PATTUKKOTTAI PALANIAPPAN MATHS

I- REVISION TEST (FULL PORTION)-2020

12th Standard

MATHS

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs

Total Marks : 90

P.A.PALANIAPPAN,MSc.,MPhil.,BEd

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9443407917

PART-I

20 x 1 = 20

Answer All the Questions

1) If A^TA^{-1} is symmetric, then $A^2 =$

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

2) Let A be a 3 x 3 matrix and B its adjoint matrix If |B|=64, then |A|=

(a) ±2 (b) ±4 (c) ±8 (d) ±12

3) If $z=\dfrac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then |z| is equal to

(a) 0 (b) 1 (c) 2 (d) 3

4) The amplitude of $\frac{1}{i}$ is equal to

(a) 0 (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{2}$ (d) π

5) If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if

(a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

6) If $x=\frac{1}{5}$, the valur of $\cos(\cos^{-1}x+2\sin^{-1}x)$ is

(a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

7) The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

8) An ellipse hasOB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

9) If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$ and is parallel to $\vec{c}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

10) Distance from the origin to the plane 3x - 6y + 2z 7 = 0 is

(a) 0 (b) 1 (c) 2 (d) 3

11) The slope of the line normal to the curve f(x) = 2cos 4x at $x=\dfrac{\pi}{12}$

(a) $-4\sqrt{3}$ (b) -4 (c) $\dfrac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

12) The point of inflection of the curve $y = (x - 1)^3$ is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

13) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

14) If $\int_a^a \frac{1}{4+x^2}dx=\frac{\pi}{8}$then a is

(a) 4 (b) 1 (c) 3 (d) 2

15) The value of $\int_{-1}^2 |x|dx$

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

16) The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$are respectively

(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

17) The solution of $\frac{dy}{dx}$+p(x)y=0 is

(a) $y=ce^{\int pdx}$ (b) $y=ce^{-\int pdx}$ (c) $x=ce^{-\int pdx}$ (d) $xce^{\int pdx}$

18) If X is a binomial randam variable with expected value 6 and variance 2.4, then P(X=5) is

(a) $\left(\frac{10}{5}\right)^2\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\left(\frac{10}{5}\right)\left(\frac{3}{5}\right)^5$ (c) $\left(\frac{10}{5}\right)\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (d) $\left(\frac{10}{5}\right)\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

19) A computer salesperson knows from his past experience that he seUs computers to one in every twenty customers who enter the showroom. What is the probability that he will seU a computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

20) The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

(a) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ (b) $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$ (c) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ (d) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

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PART-II 7 x 2 = 14

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Note: i)Answer any 7 questions only

ii)Question No.30 compuls

21) If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22) Simplify the following

$i\, i^2 i^3 \ldots i^{2000}$

23) Evaluate the following limit, if necessary use l'Hôpital Rule

$\lim\limits_{x \to \infty} e^{-x} \sqrt{x}$

24) Evaluate $\int_0^1 x^3 (1-x)^4 dx$

25) The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$.

Find the probability that exactly 3 of the 5 components tested survive.

26) Establish the equivalence property $p \rightarrow q \equiv \neg p \vee q$

27) Find the value of $\tan^{-1}(-1) + \cos^{-1}(\frac{1}{2}) + sin^{-}1(-\frac{1}{2})$

28) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $(\vec{a}+\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}+\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}+\vec{a})(\vec{a}\times\vec{b})$

29) The equation $y=\frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

30) Find the differential equation of the family of all non-vertical lines in a plane.

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PART-III 7 x 3 = 21

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Note: i)Answer any 7 questions only

ii)Question No.40 compulsary

31) Decrypt the received encoded message $[\,2 \quad -3\,]\,[\,20 \quad 4\,]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

32) Prove that a straight line and parabola cannot intersect at more than two points.

33) If the equation$3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle, find p and q . Also determine the centre and radius of the circle

34) Suppose f(x) is a differentiable function for all x with $f'(x) \le 29$ and f(2) =17. What is the maximum value of f (7)?

35) The slope of the tangent to the curve at any point is the reciprocal of four times the ordinate at that point. The curve passes through (2, 5). Find the equation of the curve.

36) A lottery with 600 tickets gives one prize of Rs.200, four prizes of noo, and six prizes of Rs. 50. If the ticket costs is Rs.2, find the expected winning amount of a ticket

37) Find the square root of $6-8i$.

38) If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines.

39) Evaluate $\int_{-\pi}^{\pi} \frac{cos^2 x}{1+a^x}$ dx

40) If u(x,y, z) =xy^2z^3, x= sin t, y= cos t, z = 1+ e^{2t}, find $\frac{du}{dt}$

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PART-IV 7 x 5 = 35

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Answer All the Questions

41) a) A boy is walking along the path y = ax^2 + bx + c through the points (−6, 8),(−2, −12) , and (3, 8) . He wants to meet his friend at P(7,60) . Will he meet his friend? (Use Gaussian elimination method.)

(OR)

b) If z=x+iy is a complex number such that Im $\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2+2y^2$+x-2y=0

42) a) Solve $cos\left(sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = sin\left\{cot^{-1}\left(\frac{3}{4}\right)\right\}$

(OR)

b) Solve: $8x^{\frac{3}{2x}} - 8x^{\frac{-3}{2x}}$=63

43) a) Two coast guard stations are located 600 km apart at points A(0,0) and B(0,600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

(OR)

b) Find the equation of a straight line passing through the point of intersection of the straight lines $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$ and perpendicular to both straight lines.

