

FIRST REVISION EXAMINATION-2019-2020 Time 3Hrs**I Choose the suitable answer (20 x 1 = 20)**

1. If $A = \begin{bmatrix} 1 & \tan \frac{\theta}{2} \\ -\tan \frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$

- a) $\left(\cos^2 \frac{\theta}{2}\right)A$ b) $\left(\cos^2 \frac{\theta}{2}\right)A^T$ c) $(\cos^2 \theta)I$ d) $\left(\sin^2 \frac{\theta}{2}\right)A$

2. The Augmented matrix of a system of a system of linear

equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$ the system has infinitely many solutions if

- a) $\lambda = 7, \mu \neq -5$ b) $\lambda = -7, \mu = 5$ c) $\lambda \neq 7, \mu \neq -5$ d) $\lambda = 7, \mu = -5$

3. If $|Z| = 1$, then the value of $\frac{1+Z}{1+\bar{Z}}$ is

- a) Z b) \bar{Z} c) $\frac{1}{Z}$ d) 1

4. If $|Z_1| = 1, |Z_2| = 2, |Z_3| = 3$ and $|9Z_1Z_2 + 4Z_1Z_3 + Z_2Z_3| = 12$ then the value of $|Z_1 + Z_2 + Z_3|$ is

- a) 1 b) 2 c) 3 d) 4

5. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

- a) -1 b) $\frac{5}{4}$ c) $\frac{4}{5}$ d) 5

6. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

- a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π

7. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

- a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

8. The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is

- a) 10 b) $2\sqrt{5}$ c) 6 d) 4

9. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

- a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ d) $(3\sqrt{3}, -2\sqrt{2})$

10. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

- a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

11. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

- a) $\tan^{-1}\left(\frac{3}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

12. The curve $y = ax^4 + bx^2$ with $ab > 0$

13. If $v(x, y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

- a) $e^x + e^y$ b) $\frac{1}{e^x + e^y}$ c) 2 d) 1

14. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

- a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$

15. The value of $\int_0^1 x(1-x)^{99} dx$ is

- a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

16. The value of $\int_0^a (\sqrt{a^2 - x^2})^3 dx$

- a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ c) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

17. The order of the differential equation of all circles with centre at (h,k) and radius 'a' is

- a) 2 b) 3 c) 4 d) 1

18. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and

it passes through (-1,1). Then the equation of the curve is

- a) $y = x^3 + 2$ b) $y = 3x^2 + 4$ c) $y = 3x^3 + 4$ d) $y = x^3 + 5$

19. If $f(x) = \begin{cases} 2x & 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a

random variable, then the value of a is

- a) 1 b) 2 c) 3 d) 4

20. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

- a) 9 b) 8 c) 6 d) 3

II Answer any SEVEN questions (Q.No 30 is compulsory)

(7 x 2 = 14)

21. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$,

by Gauss- Jordan Method.

22. Simplify: $i^{1948} - i^{-1869}$

23. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has at least 6 imaginary roots.

24. Find the value of $\sin^{-1}(\sin \frac{5\pi}{9} \cos \frac{\pi}{9} + \cos \frac{5\pi}{9} \sin \frac{\pi}{9})$.

25. Find the vertices, foci for the hyperbola $9x^2 - 16y^2 = 144$.

26. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that

$$[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = -[\vec{a}, \vec{b}, \vec{c}]$$

27. Write the Maclaurin series of the function: $\cos x$

28. If $U(x, y, z) = \frac{x^2 + y^2}{xy} + 3z^2y$, find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial z}$

29. Three fair coins are tossed simultaneously. Find the probability mass function for the number of heads occurred.

30. check whether the statement $p \rightarrow (q \rightarrow p)$ is a tautology or a contradiction without using the truth table.

III Answer any SEVEN questions (Q.No 40 is compulsory)

(7 x 3 = 21)

31. Solve the following system of equations, using matrix inversion method

$$2x_1 + 3x_2 + 3x_3 = 5, x_1 - 2x_2 + x_3 = -4, 3x_1 - x_2 - 2x_3 = 3$$

32. Find the value of the real numbers x and y, if the complex number $(2+i)x + (1-i)y + 2i - 3$ and $x + (-1+2i)y + 1+i$ are equal

33. If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P, Prove that $9pqr = 27r^3 + 2p$

34. Find the domain of $\cos^{-1}\left(\frac{2 + \sin x}{3}\right)$

35. Find the Centre, foci and eccentricity of the hyperbola

$$11x^2 - 25y^2 - 44x + 50y - 256 = 0$$

36. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{i} + 5\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{c} = -3\hat{i} + \hat{j} + 4\hat{k}$ if the base is taken as the parallelogram determined by \vec{b} and \vec{c}

37. Find the absolute extrema of the function on the given closed interval $f(x) = x^2 - 12x + 10$; $[1, 2]$.

38. Show that $f(x, y) = \frac{x^2 - y^2}{y^2 + 1}$ is continuous at every $(x, y) \in \mathbb{R}^2$

39. Evaluate : $\int_0^{\frac{\pi}{4}} \sin^6 2x \, dx$

40. A random variable X has the following probability mass function

x	1	2	3	4	5
f(x)	K^2	$2k^2$	$3k^2$	$2k$	$3k$

Find i) the value of k ii) $P(2 \leq X < 5)$ iii) $P(3 < X)$

II Answer any SEVEN questions (7 x 5 = 35)

41. a) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has i) no solution ii) a unique solution iii) an infinite number of solutions.

(Or)

b) By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$

42. a) Show that i) $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real and ii)

$\left(\frac{19 + 9i}{5 - 3i}\right)^{15} - \left(\frac{8 + i}{1 + 2i}\right)^{15}$ is purely imaginary

(Or)

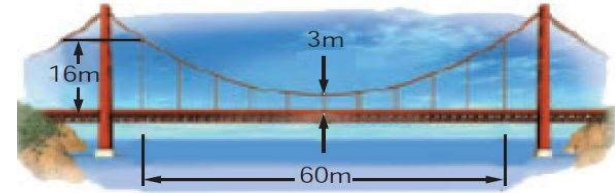
b) If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1 - 2\omega, 1 - 2\omega^2$

43. a) Solve: $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

(Or)

b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

44. a) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned. Vertical cables are to be spaced every 6m along this portion of the roadbed, calculate the lengths of first two of these vertical cables from the vertex.



(Or)

b) By Vector method, prove that $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

45. a) Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that \vec{c} is a unit vector perpendicular to both \vec{a} and \vec{b} . If the angle between \vec{a} and \vec{b} is $\frac{\pi}{6}$, show that $[\vec{a}, \vec{b}, \vec{c}] = \frac{1}{4} |\vec{a}|^2 |\vec{b}|^2$

(Or)

b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

46. a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$

(Or)

b) If X is the random variable with distribution function F(x) given by,

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}(x^2 + x), & 0 \leq x < 1 \text{ then find i) the probability} \\ 1, & 1 \leq x < \infty \end{cases}$$

density function $f(x)$ ii) $P(0.3 \leq X \leq 0.6)$

47. a) The probability density function of the random variable X is given by

$$f(x) = \begin{cases} 16xe^{-4x} \text{ for } x > 0 \\ 0 \text{ for } x \leq 0 \end{cases} \text{ find the mean and variance of } X.$$

(Or)

b) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$.

Is $*$ binary on A ? If so, examine the commutative, associative, the existence of identity existence of inverse properties satisfied by $*$ on A .

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