

HALF YEARLY COMMON EXAMINATION - 2019

11 - Std

MATHS

Time : 3.00 hrs

Marks : 90

PART - I

20 × 1 = 20

Note : i) Answer all the questions. ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$; then $n(A \cap B)$ is :
 a) ∞ b) 0 c) 1 d) 2
2. The number of constant functions from a set containing 'm' elements to a set containing 'n' elements is :
 a) mn b) m c) n d) m + n
3. The range of the function $f(x) = |[x] - x|, x \in R$ is
 a) [0, 1] b) [0, ∞) c) [0, 1) d) (0, 1)
4. If a and b are the roots of the equation $x^2 - kx + 16 = 0$ and satisfy $a^2 + b^2 = 32$ then the value of k is :
 a) 10 b) - 8 c) -8, 8 d) 8
5. If $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$ then the value of k is :
 a) 1 b) 2 c) 3 d) 4
6. $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \dots + \cos 179^\circ =$
 a) 0 b) 1 c) -1 d) 89
7. If $\sin \alpha + \cos \alpha = b$, then $\sin 2\alpha$ is equal to :
 a) $b^2 - 1$ if $b \leq \sqrt{2}$ b) $b^2 - 1$ if $b > \sqrt{2}$
 c) $b^2 - 1$ if $b \geq 1$ d) $b^2 - 1$ if $b \geq \sqrt{2}$
8. The coefficient of x^5 in the series e^{2x} is :
 a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $-\frac{4}{15}$ d) $\frac{4}{15}$
9. The length of the perpendicular from the origin to the line $\frac{x}{3} - \frac{y}{4} = 1$ is :
 a) $\frac{11}{5}$ b) $\frac{5}{12}$ c) $\frac{12}{5}$ d) $-\frac{5}{12}$
10. If the lines represented by the equation $9x^2 - 24xy + 16y^2 = 0$ makes angles θ_1 and θ_2 with x - axis, then $\tan \theta_1 \tan \theta_2 =$
 a) $-\frac{9}{16}$ b) $-\frac{16}{9}$ c) $\frac{9}{16}$ d) $\frac{16}{9}$

11. If A is a square matrix, then which of the following is not symmetric :
 a) $A + A^T$ b) AA^T c) $A^T A$ d) $A - A^T$
12. If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$, $|\vec{b}| = 5$ and the angle between \vec{a} and \vec{b} is $\frac{\pi}{6}$, then the area of the triangle formed by these two vectors as two sides is :
 a) $\frac{7}{4}$ b) $\frac{15}{4}$ c) $\frac{3}{4}$ d) $\frac{17}{4}$
13. The vectors $\hat{i} - \hat{j}$, $\hat{j} - \hat{k}$, $\hat{k} - \hat{i}$ are :
 a) parallel to each other b) unit vectors
 c) mutually perpendicular vectors d) coplanar vectors
14. $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$
 a) 0 b) 1 c) $\sqrt{2}$ d) does not exist
15. $\lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n}{n^2} \right) =$
 a) $\frac{1}{2}$ b) 0 c) 1 d) ∞
16. $\frac{d}{dx} (3^x + x^3) =$
 a) $3x^3 + (\log 3)3^x$ b) $x^3 + \log 3$
 c) $(\log 3)(3^x) + 3x^2$ d) $3^{x-1} x + (\log 3) 3^x$
17. The number of points in \mathbb{R} in which the function $f(x) = |x - 1| + |x - 3| + \sin x$ is not differentiable is :
 a) 3 b) 2 c) 1 d) 4
18. The differentiate of $y = |x - 1|$ at $x = 1$ is :
 a) 1 b) -1 c) 0 d) does not exist
19. If $f(x) = x \tan^{-1} x$ then find $f'(1)$:
 a) $\frac{1}{2} - \frac{\pi}{4}$ b) $\frac{1}{2} + \frac{\pi}{4}$ c) $\frac{\pi}{4}$ d) $\frac{1}{2}$
20. Find $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$
 a) 2 b) $\frac{1}{2}$ c) 1 d) 0

Note : Answer any seven questions. Question No. 30 is compulsory.