44) a) Find the points on the unit circle $x^2 + y^2$ =1 nearest and farthest from (1,1).

(OR)

b) Let w(x, y) = xy+$\frac{e^y}{y^2+1}$ for all (x, y) ∈ R^2. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

45) a) Father of a family wishes to divide his square field bounded by x = 0,x= 4, y=4 and y=0 along the curve y^2 x=4 and x^2y=4 into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

(OR)

b) (x^2+y^2)dy = xy dx. It is given that y(1) = 1 and y(x_0) = e. Find the value of x_0.

46) a) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

(i) P(2 < X < 6)

(ii) P(2 ≤ X < 5)

(iii) P(X ≤4)

(iv) P(3 < X)

(OR)

b) a) Define an operation*on Q as follows: a*b=$\left(\frac{a+b}{2}\right)$; a,b ∈Q. Examine the closure, commutative, and associative properties satisfied by*on Q.

b) Define an operation∗ on Q as follows: a*b=$\left(\frac{a+b}{2}\right)$; a,b ∈Q. Examine the existence of identity and the existence of inverse for the operation * on Q.

47) a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

(OR)

b) Sketch the curve y = f (x) = x^3−6x-9

"PLEASE CORRECT ANY MISTAKES"

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PART-I

20 x 1 = 20

Answer All the Questions

1) (b) $(A^T)^2$

2) (c) ±8

3) (c) 2

4) (c) $-\frac{\pi}{2}$

5) (c) a<0

6) (d) $-\frac{1}{5}$

7) (b) $2(a^2+b^2)$

8) (a) $\frac{1}{\sqrt{2}}$

9) (b) $\vec{b}$

10) (b) 1

11) (c) $\frac{\sqrt{3}}{12}$

12) (c) (1,0)

13) (b) $\frac{1}{5}$

14) (d) 2

15) (c) $\frac{5}{2}$

16) (a) 2, 3

17) (b) $y = ce^{-\int p dx}$

18) (a) $\left(\frac{10}{5}\right)^2\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$

19) (a) $\frac{57}{20^3}$

20) (d) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

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PART-II

7 x 2 = 14

Note: i)Answer any 7 questions only

ii)Question No.30 compuls

21) We compute |adj A| = $\begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$ = 9.

So, we get $A^{-1} = \pm\frac{1}{\sqrt{|adjA|}}$ adj(A) = $\frac{1}{\sqrt{9}}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \pm\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

22) $i\, i^2\, i^3 \ldots. i^{2000}$

$= i^{1+2+3+.....+2000}$

$= i^{\frac{2000\times 2001}{2}}$

[∴ 1+2+3+....n=$\frac{n(n+1)}{2}$]

$= i^{1000 \times 2001}$

$= i^{2001000}$

=1

[∴ 2001000 is divisible by 4 as its last two digits are divisible by 4]

23) $\lim\limits_{x\to\infty} e^{-x}\sqrt{x} = \lim\limits_{x\to\infty}\frac{\sqrt{x}}{e^{x}} = \frac{\infty}{\infty}$

Which is in indeterminate form. Applying L' Hopital rule we get,

$\lim\limits_{x\to\infty}\frac{\frac{1}{2}x^{\frac{1}{2}-1}}{e^{x}} = \frac{1}{2}\lim\limits_{x\to\infty}\frac{x^{\frac{1}{2}-1}}{e^{x}} = \frac{1}{2}\lim\limits_{x\to\infty}\frac{e^{-x}}{\sqrt{x}}$

$\frac{1}{2}\lim\limits_{x\to\infty} e^{-x}\sqrt{\frac{1}{x}} = \frac{1}{2}e^{-\infty}(0) = 0$ [When x → ∞, $\frac{1}{x}$ → 0 $e^{-\infty} = 0$]

24) $\int_0^1 x^m(1-x)^n dx = \frac{m!\times n!}{(m+n+1)!}$

$\therefore \int_0^1 x^3(1-x)^4 dx = \frac{3!\times 4!}{(3+4+1)!} = \frac{3!\times 4!}{8!} = \frac{3\times2\times1\times4\times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1} = \frac{1}{280}$

25) Given $p = \frac{3}{4}$

n=5

$P(X = x) = nC_x p^x (1-p)^{n-x}$

$P(X=3) = 5C_3\left(\frac{3}{4}\right)^3\left(1-\frac{3}{4}\right)^2$

$P(X=3) = 5C_2\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$ [∵$nCr = nC_{n-r}$]

$P(X=3) = \frac{5\times\cancel{4}}{2\times1}\times\frac{3\times3\times3}{\cancel{4}\times4\times4\times4\times4}$

$= \frac{135}{512}$

26)

p	q	¬p	p → q	¬p ∨ q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The entries in the columns corresponding to p → q and ¬p ∨ q are identical and hence they are equivalent.

27) Let $\tan^{-1}(-1)$=y . Then, tan y =-1=-tan$\frac{\pi}{4} = tan(-\frac{\pi}{4})$

As -$\frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2}), tan^{-}1(-1) = -\frac{\pi}{3}$

Now, $\cos^{-1}(\frac{1}{2})$=y implies cos y =$\frac{1}{2}$=cos$\frac{\pi}{3}$

As $\frac{\pi}{3}$∈[0,π], $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$

Now,$\sin^{-1}(-\frac{1}{2})$=y implies sin y =-$\frac{1}{2}$=sin(-$\frac{\pi}{3}$).

