HALF YEARLY EXAM - 2019

16|12|2019

ஜீவா கல்வி மையம், ஈரோடு வழங்கும்.

11th MATHEMATICS

தீபக்குமார் சிவக்குமார்.

PART - I

$\boxed{20 \times 1m = 20}$

1. c) 1
2. c) n
3. c) [0,1)
4. c) −8, 8
5. c) 3
6. a) 0
7. a) b^2-1, if $b \le \sqrt{2}$
8. c) $-\frac{4}{15}$
9. c) $12/5$
10. c) $9/16$ (Creative)
11. d) $A - A^T$
12. b) $15/4$ $\because \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 0$
13. d) coplanar vectors
14. d) does not exist
15. a) $\frac{1}{2}$
16. Question (Correct Answer) $(3+3x^2)$ $\boxed{\text{Free mark}}$ Creative
17. b) 2 $\frac{d}{dx}(3^x + x^3) = \log 3\,(3)^x + 3x^2$
18. d) does not exist (Creative)
19. b) $\frac{1}{2} + \frac{\pi}{4}$
20. 2 (Creative) $\because \lim_{x\to 0} \frac{\cos 2x(0)}{1} = \frac{2\cos 0}{1} = 2$

PART - II

$\boxed{7 \times 2m = 14m}$

21. minimum value is $f(-2) = -16$ and maximum value is $f(2) = 16$.
$\therefore B$ is $[-16, 16]$

22. $f(-3) = 1$
$f(0) = -3$

23. (Creative)
$$\frac{1}{(2+x)^4} = (2+x)^{-4} = 2^{-4}\left(1+\frac{x}{2}\right)^{-4} \quad \because |x|<2 \Rightarrow \left|\frac{x}{2}\right|<1$$
$$= \frac{1}{16}\left[1-(4)\left(\frac{x}{2}\right) + \frac{(4)(4+1)}{1\times 2}\left(\frac{x}{2}\right)^2 - \frac{4(4+1)(4+2)}{1\times2\times3}\left(\frac{x}{2}\right)^3 + \dots\right]$$
$$= \frac{1}{16} - \frac{x}{8} + \frac{5}{32}x^2 - \frac{5}{32}x^3 + \dots \infty$$

24. Combined equation is $(x-2y)(2x+y) = 0 \Rightarrow 2x^2 - 3xy - 2y^2 = 0$.
Given lines represents the pair of straight lines

25. $|A| = 0 \Rightarrow b = 49/8$.

26. (Creative) $A - B + C = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$
W.K.T. $\sec^2 x - \tan^2 x = 1$; $\cot^2\theta - \csc^2\theta = -1$
$\sin^2\theta + \cos^2\theta = 1$.

(27) Let $y = \sin^{-1}(3x - 4x^3)$

Take $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$.

$\therefore y = \sin^{-1}(\sin 3\theta) = 3\theta = 3\sin^{-1}x = \dfrac{3}{\sqrt{1-x^2}}$

(28) Let $y = x - 2$, as $x \to \infty$, $y \to \infty$

$$LHS = \lim_{x \to \infty}\left(\frac{x+2}{x-2}\right)^x = \lim_{x \to \infty}\left(1 + \frac{4}{x-2}\right)^{(x-2)+2}$$

$$= \lim_{y \to \infty}\left(1 + \frac{4}{y}\right)^{y+2} = \lim_{y \to \infty}\left(1+\frac{4}{y}\right)^y \cdot \lim_{y \to \infty}\left(1+\frac{4}{y}\right)^2$$

$$= e^4(1) = e^4 = RHS.$$

(29) f is defined if $1 - x^2 \geq 0$.

For any point $c \in (-1, 1)$

$$\lim_{x \to c} f(x) = \lim_{x \to c}\sqrt{1-x^2} = \left[\lim_{x \to c}(1-x^2)\right]^{1/2}$$

$$= (1-c^2)^{1/2} = f(c)$$

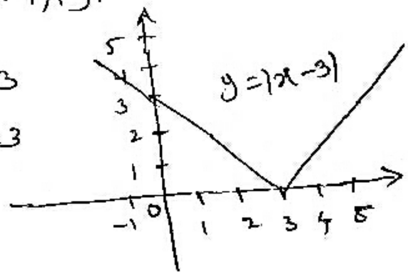
$$\lim_{x \to -1^+} f(x) = 0 = f(1)$$

$$\lim_{x \to -1^-} f(x) = 0 = f(-1)$$

Thus f is continuous on $[-1, 1]$.