21. If $f: [-2, 2] \rightarrow B$ is given by $f(x) = 2x^3$, then find B so that f is onto.

2. Write the value of f at -3 and 0 if: $f(x) = \begin{cases} x^2 + x - 5; & x \in (-\infty, 0) \\ x^2 + 3x - 2; & x \in (3, \infty) \\ x^2; & x \in (0, 2) \\ x^2 - 3; & \text{otherwise} \end{cases}$

23. Find the expansion of $\frac{1}{(2+x)^4}$ where $|x| < 2$ upto the fourth term.

24. Find the combined equation of the straight lines whose separate equation are $x - 2y = 0$ and $2x + y = 0$. What can you say about the two lines?

25. Find b if the matrix $\begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$ is singular.

26. Find the sum $A - B + C$ if $A = \begin{bmatrix} \sec^2 \theta & \sin^2 \theta \\ \cot^2 \theta & 0 \end{bmatrix}$, $B = \begin{bmatrix} \tan^2 \theta & -\cos^2 \theta \\ \operatorname{cosec}^2 \theta & -1 \end{bmatrix}$,

$$C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

27. Find the derivative of $\sin^{-1}(3x - 4x^3)$.

28. Show that $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x-2} \right)^x = e^4$.

29. Show that $f(x) = \sqrt{1-x^2}$ is continuous on $[-1, 1]$.

30. Find $f'(2)$ and $f'(4)$ if $f(x) = |x-3|$.

PART - III

Note : Answer any seven questions. Question No. 40 is compulsory.

31. If $f: R \rightarrow R$ is defined as $f(x) = 2x^2 - 1$, find the pre-images of 17, 4 and -2.

32. Find the domain of $\frac{1}{1-2\sin x}$.

33. If $nPr = 720$ and $nCr = 120$ find n, r.

34. If $a_1, a_2, a_3, \dots, a_n$ is an arithmetic progression, prove that every term a_k ($k > 1$) is the arithmetic mean of its immediate predecessor a_{k-1} and immediate successor a_{k+1} .

35. The length of the perpendicular draw from the origin to the line is 12 and makes an angle 150° with the positive direction of the x-axis. Find the equation of the line.

36. Find the vectors magnitude $10\sqrt{3}$ that are perpendicular to the plane which contains $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 4\hat{k}$.
37. Does the limit for the function $\frac{\sin|x|}{x}$ exists when $x \rightarrow 0$. State reasons for your answer.
38. If $y = \sin^4 x + \cos^4 x$ find $\frac{dy}{dx}$.
39. In the set Z of integers, define mRn if m-n is a multiple of 12. Prove that R is an equivalence relation.
40. Solve : $\sqrt{3} \sin x + \cos x = 2$.

PART - IV

7 X 5 = 35

Note : Answer the following questions.

41. a) Let $f, g: R \rightarrow R$ be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$ find $f \circ g$. (OR)

b) Solve : $\frac{x^2 - 4}{x^2 - x - 15} \leq 0$.

42. a) If $A + B + C = \frac{\pi}{2}$ prove that $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cos B \cos C$

(OR) b) In a ΔABC , prove that $\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \cos \frac{A}{2}$.

43. a) By principle of mathematical induction, prove that, for $n \geq 1$

$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ (OR)

b) Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them.

44. a) Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$. (OR)

b) Prove using vectors that the medians of a triangle are concurrent.

45. a) Show that the function $\begin{cases} \frac{x^3-1}{x-1} & ; x \neq 1 \\ 3 & ; x = 1 \end{cases}$ is continuous on $(-\infty, \infty)$ (OR)

b) If $y = e^{\tan^{-1} x}$ show that $(1+x^2)y'' + (2x-1)y' = 0$.

46. a) Prove that $\sqrt{x^2+25} - \sqrt{x^2+9} = \frac{8}{x}$ nearly when x is large. (OR)

b) Evaluate : $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - x}{x}$.

47. a) Find the cosine and sine angle between the vectors $\hat{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{b} = 4\hat{i} - 2\hat{j} + 2\hat{k}$. (OR)

b) Differentiate $\sin(ax^2 + bx + c)$ with respect to $\cos(lx^2 + mx + n)$.