As -$\frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}], sin^{-}1(-\frac{1}{2}) = -\frac{\pi}{6}$

Therefore, $\tan^{-1}(-1)$+$\cos^{-1}(\frac{1}{2}) + sin^{-}1(-\frac{1}{2}) = -\frac{\pi}{4} + \frac{\pi}{3} - \frac{\pi}{6} = -\frac{\pi}{12}$

28) Given $\vec{a}, \vec{b}, \vec{c}$ are concurrent edges of a parallelepiped, and its volume is 4 cubic units.

$\vec{a}.(\vec{b}\times\vec{c})$ =±4 ......(1)

Consider

$(\vec{a}+\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}+\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}+\vec{a})(\vec{a}\times\vec{b})$

$=\vec{a}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{b}\times\vec{c}) + \vec{b}.(\vec{c}\times\vec{a}) + \vec{c}.(\vec{c}\times\vec{a}) + \vec{c}.(\vec{a}\times\vec{b}) + \vec{a}.(\vec{a}\times\vec{b})$

$=\vec{a}.(\vec{b}\times\vec{c}) + 0 + \vec{b}.(\vec{b}\times\vec{c}) + 0 + \vec{b}.(\vec{c}\times\vec{a}) + 0$

$[\because \vec{a}.(\vec{a}\times\vec{b}) = 0]$

$=\vec{a}.(\vec{b}\times\vec{c}) + \vec{a}.(\vec{b}\times\vec{c}) + \vec{a}.(\vec{b}\times\vec{c})$

$\left[\because [\vec{a}\vec{b}\vec{c}] = [\vec{a}\vec{b}\vec{c}] = [\vec{a}\vec{b}\vec{c}]\right]$

$=3[\vec{a}.(\vec{b}\times\vec{c})] = 3(\pm4)$ using (1)

=±12

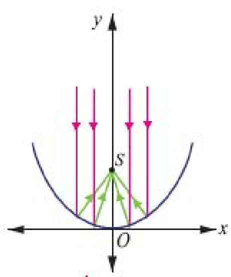
29) Equation of the parabola is y=$\frac{1}{32}x^2$

That is x^2 =32y ; the vertex is (0,0)

= 4 (8)y

$\Rightarrow a = 8$

So the heating tube needs to be placed at focus (0,a) . Hence the heating tube needs to be placed 8 units above the vertex of the parabola.



30) Equation of the family of non-vertical lines in a plane is ax + by = 1, b ≠ 0, a ∈ R.

Differentiating with respect to 'x' we get,

a + b $\frac{dy}{dx} = 0$

Differentiating again with respect to 'x' we get,

$b\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0 \quad [\because b \neq 0]$

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PART-III

7 x 3 = 21

Note: i)Answer any 7 questions only

ii)Question No.40 compulsary

31) Let the encryption matrix be A=$\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

|A|=-1+2=1≠0

∴ A-1=$\frac{1}{|A|} adjA = \frac{1}{1}\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

Hence the decryption matrix is $\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

Coded row matrix	Decoding matrix	Decoded row matrix
[2 -3]	$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$	=[2+6 2+3]=[8 5]
[20 4]	$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$	=[20-8 20-4] =[12 16]

So, the sequence of decoded row matrices is

[8 5], [12 16]

Now the 8th English alphabet is H.

5th English alphabet is E.

12th English alphabet is L.

and the 16th. English alphabet is P.

Thus the receiver reads the message as "HELP".

32) By choosing the co-ordinate axes suitably, we take the equation of the straight line as

y=mx+c ...(1)

and equation of parabola as y^2=4ax ...(2)

Substituting (1) in (2),we get

$(mx + c)^2 = 4ax$

⇒ $m^2x^2 + c^2 + 2mcx = 4ax$

⇒ $m^2x^2+x(2mc - 4a) + c^2 = 0$

Which is a quadratic equation in x.

This equation cannot have more than two solution. Hence, a straight line and a parabola cannot intersect at more than two points.

33) Given equation of the circle is

$3x^2 + (3 - p)xy + qy^2 - 2px = 8pq$

For the circle, co-efficient of xy = 0

⇒ 3-p = 0 ⇒ p=3

Also, co-efficient of x^2 = co-efficient of y^2

⇒ 3 = q

∴ Equation of the circle is

$3x^2 + 3y^2 - 6x = 8(3)(3)$

$3x^2 + 3y^2 - 6x - 72 = 0$

Dividing by 3, we get

$x^2 + y^2 - 2x - 24 = 0$

Here 2g = - 2⇒ g = - 1

1= 0 and c = - 24

Centre is (-g,-f) (1,0)

and r = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-1)^2 + 0 + 24}$

$= \sqrt{25}$ = 5 units.

34) By the mean value theorem we have, there exists 'c' ∈(2, 7) such that,

$\frac{f(7)-f(2)}{7-2}$ = f'(c) ≤ 29

Hence, f(7) ≤ 5× 29 +17 = 162

Therefore, the maximum value of f (7) is 162.

35) Given slop at any point = $\frac{1}{4(odinate)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{4y}$

$\Rightarrow 4ydy \quad = \quad dx$

integrating both sides we get,

$4\int y \quad dy = \int dx$

$\Rightarrow \quad 4.\frac{y^2}{2} = x + c$

$\Rightarrow 2y^2 = x + c$ ...(1)

Since the curve passes through (2, 5),

we get $2(5)^2 = 2 + c$

⇒ 50 - 2 = c

⇒ c = 48.