(30) Given $f(x) = |x-3| = \begin{cases} (x-3) & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$

Creative



$$f(x) = \begin{cases} x-3 & \text{if } x \geq 3 \\ 3-x & \text{if } x < 3 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 1 & \text{if } x \geq 3 \\ -1 & \text{if } x < 3 \end{cases}$$

We get $f'(2) = -1$ and $f'(4) = 1$

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Using the chain and power rule.

$$f'(x) = \frac{x-3}{|x-3|}$$

$$f'(2) = \frac{2-3}{|2-3|} = \frac{-1}{1} = -1$$

$$f'(4) = \frac{4-3}{|4-3|} = \frac{1}{1} = 1.$$

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PART – III

$\boxed{7 \times 3m = 21m}$

(31)

Pre image of 17 are 3 and −3

" of 4 are $\sqrt{5/2}$ and $-\sqrt{5/2}$

Pre image of −2 :- $x^2 = -1/2$

which has no solution

So, −2 has no Preimage under $f(x) = 2x^2 - 1$.

(32) Let $f(x) = \dfrac{1}{1 - 2\sin x}$ எனக.

$1 - 2\sin x = 0$

$1 = 2\sin x$

$x = n\pi \cancel{\pm} (-1)^n \pi/6 \ ; n \in Z$

Domain of $f(x)$ is $R - \{n\pi + (-1)^n \pi/6\} ; n \in Z$.

(33) $^nP_r = \dfrac{n!}{(n-r)!} = 720 \rightarrow (1)$

$^nC_r = \dfrac{n!}{r!\,(n-r)!} = 120 \rightarrow (2)$

$\dfrac{(1)}{(2)} \Rightarrow \quad r! = 6 = 3 \times 2 \times 1 \Rightarrow \boxed{r = 3}$

Put r=3 in (1) we get $\boxed{n = 10}$

(34) Let $a_1, a_2, a_3, \ldots, a_n$ be a geometric Progression with initial term 'a' and common ratio 'r' then

$a_k = ar^{k-1}$, $a_{k-1} = ar^{k-2}$ and $a_{k+1} = ar^k$

$\sqrt{a_{k-1}a_{k+1}} = \sqrt{ar^{k-2}ar^k} = \sqrt{a^2r^{2k-2}} = \sqrt{a^2r^{2(k-1)}} = ar^{k-1} = a_k$

∴ a_k is a geometric mean of a_{k-1} and a_{k+1}.

(35) Given

$P = 12$

$\alpha = 150^\circ$

W.K.T required line is of the form

$$x\cos\alpha + y\sin\alpha = P$$

$$x\cos 150^\circ + y\sin 150^\circ = 12$$

$$\Rightarrow x\sqrt{3} - y + 24 = 0$$

(36) Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 4\hat{k}$ $\quad \vec{a} \times \vec{b} = 5\hat{i} - 3\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{35}$ units

$\therefore$ A unit vector is perpendicular to the vector $\vec{a}$ and $\vec{b}$ is $\dfrac{5\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{35}}$

Hence, a vector of magnitude $10\sqrt{3}$ which is perpendicular to $\vec{a}$ and $\vec{b}$ is $\pm\hat{n} = \pm 10\sqrt{3}\left(\dfrac{5\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{35}}\right)$

(37)
$$f(x) = \begin{cases} \dfrac{\sin(-x)}{x} & \text{if } -1 < x < 0 \\ \dfrac{\sin x}{x} & \text{if } 0 < x < 1 \end{cases}$$

$\therefore \lim_{x \to 0^-} f(x) = -1$ and $\lim_{x \to 0^+} f(x) = +1$

Hence, the limit does not exist

$$f(0^-) \neq f(0^+)$$

(38) Creative

$$\frac{dy}{dx} = \frac{d}{dx}\left[(\sin x)^4 + (\cos x)^4\right]$$

$$= 4\sin^3 x \cos x + 4\cos^3 x\,(-\sin x)$$

$$= 4\sin x \cos x\left[\sin^2 x - \cos^2 x\right]$$

(39) As $m - m = 0$ and $0 = 0 \times 12 \Rightarrow R$ is Reflexive

$m - n = 12k \Rightarrow mRn$

Also, $n - m = 12(-k) \Rightarrow nRm$ $\Big\} \Rightarrow R$ is symmetric

Let $m - n = 12k$ and let $n - p = 12l$ for some integers k and l.

