DIRECTORATE OF GOVERNMENT EXAMINATIONS

HIGHER SECONDARY (FIRST YEAR) EXAMINATION - MARCH / APRIL 2018

BUSINESS MATHEMATICS - ENGLISH MEDIUM – ANSWER KEY

General Instructions

1. For objective type questions, award 1 mark for "writing the correct option's code and the corresponding option's answer".
2. One mark shall be awarded either for the correct option code or for the correct corresponding answer in multiple choice questions. **(This year only)**
3. Award "0 mark" for one who wrote both "option's code" and "option's answer" with one of them is not correct.
4. Marks should be awarded for suitable alternative method also.
5. Mark(s) should not be reduced for the correct answer / stage, if it is written without formula / properties also.
6. Award full mark directly, if the solution is arrived with nil mistakes without giving weightage for the stages.
7. The stage mark is essential, only if the part of the solution is incorrect.
8. Award marks, if the answer is in decimal value and also approximately equal to the key answer.
9. For a particular stage in which the stage mark is greater than ½ and one who begins with correct step but reaches with incorrect solution, for such cases, the suitable credits should be given by breaking the stage marks.

388-62_(2,000 Cps)-1

SECTION – I					$20 \times 1 = 20$
Q.NO	OPTION	ANSWER	Q.NO	OPTION	ANSWER
1	(d)	I	11	(b)	$\{x / x \neq -3\}$
2	(d)	0	12	(a)	e^x
3	(d)	$\frac{5}{24}$	13	(b)	0
4	(a)	20	14	(d)	8
5	(d)	$\frac{2c}{b}$	15	(d)	$\frac{\left(\frac{2}{3}\right)^x}{\log_e\left(\frac{2}{3}\right)} + c$
6	(a)	₹ 1000	16	(c)	$\frac{-1}{3}$
7	(d)	*negative*	17	(d)	₹ 11,000
8	(a)	± 10	18	(c)	25 %
9	(a)	1	19	(d)	0
10	(c)	$\sin\theta$	20	(a)	0

SECTION – II		7 × 2 = 14
Q. NO.	KEY STEPS - ANSWER	STEPS MARKS
21	y' (or) $\frac{dy}{dx} = x + 2x \log x$	1
	y'' (or) $\frac{d^2y}{dx^2} = 1 + 2(1 + \log x)$	½
	$= 3 + 2 \log x$	½
22	$n = 9, \quad d_1 = 4, \quad d_2 = 2$	½
	Number of arrangements $= \frac{n!}{d_1!\, d_2!}$	½
	$= \frac{9!}{4! \cdot 2!}$	1
23	$a^x = b^y = c^z = k$ (or) $a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$	½
	a, b, c are in G.P. $\Rightarrow b^2 = ac$	½
	$\left(k^{\frac{1}{y}}\right)^2 = \left(k^{\frac{1}{x}}\right)\left(k^{\frac{1}{z}}\right)$	½
	$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$ (or) $y = \frac{2xz}{x+z}$	½
24	$g = -2, \quad f = -3, \quad c = -9$	
	(i) Center: $C = (-g, -f) = (2, 3)$	½+½
	(ii) Radius: $r = \sqrt{g^2 + f^2 - c} = \sqrt{22}$ units	½+½
25	$\tan 105° = \tan(45 + 60)°$	½
	$= \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}$	1
	$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$ (or) $-(2 + \sqrt{3})$	½

26*	$LHS = \begin{vmatrix} a+b+c & a & 1 \\ a+b+c & b & 1 \\ a+b+c & c & 1 \end{vmatrix}$ $= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ $= 0 = RHS$	1 ½ ½
27	$f(-x) = (-x)^7 + 6(-x)^5 - 5(-x)^3 - 2(-x)$ $= -x^7 - 6x^5 + 5x^3 + 2x$ $= -(x^7 + 6x^5 - 5x^3 - 2x)$ $= -f(x) \quad \Rightarrow$ $f(x)$ is odd function.	½ ½ ½ ½
28	$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} sin^{-1}\left(\frac{x}{a}\right) + c$ $= \frac{x}{2}\sqrt{81 - x^2} + \frac{81}{2} sin^{-1}\left(\frac{x}{9}\right) + c$ (or) $= \frac{1}{2}\left[x\sqrt{81 - x^2} + 81\, sin^{-1}\left(\frac{x}{9}\right)\right] + c$	1 1
29	Face value (Rs.) / Brokerage (Rs.): 70×100 — 105 100 — ? Brokerage % $= 105 \times \frac{100}{70 \times 100} = \frac{3}{2}\%$ (or) $1\frac{1}{2}\%$	1 ½+½
30	$P(A) = 0.5, P(A \cup B) = 0.8$ *and* $A\&B$ are mutually exclusive. $P(A \cup B) = P(A) + P(B)$ $0.8 = 0.5 + P(B)$ $P(B) = 0.3$	1 ½ ½