∴ (1) becomes, $2y^2 = x + 48$ which is the requaired equation of the curve.

36) $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

$E(X) = 200 \times \frac{1}{600} + 100 \times \frac{4}{600} + 50 \times \frac{6}{600} - 2 \times \frac{600}{600}$

$= \frac{200}{600} + \frac{400}{600} + \frac{300}{600} - 2$

$= \frac{900}{600} - 2 = \frac{3}{2} - 2 = \frac{3-4}{2}$

$= \frac{-1}{2} = Rs. -0.50$

37) We compute $|6 - 8i| = \sqrt{6^2 + (-8)^2} = 10$

and applying the formula for square root, we get

$\sqrt{6-8i} = \pm\left(\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right)$

$= \pm\left(\sqrt{8} + i\sqrt{2}\right)$

$= \pm\left(2\sqrt{2} - i\sqrt{2}\right)$

38) $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{\lambda}$

$\therefore \vec{a} = \hat{i} - \hat{j}, \vec{b} = 2\hat{i} + \lambda\hat{j} + 2\hat{k}$

$\vec{c} = -\hat{i} - \hat{j}, \vec{d} = 5\hat{i} + 2\hat{j} + \lambda\vec{k}$

$(\vec{c} - \vec{a}) = -2\hat{i},$

and $\left(\vec{b} \times \vec{d}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & \lambda & 2 \\ 5 & 2 & \lambda \end{vmatrix}$

$= \hat{i}\left(\lambda^2 - 4\right) - \hat{j}\left(2\lambda - 10\right) + \hat{k}\left(4 - 5\lambda\right)$

Since the given lines are co-planar,

$(\vec{c} - \vec{a}) . \left(\vec{b} \times \vec{d}\right) = 0$

$\Rightarrow \left(-2\hat{i}\right) . \left[\left(\lambda^2 - 4\right)\hat{i} - \hat{j}\left(2\lambda - 10\right) + \hat{k}\left(4 - 5\lambda\right)\right] = 0$

$\Rightarrow -2\left(\lambda^2 - 4\right) = 0$

$\Rightarrow \lambda^2 = 4 \qquad [\because -2 \neq 0]$

$\Rightarrow \lambda = \pm\sqrt{4} = \pm 2$

The Cartesian equation of the plane containing the given lines is

$\begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} x+1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0 \, [\because \lambda = 2]$

$\Rightarrow (x+1)(4-4) - (y+1)(4-10) + z(4-10) = 0$

$\Rightarrow (x+1)(0) - (y+1)(-6) + z(-6) = 0$

$\Rightarrow 6(y+1) - 6z = 0$

$\Rightarrow y+1-z=0$

$\Rightarrow y-z+1=0$ which is the required equation of the plane containing the given lines

39) Let I=$\int_{-\pi}^{\pi} \frac{cos^2 x}{1+a^x} dx$ --- (1)

Using $\int_a^b$ f(x) dx =$\int_a^b$ f(a+b -x)dx we get,

I=$\int_{-\pi}^{\pi} \frac{cos^2(\pi-\pi-x)}{1+a^{\pi-\pi-x}} dx$

= $\int_{-\pi}^{\pi} \frac{cos^2(-x)}{1+a^{-x}} dx$

= $\int_{-\pi}^{\pi} a^x (\frac{cos^2 x}{a^x+1}) dx$ --- (2)

Adding (1) and (2) we get

2I = $\int_{-\pi}^{\pi} \frac{cos^2 x}{a^x+1}(a^x+1)dx$ = $\int_{-\pi}^{\pi} cos^2 x dx$

= 2 $x = \int_{-\pi}^{\pi} cos^2 x dx$ (since $\cos^2$ x is an even function)

Hence, I = $\int_0^{\pi} (\frac{1+cos 2x}{2}) dx$ = $\frac{1}{2}[x+\frac{sin 2x}{2}]_0^{\pi}$ = $\frac{1}{2}[\pi]$=$\frac{\pi}{2}$

40) Given u(x,y, z) =xy^2z^3, x= sin t, y= cos t, z = 1+ e^{2t}

$\frac{\partial u}{\partial x} = y^2 z^3; \frac{\partial u}{\partial y} = 2xyz^3$

$\frac{\partial u}{\partial z} = 3xy^2 z^2$

$\frac{\partial u}{\partial x} = cos^2 t + (1+e^{2t})^3;$

$\frac{\partial u}{\partial y} = 2 sin\ t\ cos\ t (1+e^{2t})^3;$

$\frac{\partial u}{\partial z} = 3\ sin\ t\ cos^2 t (1+e^{2t})^3$

$\frac{dx}{dt} = cos\ t; \frac{dy}{dt} = -sin\ t$

$\frac{dz}{dt} = 2e^{2t}$

By chain rule,

$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$

= $\cos^2$ t (1 + et)^3 (cos t) + 2 sin t cos t (1+ e^2t)^3 (- sin t) + 3 sin t $\cos^2$t (1+ e^2t)^2 (2e^2t)

= (1 + e^2t)2 [$\cos^3$ t(1 + e^2t) - 2 $\sin^2$ t cos t (1+ e^{2t}) + 6 sin ($\cos^2$ t e^{2t}]

$\frac{du}{dt}$ = (1 + e^{2t})^2 [$\cos^3$t (1 + e^{2t}) - sin t sin 2t (1 e^{2t})+6 sin t $\cos^2$t. e^{2t}

[∵ sin 2t = 2 sin t cos t]