Adding we get $m - p = 12(k + l)$ $\therefore R$ is transitive.

So, R is an equivalence relation.

(47) (a) $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{12}{\sqrt{14}\sqrt{24}} = \frac{12}{\sqrt{7\times2}\sqrt{2\times2\times2\times3}} = \frac{12}{2\sqrt{7}\sqrt{2}\sqrt{2}\sqrt{3}} = \sqrt{\frac{3}{7}}$

$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{4\sqrt{3}}{\sqrt{7}\sqrt{12}} = \frac{2}{\sqrt{7}}$

(b) Let $u = \sin(ax^2+bx+c)$.

$v = \cos(lx^2+mx+n)$

$\frac{du}{dx} = \cos(ax^2+bx+c).(2ax+b)$

$\frac{dv}{dx} = -\sin(lx^2+mx+c).(2lx+m)$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\cos(ax^2+bx+c).(2ax+b)}{-\sin(lx^2+mx+c).(2lx+m)}$

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Part-III
3mark
Creative

(40) $\sqrt{3}\sin x + \cos x = 2$

$\cos x + \sqrt{3}\sin x = 2$

here $a=1$, $b=\sqrt{3}$, $r=\sqrt{a^2+b^2}=\sqrt{1+3}=\sqrt{4}=2$ unit

$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = 1$

$\sin 30^\circ \cos x + \cos 30^\circ \sin x = \sin \pi/2$

$\sin(x+\pi/6) = \sin \pi/2$

$x+\pi/6 = n\pi \pm (-1)^n \pi/2, n\in z$

$x = n\pi + \pi/6 \pm (-1)^n\pi/2, n\in z$

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$\sin 60^\circ \sin x + \cos 60^\circ \cos x = \cos 0^\circ$

$\cos(x-\pi/3) = \cos 0^\circ$

$x-\pi/3 = 2n\pi \pm 0^\circ$

$x = 2n\pi + \pi/3, n\in z.$

$\boxed{7 \times 5m = 35m}$

PART - IV

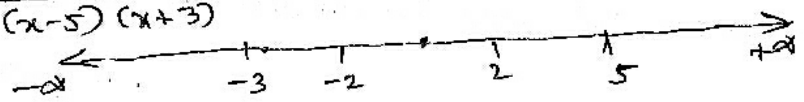
(41) a) $f(x) = \begin{cases} 3x & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$

$g(x) = \begin{cases} x & \text{if } x \le 0 \\ 3x & \text{if } x > 0 \end{cases}$

Let $x \le 0$ then $(g \circ f)(x) = 3x$

Let $x > 0$ then $(g \circ f)(x) = 3x$. for all x.

b) $f(x)$ $\dfrac{(x+2)(x-2)}{(x-5)(x+3)} \le 0 \Rightarrow$ critical numbers are $\pm 2, 5, -3$



If $x \in (-\infty, -3)$ then $f(x) > 0$

If $x \in (-3, -2)$ then $f(x) < 0$

If $x \in (-2, 2)$ then $f(x) > 0$

If $x \in (2, 5)$ then $f(x) < 0$

If $x \in (5, \infty)$ then $f(x) > 0$

$x^2 - 4 = 0$

$x^2 = 4$

$\boxed{x = \pm 2}$

$\therefore$ Solution set is $(-3, -2] \cup [2, 5)$.

(42) a) $LHS = 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cos C$

$= 2\sin(\pi/2 - C)\cos(A-B) + 2\sin C\cos C$

$= 2\cos C\left\{\cos(A-B) + \sin\left[\pi/2 - (A+B)\right]\right\}$

$= 2\cos C\left\{\cos(A-B) + \cos(A+B)\right\}$

$= 2\cos C \; 2\cos A\cos B = RHS.$

b) $RHS = \dfrac{2R\sin B - 2R\sin C}{2R\sin A}\cos A/2$

$= \dfrac{2\sin\left(\frac{B-C}{2}\right)\cos\left(\frac{B+C}{2}\right)}{2\sin A/2\cos A/2}\cos A/2$

$= \dfrac{\sin\left(\frac{B-C}{2}\right)\cos\left(\pi/2 - A/2\right)}{\sin A/2}$

$= \sin\left(\frac{B-C}{2}\right) = RHS.$

(43) (a) $P(n): 1^3+2^3+\dots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$P(1)$ is true.