SECTION – III		7 × 3 = 21
Q. NO.	**KEY STEPS - ANSWER**	**STEPS MARKS**
31	$A - I = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$	1
	$A - 4I = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$	1
	$(A - I)(A - 4I) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (OR) 0	1
32	${}_{10}P_r = 5040$	
	$= 10 \times 9 \times 8 \times 7$	2
	$\Rightarrow r = 4$	1
33	$a, b. c$ are in $H.P. \Rightarrow b = \frac{2ac}{a+c}$	½
	$\frac{b}{a} = \frac{2c}{a+c} \Rightarrow \frac{b+a}{b-a} = \frac{3c+a}{c-a}$	½
	$\frac{b}{c} = \frac{2a}{a+c} \Rightarrow \frac{b+a}{b-a} = \frac{3a+c}{a-c}$	½
	$\frac{b+a}{b-a} + \frac{b+a}{b-a} = \frac{3c+a}{c-a} - \frac{3a+c}{c-a}$	½
	$= \frac{2(c-a)}{c-a} = 2$	1
34	$A = \pi r^2 \Rightarrow \pi r^2 = 16\pi$	½
	$\Rightarrow r = 4$ units (or) $r^2 = 16$ units	½
	Equation : $(x - h)^2 + (y - k)^2 = r^2$	1
	$(x - 7)^2 + (y + 3)^2 = 4^2$	½
	$x^2 + y^2 - 14x + 6y + 42 = 0$	½

35	$LHS = \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\right]^2 + \left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]^2$	1
	$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\left[\cos^2\left(\frac{\alpha-\beta}{2}\right) + \sin^2\left(\frac{\alpha-\beta}{2}\right)\right]$	1
	$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right) = RHS$	1
36	$(fog)(x) = f(g(x))$	½
	$= f\left(\frac{x-1}{2}\right)$	½
	$= 2\left(\frac{x-1}{2}\right) + 1 = x$ ....(i)	½
	$(gof)(x) = g(f(x))$	
	$= g(2x+1)$	½
	$= \frac{(2x+1)-1}{2} = x$ ....(ii)	½
	From (i) and (ii), we get	
	$(fog) = (gof)$	½
37	$y = e^{4x}\sin 4x$	
	$\frac{dy}{dx} = e^{4x}(4\cos 4x) + (4e^{4x})\sin 4x$	1+1
	$= 4e^{4x}(\cos 4x + \sin 4x)$	1
38	$I = \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx$	
	$t = \sin^{-1}x \Rightarrow dt = \frac{1}{\sqrt{1-x^2}}dx$	1
	$\therefore I = \int e^t\,dt$	1
	$= e^t + c$	½
	$= e^{\sin^{-1}x} + c$	½

39	Investment $= 120 \times 95$	½
	Case (i)	
	<table><tr><th>Investment (₹.)</th><th>Income (₹.)</th></tr><tr><td>120</td><td>6</td></tr><tr><td>120×95</td><td>?</td></tr></table>	½
	Income 1 $= \frac{6}{120} \times 120 \times 95 =$ ₹ 570	½
	Case(ii)	
	<table><tr><th>Investement (₹.)</th><th>Income (₹.)</th></tr><tr><td>95</td><td>5</td></tr><tr><td>120×95</td><td>?</td></tr></table>	½
	Income 2$= \frac{5}{95} \times 120 \times 95 =$ ₹ 600	½
	Result: 5 % stock at Rs. 95 is better.	½
40*	$S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$	½
	(i) $A =$ exactly one tail	
	$= \{HT, TH\} \Rightarrow n(A) = 2$	½
	$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{2}{4}$ (or) $\frac{1}{2}$	½
	(ii) $B =$ atleast one tail	
	$= \{HT, TH, TT\} \Rightarrow n(B) = 3$	½
	$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$	½
	(iii) $C =$ atmost one tail	
	$= \{HH, HT, TH\} \Rightarrow n(C) = 3$	
	$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{3}{4}$	½