P.A.PALANIAPPAN,MSc.,MPhil.,BEd

PART-IV

7 x 5 = 35

PG ASST IN MATHS

PATTUKKOTTAI

9443407917

Answer All the Questions

41) a)

Giveny = ax^2 + bx + c ...(1)

(-6,8) lies on (1)

⇒ 8=a(-6)^2+b(-6)+c

⇒ 8=36z-6b+c ....(2)

(-2,12) lies on (1)

⇒ -12=a(-2)^2+b(-2)+c

⇒ -12=4a-2b+c ......(3)

Also (3,8) lies on (1)

⇒ 8=a(3)^2+b(3)+c

⇒ 8=9a+3b+c ....(4)

Reducing the augment matrix to an equivalent row-echelon form by using elementary. row operations, we get,

$$\left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 0 & 3 & 1 & 8 \end{array}\right] \xrightarrow[R_3 \to 4R_3 - R_1]{R_2 \to 9R_2 - R_1} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -12 & 8 & -116 \\ 0 & 18 & 3 & 24 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 \div 3]{R_2 \to R_2 \div 4} \left[\begin{array}{ccc|c} 36 & -6 & 1 & -8 \\ 0 & -3 & 2 & -29 \\ 0 & 0 & 5 & -8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & -8 \\ 0 & -3 & 2 & -29 \\ 0 & 0 & 5 & -50 \end{array}\right]$$

Writing the equivalent equation from the row echelon matrix, we get 36a - 6b + c = 8 ....(1)

-3b+2c=-29 ....(2)

5c=-50

⇒ c=$\frac{-50}{5}$=-10

Substituting c = -10 in (2) we get,

-3b+2(-10)=-29

⇒ -3b+2(-10)=-29

⇒ -3b-20=-29

⇒ -3b=-9

⇒ b=$\frac{-9}{-3}$=3

Substituting b = 3 and c = -10 in (1) we get,

36a-6(3)-10=8

⇒ 36a-18-10=8

⇒ 36a-28=8

⇒ 6a=8+28=36

⇒ a=$\frac{36}{36}$=1

∴ a=1, b=3, c=-10

Hence the path of the boy is

y=1(x^2)+3(x)-10

⇒ y=x^2+3x-10

Since his friend is at P(7, 60),

60=$(7)^2$+3(7)-10

⇒ 60=49+21-10

⇒ 60=70-10=60

⇒ 60=60

Since (7, 60) satisfies his path, he can meet his friend who is at P(7, 60)

(OR)

b)

Given z=x+iy

Im $\left(\frac{2z+1}{iz+1}\right)$=0

⇒ Im$\left(\frac{2(x+iy)+1}{i(x+iy)+1}\right)$=0

⇒ Im$\left(\frac{(2x+1)+2iy}{ix+i^2y+1}\right)$

⇒ Im$\left(\frac{(2x+1)+2iy}{ix-y+1}\right)$

$\left(\frac{(2x+1)+iy}{(1-y)+ix}\right)$

Multiply and divide by the conjugate of the denominator

We get Im$\left(\frac{(2x+1)+2iy}{(1-y)+ix}\times\frac{(1-y)-ix}{(1-y)-ix}\right)$=0

⇒ Im$\left(\frac{(2x+1)+2iy\times(1-y)-ix}{(1-y)^2+x^2}\right)$

Choosing the imaginably part we get,

$\frac{(2x+1)(-x)+2y(1-y)}{(1-y)^2+x^2}$

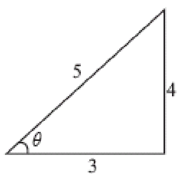
⇒ (2x+1)-x+2y(1-y)=0

⇒ -2x^2-x+2y-2y^2=0

⇒ 2x^2+2y^2+x-2y=0

Hence, locus of z is 2x^2+2y^2+x-2y=0

42) a)



We know that $sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

Thus, $cos\left(sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\frac{1}{\sqrt{1+x^2}}$ ...(1)

From the diagram, we have$cot^{-1}\left(\frac{3}{4}\right)=sin^{-1}\left(\frac{4}{5}\right)$

Hence, $sin\left\{cot^{-1}\left(\frac{3}{4}\right)\right\}=\frac{4}{5}$ ...(2)

Using (1) and (2) in the given equation, we $\frac{1}{\sqrt{1+x^2}}=\frac{4}{5}$ $\sqrt{1+x^2}=\frac{5}{4}$

Thus, x=$\pm\frac{3}{4}$

(OR)

b)

$8x^{\frac{3}{2x}}-8x^{\frac{-3}{2x}}$=63

$\Rightarrow 8\left[\left(x^{\frac{1}{2n}}\right)^3 - \left(x^{\frac{-1}{2n}}\right)^3\right] = 63$

Put $x^{\frac{1}{2n}} = y$

$\Rightarrow 8\left(y^2 - \frac{1}{y^3}\right) = 63$

$\Rightarrow y^3 - \frac{1}{y^3} = \frac{63}{8} \Rightarrow \frac{y^6-1}{y^3} = \frac{63}{8}$

$\Rightarrow 8y^6 - 8 = 63y^3$

$\Rightarrow 8y^6 - 63y^3 - 8 = 0$

$\Rightarrow 8t^2 - 63t - 8 = 0 \quad [where \quad t = y^3]$

$\Rightarrow (8t-1)(t-8) = 0$

$\Rightarrow t = \frac{1}{8}, 8$

Case (i)when $t = 8, \Rightarrow y^3 = 8 \Rightarrow y^2 = 2^3$

$\Rightarrow y = 2$

Case (ii)when $t = \frac{1}{8}, y^3 = \frac{1}{8} \Rightarrow y = \frac{1}{2}$

When $y = 2, x^{\frac{1}{2n}} = 2$

$\Rightarrow x = (2)^{2n} \quad \Rightarrow x = (2^2)^n$

$\Rightarrow x = 4^n$

When $y = \frac{1}{2}, x^{\frac{1}{2n}} = \frac{1}{2} \Rightarrow x = \left(\frac{1}{2}\right)^{2n}$

$\Rightarrow x = \left(\frac{1}{2^2}\right)^n = \frac{1}{4^n}$

Hence the roots are 4^n.

