Part - I 20 x 1 = 20

Note: i) All questions are compulsory. ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer:

1. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of fx

a) $-\frac{4}{5}$ b) $-\frac{3}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

2. If A, B are invertible matrices of some order then which one of the following is true?

a) $|adjA| = |A|^n$ b) $(adj\ A)^T = adjA$ c) $(adj\ A)^{-1} = A$ d) $adj\ (adjA) = |A|^{n-2} A$

3. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

a) (0, 1) b) (1, 0) c) (-1, 1) c) (1, 1)

4. If the roots of $x^3 + px^2 + q^{x+r} = 0$ are in A.P then

a) $9pq = 2p^3 + 27\ r$ b) $9pq = 27r^3 + 2p$ c) $9pq = 2p^3 - 27r$ d) $9pq = 27r^3 - 2p$

5. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ then $\cos^{-1} x + \cos^{-1} y =$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) π d) $\frac{2\pi}{3}$

6. If x + y = k is a normal to the parabola $y^2 = 12x$, then the value of k is

a) -1 b) 3 c) 9 d) 1

7. The locus of a point where distance from (-2, 0) iis 2/3 times its distance from the line x = -9/2 is

a) a circle b) a parabola c) an ellipse d) a hyperbola

8. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors then $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] =$

a) $[\vec{a}\ \vec{b}\ \vec{c}]$ b) 0 c) $-[\vec{a}\ \vec{b}\ \vec{c}]$ d) $2[\vec{a}\ \vec{b}\ \vec{c}]$

9. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

a) $c > 0$ b) $c = \pm\sqrt{3}$ c) $c = \pm 3$ d) $0 < c < 1$

10. If $\lim_{\theta \to 0} \left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$ then which of the following is true

a) m = n b) $\frac{m}{n} = -1$ c) mn = -1 d) $m = \pm n$

11. The point of inflection of the curve $y = (x - 1)^3$ is

a) (0, 1) b) (1, 0) c) (1, 1) d) (0, 0)

12. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

a) 31 b) 5 c) 1/5 d) 1/31

13 $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{x^3 + y^2}{x + y + 2}\right) =$

a) 0 b) 1 c) ∞ d) $\pi/2$

14. The value of $\int_0^\infty e^{-3x}x^2dx$ is

a) 5/27 b) 4/27 c) 2/27 d) 7/27

15. The value of $\int_0^1 x(1-x)^{99}dx$ is

a) $\dfrac{1}{10100}$ b) $\dfrac{1}{11000}$ c) $\dfrac{1}{10001}$ d) $\dfrac{1}{10010}$

16. The order and degree of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$ is

a) 1, 1 b) 2,1 c) 2, 2 d) 1, 2

17. Solution of the differential equation $2x\dfrac{dy}{dx} - y = 3$ represents

a) circle b) parabola c) ellipse d) straight line

18. Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6, the player wins ` 36, otherwise he loses ` k^2, where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$. The expected amount to win at this game in ` is

a) $-\dfrac{19}{6}$ b) $\dfrac{19}{6}$ c) $-\dfrac{3}{2}$ d) $\dfrac{3}{2}$

19. Vanance of one point distribution is

a) 1 b) 0 c) npq d) none

20. The operation * defined by $a * b = \dfrac{ab}{7}$ is not a binary operation on

a) Z b) R c) C d) Q^+

Part - B

II. Answer any 7 questions. Question No. 30 is compulsory. **7 x 2 = 14**

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

22. Prove that a complex number is purely imaginary if and only if $Z = -\bar{Z}$

23. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$

24. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

25. Obtain the equation of the circle for which (3, 4) and (2, -7) are the ends of a diameter

26. For any four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ prove that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

27. Find the local extrema of the function $f(x) = x^4 + 32x$

28. If $u(x, y) = x^2 + y^2$; $x = e^t$; $y = e^{-t}$, find $\frac{du}{dt}$

29. The probability mass function is defined by $f(x) = \begin{cases} \frac{x^2+1}{k}, & x = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$, find the value of k.

30. Evaluate : $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{(1-x^2)^{1/2}}dx$

Part - C

III. Answer any seven questions. Question No.40 is compulsory **7 x 3 = 21**

31. Four men and four women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

32. Obtain the cartesian form of the locus of $z = x + iy$ if $|z+i| = |z-1|$

33. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

34. Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 7 = 0$ and $x + y - 2z + 5 = 0$ and is perpendicular to the plane $x + y - 3z - 5 = 0$

35. Expand sinx in ascending powers $x - \frac{\pi}{4}$ upto three non-zero terms.

36. Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq 1$ at $x_0 = 3$ use the linear approximation to estimate f (3.2)

37. Find the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = 2x^2$; $y = 0$ and $x = 1$.

38. Solve : $\frac{dy}{dx} + 2y\cot x = 3x^2 \operatorname{cosec}^2 x$

39. Construct the truth table for $(\neg p \to r) \wedge (p \leftrightarrow q)$

40. If $u(x, y) = \cos^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then find $x\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y}$

Part - D

IV. Answer all the questions: $7 \times 5 = 35$

41 a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has i) no solution ii) a unique solution iii) an infinite number of solutions.

(OR)

b) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

42. a) Solve the equation $z^3 + 8i = 0$, where $Z \in C$ **(OR)**

b) Two coast guard stations are located 600km apart at points A (0, 0) and B (0, 600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

43. a) Solve $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ **(OR)**

b) Use integration find the area of the region bounded by triangle ABC whose vertices A, B and C are (-1, 1), (3, 2) and (0, 5) respectively.

44. a) Identify the type of conic $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ and find centre, foci, vertices and directrices. **(OR)**

b) Verify i) closure property ii) commutative property iii) associative property iv) existence of identity and v) existence of inverse for the operation X_{11} on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(OR)**

45. a) Sketch the curve $y = f(x) = x^2 - x - 6$

b) Find the non-parametric from of vector equation and cartesian equation of the plane through the point (1, -2, 4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

46. a) If $w(x, y) = xy + \sin(xy)$ then prove that $\frac{\partial^2 \omega}{\partial x \partial y} = \frac{\partial^2 \omega}{\partial y \partial x}$ **(OR)**

b) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall i) how fast is the top of the ladder moving down the wall? ii) at what rate, the area of the triangle formed by the ladder, wall and the floor is changing?

47. a) On the average 20% of the products manufactured by ABC company are found to be defect. If we select 10 of these products at random and x denotes the number of defective products are defective ii) atmost three products are defective iii) atleast four products are defective. **(OR)**

b) A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at a rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.