SECTION – I V		$7 \times 5 = 35$
Q. NO.	**KEY STEPS - ANSWER**	**STEPS MARKS**
41(a)		
	(i) Initial stock matrix: $P = \begin{pmatrix} 43 & 62 & 36 \\ 24 & 18 & 60 \end{pmatrix}$	½
	(ii) The order matrix: $Q = \begin{pmatrix} 30 & 30 & 20 \\ 10 & 6 & 4 \end{pmatrix}$	½
	(iii) The supply matrix: $R = \frac{1}{2}Q = \begin{pmatrix} 15 & 15 & 10 \\ 5 & 3 & 2 \end{pmatrix}$	1
	(iv) Final stock matrix: $S = P + R = \begin{pmatrix} 58 & 77 & 46 \\ 29 & 21 & 62 \end{pmatrix}$	1
	(v) Cost matrix: $T = \begin{pmatrix} 42 \\ 38 \\ 36 \end{pmatrix}$	1
	(vi) Total cost of the stock matrix: $U = S\,T$ $= \begin{pmatrix} 7018 \\ 4248 \end{pmatrix}$	1
41(b)	$m \log x + n \log y = (m+n) \log(x+y)$	1
	$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left(1 + \frac{dy}{dx}\right)$	1
	$\frac{dy}{dx}\left\{\frac{(m+n)}{(x+y)} - \frac{n}{y}\right\} = \left\{\frac{m}{x} - \frac{(m+n)}{(x+y)}\right\}$	1
	$\frac{dy}{dx} = \frac{\left\{\frac{m}{x} - \frac{(m+n)}{(x+y)}\right\}}{\left\{\frac{(m+n)}{(x+y)} - \frac{n}{y}\right\}}$	1
	$= \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{y}\right)} = \frac{y}{x}$	1

42(a)	$n = 10\,, x = \sqrt{x}\,, a = -\dfrac{2}{x^2}$	½
	$T_{r+1} = {}_nC_r\, x^{n-r} a^r$	1
	$= {}_{10}C_r \left(\sqrt{x}\right)^{10-r}\left(-\dfrac{2}{x^2}\right)^r$	½
	$= (-2)^r\, {}_{10}C_r\, x^{\frac{10-5r}{2}}$	1
	Equate, $\dfrac{10-5r}{2} = 0 \Rightarrow r = 2$	1
	Independent term of x = $(-2)^2\, {}_{10}C_2 = 180$	1
42(b)		

Units (x)	Cost (y) (₹)
10000	16000
20000	29000

	Equation is	
	$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$ (or) $y = ax + b$	1
	$\dfrac{y-16000}{29000-16000} = \dfrac{x-10000}{20000-10000}$ (or) $10000a + b = 16000$	1
	$\dfrac{y-16000}{13000} = \dfrac{x-10000}{10000}$ (or) $20000a + b = 29000$	1
	$y - 16000 = 1.3(x - 10000)$ (or) $a = 1.3$ & $b = 3000$	1
	$y = 1.3x + 3000$	1

43(a)	$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{4}$	1
	$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$	2
	$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{2 \times 3} - \frac{1}{3 \times 4} - \frac{1}{2 \times 4} + \frac{1}{2 \times 3 \times 4}$	1
	$= \frac{18}{24} (or) \frac{3}{4} (or) 0.75$	1

43(b)	Investment in 8 % stock = ₹ x and Investment in $7\frac{1}{2}$% stock = ₹ $(34000 - x)$	½
	Case (i)	
	Investment (₹) / Income (₹): 80 → 8; x → ?	½
	Income 1 $= 8 \times \frac{x}{80} = \frac{x}{10}$	½
	Case (ii)	
	Investment (₹) / Income (₹): 90 → $\frac{15}{2}$; $(34000 - x)$ → ?	½
	Income 2 $= \frac{15}{2} \times \left(\frac{34000-x}{90}\right) = \frac{34000-x}{12}$	½
	By data, $\frac{x}{10} + \frac{34000-x}{12} = 3000$	1
	$x = 10000$	½
	Investment in 8 % stock = ₹ 10000 and Investment in $7\frac{1}{2}$% stock = ₹ 24000	1

Case (i)

Investment (₹)	Income (₹)
80	8
x	?

Case (ii)

Investment (₹)	Income (₹)
90	$\frac{15}{2}$
$(34000 - x)$	?

44(a)	$y = (\sin x)^{\sin x}$	1
	$\log y = \sin x \log(\sin x)$	1
	$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{\cos x}{\sin x} + \cos x \log(\sin x)$	1
	$\frac{dy}{dx} = y[\cos x + \cos x \log(\sin x)]$	1
	$\frac{dy}{dx} = (\sin x)^{\sin x}[\cos x + \cos x \log(\sin x)]$ (or)	1
	$= (\sin x)^{\sin x}\cos x\,[1 + \log(\sin x)]$	
44(b)	Formula	1
	$LHS = \tan^{-1}\left\{\frac{\left(\frac{m}{n}\right) - \left(\frac{m-n}{m+n}\right)}{1 + \left(\frac{m}{n}\right)\left(\frac{m-n}{m+n}\right)}\right\}$	1
	$= \tan^{-1}\left\{\frac{m(m+n) - n(m-n)}{n(m+n) - m(m-n)}\right\}$	1
	$= \tan^{-1}\left\{\frac{m^2+n^2}{m^2+n^2}\right\} \quad = \tan^{-1}\{1\}$	1
	$= \frac{\pi}{4} = RHS$	1