43) a)

Since the centre is located at (0,300) , midway between the two foci, which are the coast guard stations, the equation is

$\frac{(y-300)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1$ .... (1)

To determine the values of a and b , select two points known to be on the hyperbola and substitute each point in the above equation. The point(0,400) lies on the hyperbola, since it is 200 km further from Station A than from station B.

$\frac{(400-300)^2}{a^2} - \frac{O}{b^2} = 1 \frac{100^2}{a^2} = 1, a^2 = 10000.$ There is also a point (x,600) on the hyperbola such that $600^2 + x^2 = (x+200)^2$

$360000 + x^2 = x^2 + 400x + 40000$

x = 800

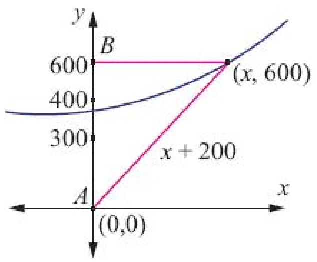
Substituting in (1), we have $\frac{(600-300)^2}{10000} - \frac{(800-0)^2}{b^2} = 1$

$9 - \frac{640000}{b^2} = 1$

$b^2 = 80000$

Thus the required equation of the hyperbola is $\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1$

The ship lies somewhere on this hyperbola. The exact location can be determined using data from a third station.



(OR)

b)

The Cartesian equations of the straight line $\vec{r} = (\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ is

$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$ = s(say)

Then any point on this line is of the form (2s + 1, 3s + 3, 2s -1) ....(1)

The Cartesian equation of the second line is $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4} = t$ (say)

Then any point on this line is of the form (t + 2, 2t + 4, 4t - 3) ....(2)

If the given lines intersect, then there must be a common point. Therefore, for some s, t ∈ R

we have (2s + 1, 3s + 3, 2s −1) = (t + 2, 2t + 4, 4t − 3)

Equating the coordinates of x, y and z we get

2s − t = 1, 3s − 2t = 1 and s − 2t = −1.

Solving the first two of the above three equations, we get s = 1 and t = 1. These values of s and t satisfy the third equation. So, the lines are intersecting.

Now, using the value of s in (1) or the value of t in (2), the point of intersection (3,6,1) of these two straight lines is obtained.

If we take $\vec{b} = (2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\vec{d} = \hat{i} + 3\hat{j} - \hat{k}$, then $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 1 & 2 & 4 \end{vmatrix} = 8\hat{i} - 6\hat{j} + \hat{k}$ is a vector perpendicular to both the given straight lines. Therefore, the required straight line passing through (3,6,1)

and perpendicular to both the given straight lines is the same as the straight line passing through (3,6,1) and parallel to

$8\hat{i}-6\hat{j}+\hat{k}$. Thus, the equation of the required straight line is

$\vec{r}=(3\hat{i}+6\hat{j}-\hat{k})+m(8\hat{i}-6\hat{j}+\hat{k})$, k ∈ R.

44) a)

The distance from the point (1,1) to any point (x, y) is d = $\sqrt{(x-1)^2+(y-1)^2}$. Instead of extremising d, for convenience we extremise D = d^2 = $(x-1)^2+(y-1)^2$ subject to the condition, $x^2+y^2=1$ Now, $\frac{dD}{dx}$ = 2(x −1)+2(y −1) x $\frac{dy}{dx}$ where the $\frac{dy}{dx}$ will be computed by differentiating x^2 y^2 + =1 with respect to x . Therefore we get, 2x + 2y $\frac{dy}{dx}$ = 0 ⇒ $\frac{dy}{dx}$ =- $\frac{x}{y}$ which gives us $\frac{dD}{dx}$ =2(x −1) +2(y −1)$\left(\frac{x}{y}\right)$

$=\frac{2[xy-y-xy+x]}{y}$

Substituting this, we get $\frac{dD}{dx}$ =2$\left[\frac{x-y}{y}\right]$=0

⇒x = y

Since (x, y) lie on the circle x^2+y^2 + =1 we get, $2x^2=1$ gives x= $\pm\frac{1}{\sqrt{2}}$.