$P(k)$ is true then $1^3+2^3+\dots+k^3 = \left[\frac{k(k+1)}{2}\right]^2$

$\therefore P(k+1)$ is true

By the principle of mathematical induction

$P(n)$ is true for all value of n.

(b) condition to represent a pair of parallel lines

is $h^2-ab=0 \Rightarrow 4-4(1)=0 \Rightarrow 0=0$.

$\therefore$ The given lines are parallel.

$\therefore$ Distance between the parallel lines $= 2\sqrt{\frac{g^2-ac}{a(a+b)}}$

$= 2\sqrt{\frac{9+16}{4(5)}}$

$= \sqrt{5}$ units

(44) (a) Let $\Delta = \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} =$

$\Delta=0 \Rightarrow (a-b)$ is a factor of Δ

lllly $(b-c)$ and $(c-a)$ are factor of Δ

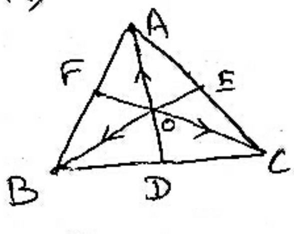
$m = 4-3 = 1 \Rightarrow k(a+b+c)$ is another factor of Δ.

$\therefore \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = k(a+b+c)(a-b)(b-c)(c-a)$

put $a=1, b=2, c=3$ we get $k=1$

$\therefore \Delta = (a+b+c)(a-b)(b-c)(c-a)$

(b) diagram



$\vec{OG_1} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \rightarrow (1)$

$\vec{OG_2} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \rightarrow (2)$

$\vec{OG_3} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} \rightarrow (3)$ from (1), (2) and (3)

Hence, the three medians are concurrent at G.

(45) (a) ~~lim~~ lt $f(x) = \begin{cases} \frac{x^3-1}{x-1} ; & \text{if } x \neq 1 \\ 3 ; & \text{if } x = 1 \end{cases}$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^3-1}{x-1} = \lim_{x\to 1^-} \frac{(x-1)(x^2+x+1)}{(x-1)} = 3$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2+x+1 = 3$$

Also $f(1) = 3$.

$\therefore f(x)$ is continuous on $(-\infty, \infty)$.

(b) $y' = e^{\tan^{-1}x} \frac{d}{dx}(\tan^{-1}x) = e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right) = \frac{y}{1+x^2}$

$(1+x^2)y' = y$

Differentiate again w.r.to 'x'.

$(1+x^2)y'' + y'(2x) = y'$

$(1+x^2)y'' + (2x-1)y' = 0.$

(46) (a) Creative

$LHS = \sqrt{x^2\left(1+\frac{25}{x^2}\right)} - \sqrt{x^2\left(1+\frac{9}{x^2}\right)}$ when x is large, $\frac{1}{x}$ is small.

$= x\sqrt{1+\frac{25}{x^2}} - x\sqrt{1+\frac{9}{x^2}}$ $\because \left|\frac{25}{x^2}\right| \leq 1$

$= x\left[1+\frac{1}{2}\frac{25}{x^2}+\cdots\right] - x\left[1+\frac{1}{2}\frac{9}{x^2}+\cdots\right]$ $\because \left|\frac{9}{x^2}\right| \leq 1$

$= x+\frac{25}{2x}+\cdots - x - \frac{9}{2x} - \cdots$

$= \frac{25}{2x} - \frac{9}{2x}$ (approxi.)

$= \frac{16}{2x} = \frac{8}{x}$ (approxi.)

$= RHS.$

Creative (b) $\lim_{x\to 0} \frac{\sqrt{1+x^2}-x}{x} \times \frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)} = \lim_{x\to 0} \frac{1+\cancel{x^2}-\cancel{x^2}}{x(\sqrt{1+x^2}+x)}$

$= \lim_{x\to 0} \frac{1}{x(\sqrt{1+x^2}+x)} = \frac{1}{0} = \infty.$