45(a)	$\int_a^b f(x)\,dx = \lim_{\substack{n\to\infty \\ h\to 0}} h[f(a+h)+f(a+2h)+f(a+3h)$ $+ \ldots + f(a+nh)]$ where $nh = b-a$	2
	$a=0, b=1, nh=1, f(x)=e^x$	
	$\int_0^1 e^x\,dx = \lim_{\substack{n\to\infty \\ h\to 0}} h[e^h+e^{2h}+e^{3h}+\ldots+e^{nh}]$	1
	$= \lim_{\substack{n\to\infty \\ h\to 0}} he^h \left\{\frac{(e^h)^n-1}{e^h-1}\right\}$	1
	$=(e-1)$	1
45(b)	$\frac{1}{9}, x_1, x_2, x_3, x_4, x_5, \frac{1}{10}$ are in H.P.	½
	$9, \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \frac{1}{x_4}, \frac{1}{x_5}, 10$ are in A.P.	½
	$a=9, a+6d=10 \Rightarrow d=\frac{1}{6}$	1
	$\frac{1}{x_1}=a+d=\frac{55}{6}$	½
	$\frac{1}{x_2}=a+2d=\frac{56}{6}$	½
	$\frac{1}{x_3}=a+3d=\frac{57}{6}$	½
	$\frac{1}{x_4}=a+4d=\frac{58}{6}$	½
	$\frac{1}{x_5}=a+5d=\frac{59}{6}$	½
	H.M.'s are $\frac{6}{55}, \frac{6}{56}, \frac{6}{57}, \frac{6}{58}, \frac{6}{59}$	½

46(a)	$u = \log x \quad dv = x\,dx$	½
	$du = \frac{1}{x}\,dx \quad v = \frac{x^2}{2}$	½
	$\int_a^b u\,dv = [u\,v]_a^b - \int_a^b v\,du$	1
	$\int_1^e x \log x\,dx = \left[\frac{x^2}{2}\log x\right]_1^e - \int_1^e \frac{x}{2}\,dx$	1
	$= \left[\frac{x^2}{2}\log x\right]_1^e - \left[\frac{x^2}{4}\right]_1^e$	1
	$= \frac{e^2}{2} - \frac{e^2-1}{4} = \frac{e^2+1}{4}$	½+½
46(b)	$P(A_1) = \frac{20}{100}$; $P(B/A_1) = \frac{90}{100}$	½
	$P(A_2) = \frac{30}{100}$; $P(B/A_2) = \frac{80}{100}$	½
	$P(A_3) = \frac{50}{100}$; $P(B/A_3) = \frac{95}{100}$	½

	(i) $P(B) = P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)$ $= \frac{1800}{10000} + \frac{2400}{10000} + \frac{4750}{10000}$ $= \frac{8950}{10000}$ (or) $\frac{895}{1000}$ (or) $\frac{179}{200}$ (or) 0.895	1 ½
	(ii) $P(A_2/B) = \frac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$ $= \frac{2400}{8950}$ (or) $\frac{48}{179}$ (or) 0.268	1 1
47(a)	$\frac{x}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$ $x = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$ $\Rightarrow A = \frac{1}{4}, B = -\frac{1}{4}, C = \frac{1}{2}$ $\frac{x}{(x-1)(x+1)^2} = \frac{\left(\frac{1}{4}\right)}{(x-1)} + \frac{\left(-\frac{1}{4}\right)}{(x+1)} + \frac{\left(\frac{1}{2}\right)}{(x+1)^2}$ (or) $\frac{x}{(x-1)(x+1)^2} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} + \frac{1}{2(x+1)^2}$	½+½+½ 1 ½+½+½ 1

47(b)	<table><tr><th>Years (x)</th><th>Salary (y) (Rs.)</th></tr><tr><td>2002</td><td>7500</td></tr><tr><td>2004</td><td>7750</td></tr><tr><td>2005</td><td>?</td></tr></table>	1
	Equation is	
	$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$	1
	$\frac{y-7500}{250}=\frac{x-2002}{2}$	1
	$y-7500=125(x-2002)$	
	(or)	1
	$y=125(x-2002)+7500$	
	When $x=2005$, $y=125(3)+7500=₹7\,875$	1
Aliter	$y=ax+b$	1
	$2002a+b=7500$	1
	$2004a+b=7750$	1
	$a=125$ & $b=-242750$	½+½
	When $x=2005$, $y=₹7\,875$	1