Hence the points at which the extremum distance occur are $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right),\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

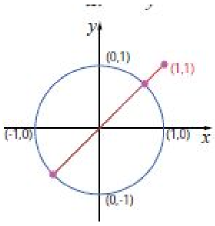
To find the extrema, we apply second derivative test. So,

$\frac{d^2D}{dx^2}=2\frac{y^2+x^2}{y^3}$

The value of $\left(\frac{d^2D}{dx^2}\right)_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)}>0;\left(\frac{d^2D}{dx^2}\right)_{\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)}<0$

This implies the nearest and farthest points are $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

Therefore, the nearest and the farthest distances are respectively $\sqrt{2}$-1 and $\sqrt{2}$+1



(OR)

b)

First we calculate

$$\frac{\partial w}{\partial x}(x,y)=\frac{\partial(xy)}{\partial x}+\frac{\partial\left(\frac{e^y}{y^2+1}\right)}{\partial x}$$

This gives $\frac{\partial w}{\partial x}$ (x,y)=y + 0 and hence $\frac{\partial^2 w}{\partial y\partial x}$ (x,y)=1 On the other hand,

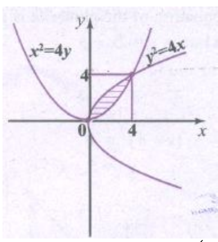
$\frac{\partial w}{\partial y}(x,y)=\frac{\partial(xy)}{\partial y}+\frac{\partial\left(\frac{e^y}{y^2+1}\right)}{\partial y}$

$=x+\frac{(y^2+1)e^y-e^y2y}{(y^2+1)}$

Hence, $\frac{\partial^2 w}{\partial x\partial y}$(x,y)=1

45) a)

Equation of the given curves are $y^2=4x$ and $x^2=4y$



$\therefore$ Required area $=\int_0^4\left(\sqrt{4x}-\frac{x^2}{4}\right)dx$

$=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx$

$=\left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4\left[\frac{4}{3}x\sqrt{x}-\frac{x^3}{12}\right]_0^4$

$=\frac{4}{3}(4)(2)-\frac{64}{12}$

$=\frac{32}{3}-\frac{32}{6}=\frac{64-32}{6}=\frac{32}{6}$

$=\frac{16}{3}$ sq.units

Yes the area can be divided into 3 equal parts and the area to the divided among his, wife daughter and son is $=\frac{16}{3}$ sq.units

(OR)

b)

$(x^2+y^2)dy = xy\,dx$

$\frac{dy}{dx}=\frac{xy}{x^2+y^2}\dots(1)$

$\therefore put = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$(1) becomes,

$v + x\frac{dv}{dx} = \frac{xvx}{x^2+v^2x^2}$

$= \frac{x^2v}{x^2(1+v^2)} = \frac{v}{1+v^2}$

$x\frac{dv}{dx} = \frac{v}{1+v^2}v = \frac{v-v-v^3}{1+v^2} = \frac{-v^3}{1+v^2}$

Separating the variables we get,

$\frac{1+v^2}{v^3}dv = \frac{-dx}{x}$

$\Rightarrow \frac{1}{v^3} + \frac{v^2}{v^3}dv = \frac{-dx}{x}$

$\Rightarrow \int v^{-3}dv + \int\frac{dv}{v} = -\int\frac{dx}{x}$

$\Rightarrow \frac{v^{-2}}{-2} + log \quad v = -logx + logc$

$\Rightarrow -\frac{1}{2v^2} + log \quad v = -log \quad x + log \quad c$

$\Rightarrow \frac{1}{2v^2} - log \quad v = logx - logc$

$\Rightarrow \frac{1}{2v^2} = logv + logx - logc$

$\Rightarrow \frac{1}{2v^2} = logv + logx - logc$

$\Rightarrow \frac{1}{2v^2} = log\left(\frac{vx}{c}\right)$

$\Rightarrow \frac{x^2}{2y^2} = log\left(\frac{y}{c}\right) \Rightarrow e^{\frac{x^2}{e^{2y^2}}} = \frac{y}{c}$

$\Rightarrow y = ce^{\frac{x^2}{2y^2}} \qquad \ldots(2)$

Given y(1)=1

$1 = ce^{\frac{1}{2}} \Rightarrow 1 = c\sqrt{e}$

$\Rightarrow \quad c = \frac{1}{\sqrt{e}}$

$\therefore$(2) becomes,

$y = \frac{1}{\sqrt{e}}e^{\frac{x^2}{2y^2}}$

$Also \quad y(x_0) = e \Rightarrow e = \frac{1}{\sqrt{e}}e^{\frac{x_0^2}{2e^2}}$

$\Rightarrow e\sqrt{e} = e^{\frac{x_0^2}{2e^2}}$

$\Rightarrow \frac{x_0^2}{2e^2} = log \quad e\sqrt{e} = loge^{\frac{3}{2}}$

$\Rightarrow \frac{x_0^2}{2e^2} = \frac{3}{2}log_e^e = \frac{3}{2}(1)$

$[\because log_e^e = 1]$

$\Rightarrow x_0^2 = \frac{3}{2}(2e^2) = 3e^2$

$\Rightarrow x_0 = \pm\sqrt{3e^2} = \pm\sqrt{3}.e$

$\therefore x_0 = \pm\sqrt{3}.e$

46) a)

Since the given function is a probability mass function, the total probability is one. That is $\sum_x f(x) = 1$

From the given data k + 2k + 6k + 5k + 6k +10k+1

$30k = 1 \Rightarrow k = \frac{1}{30}$

Therefore the probability mass function is

x	1	2	3	4	5	6
f(x)	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$	$\frac{10}{30}$

(i) P(2 < X < 6) = f(3)+ f(4)+ f(5) = $\frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$

(ii) P(2≤X≤5)=f(2)+f(3)+f(4)= $\frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{13}{30}$

(iii) P(2≤4)=f(1)+f(2)+f(3)+f(4)= $\frac{1}{30} + \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{14}{30}$

(iv) P(3>X)=f(4)+f(5)+f(6)= $\frac{5}{30} + \frac{6}{30} + \frac{10}{30} = \frac{21}{30}$

(OR)

b) a)

Given $a * b = \frac{a+b}{2} \forall \in Q$

i) Closure property:

Let a,b ∈ Q

∴ a*b=$\frac{a+b}{2}$∈Q

[∵ addition and division are closed on Q]

* is closed on Q.

(ii) Commutative property:

Let a, b ∈ Q

Then a+b $= \frac{a+b}{2} = \frac{b+a}{2} = b * a$

∴ a*b =b*a ∀a,b∈Q

∴ * is commutative on Q.

Associative property

Let a, b, c ∈ Q

a*(b*c)=(a*b)*c

Let a=2,b=3, c-5

∴ a*(b*c)=2*(3*-5)

$= * \left(\frac{3-5}{2}\right)$

$= 2 * (-1) = \frac{2+(-1)}{2}$

$= \frac{1}{2} \quad \ldots(1)$

Now (a*b)*c=(2*3)*(-5)

$= \left(\frac{2+3}{2}\right) * (-5)$

$= \frac{5}{2} * -5 = \frac{\frac{5}{2}+(-5)}{2}$

$= \frac{5-10}{4} = \frac{-5}{4} \ldots(2)$

From (1)&(2), a*(b*c)≠(a*b)*c

∴ * is not associative on Q.

b)

Given $a * b = \frac{a+b}{2}$, where a.b ∈Q Let a,b ∈Q

An element e has to found out such that

a*e=e*a=a

Let a=5, Then 5*e=5

$\Rightarrow \frac{5+e}{2}$ =5⇒5+e=10

Let a=$\frac{2}{3}$. Then $\frac{2}{3}$*e=$\frac{2}{3}$

$\Rightarrow \frac{\frac{2}{3}+e}{2} = \frac{2}{3}$

$\Rightarrow \frac{2}{3} + e = \frac{4}{3}$

$\Rightarrow e = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

Since identity differs for every element, the identity does not exist for Q.

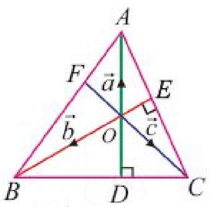
∴ * has no identity on Q.

∴ * has no inverse on Q.

Hence, identity and inverse does not exist for Q under the given binary operation *.

47) a)

Consider a triangle ABC in which the two altitudes AD and BE intersect at O. Let CO be produced to meet AB at F. We take O as the origin and let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$



Since $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BC}$, we have $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$, and hence we get $\overrightarrow{OA} \cdot \overrightarrow{BC}$ = 0. Thnat is,

$\vec{a}.(\vec{c} - \vec{b}) = 0$, which means

$\vec{a}.\hat{c} - \hat{a}.\hat{b} = 0$ ....(1)

Similarly, since $\overrightarrow{BE}$ is perpendicular to $\overrightarrow{CA}$, we have $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{CA}$, and hence we get $\overrightarrow{OB}.\overrightarrow{CA}$ = 0. That is,

$\vec{a}.\hat{c} - \hat{b}.\hat{c} = 0$ ....(2)

Adding equations (1) and (2), gives $\vec{a}.\hat{c} - \hat{b}.\hat{c} = 0$. That is, $\hat{c}(\hat{a} - \hat{b}) = 0$

That is $\overrightarrow{OC} \cdot \overrightarrow{BA}$ = 0. Therefore, $\overrightarrow{BA}$ is perpendicular to $\overrightarrow{OC}$. Which implies that $\overrightarrow{CF}$ is perpendicular to $\overrightarrow{AB}$. Hence, the perpendicular drawn from C to the side AB passes through O. Therefore, the altitudes are concurrent

(OR)

b)

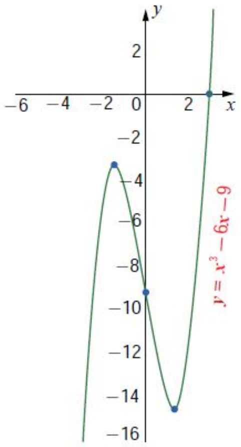


Fig.7.36

Factorising the given function, we have

y = f (x) = (x − 3)(x^2 + 3x + 3).

(1) The domain and the range of the given function f (x) are the entire real line.

(2) Putting y = 0, we get the x = 3. The other two roots are imaginary. Therefore, the x -intercept is (3,0) . Putting x = 0, we get y = −9. Therefore, the y-intercept is (0,−9)

(3) f'(x) = 3(x^2 -2) and hence the critical points of the curve occur at x =$\pm\sqrt{2}$

(4) f''(x) = 6x . Therefore at x =$\sqrt{2}$ the curve has a local minimum because f''($\sqrt{2}$)=6$\sqrt{2}$>0. Then local minimum is f($\sqrt{2}$)=-4 $\sqrt{2}$-9. Similarly x=-$\sqrt{2}$ the curve has a local maximum because f''(-$\sqrt{2}$)=-6$\sqrt{2}$<0. The local maximum is f (-$\sqrt{2}$)=4$\sqrt{2}$-9.

(5) Since f ''(x) = 6x > 0, ∀x > 0 the function is concave upward in the positive real line. As f ''(x) = 6x < 0,∀x < 0 the function is concave downward in the negative real line.

(6) Since f ''(x) = 0 at x = 0 and f ″(x) changes its sign when passing through x = 0 . Therefore the point of inflection is (0, f (0)) = (0,9) .

(7) The curve has no asymptotes.

The rough sketch of the curve is shown on the right side.