STD: XI Half yearly answer key - 2019

SUBJECT: BUSINESS MATHEMATICS

PART - I

1.) c.) $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

2) b) $|A|^{n-1}$

3) a) 2^n

4) a) 20

5) b) latus rectum

6) c) $a+b=0$

7) b) $-\sqrt{3}/2$

8) b) $[-\frac{\pi}{2}, \pi/2]$

9) d) (0, 1)

10) a) 5

11.) b) $\frac{1}{5}e^{5x}$

12) a) I quadrant

13) a) Annuity due

14) b) added

15) a) speed or rates

16) b) 0

17) d) -0.97

18) c) No correlation

19) b) Independent variable

20) b) Minimize total project duratio

PART - II

21.) $\begin{vmatrix} x & x+2 \\ x-2 & x \end{vmatrix} = x^2 - (x-2)(x+2)$

$= x^2 - (x^2 - 2^2)$

$= x^2 - x^2 + 4$

$\begin{vmatrix} x & x+2 \\ x-2 & x \end{vmatrix} = 4$

22) WKT distance of the point (x_1, y_1) to the line $ax+by+c=0$

is $d = \left| \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|$ here $(x_1, y_1) = (4, 1)$

line $3x-4y+12=0$

distance $d = \left| \frac{3(4)-4(1)+12}{\sqrt{3^2+(-4)^2}} \right| = \left| \frac{12-4+12}{\sqrt{9+16}} \right| = \left| \frac{20}{5} \right|$ 4

$d = 4$ units

23.) Combined equation of st. line

$(2x+y-7)(x+2y-1)=0$

$2x^2+4xy-2x+xy+2y^2-y-7x-14y+7=0$

$2x^2+5xy+2y^2-9x-15y+7=0$

24.) Let $y = x^6 - 4\sin x + 7\cos x + e^{-4x}$

$$\frac{dy}{dx} = 6x^5 - 4\cos x - 7\sin x - 4e^{-4x}$$

25.)

Investment in both the case $= 140 \times 70$

Income of 20% stock at Rs. 140

$$= \frac{20}{140} \times (140 \times 70)$$

$$= ₹1400$$

Income of 10% stock at Rs. 70

$$= \frac{10}{70} \times (140 \times 70)$$

$$= ₹1400$$

Both cases are same income.

∴ They are equivalent share

6.)

28.) Correlation coefficient $r(x,y) = \frac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}}$

$$= \frac{120}{\sqrt{90}\sqrt{640}} = \frac{120}{3 \times 8 \times 10} = \frac{1}{2}$$

$$\boxed{r(x,y) = 0.5}$$

29.) Regression Equation x on y

$$X - \bar{X} = b_{xy}(Y - \bar{Y})$$

$$b_{xy} = \frac{N\Sigma dx\,dy - \Sigma dx\,\Sigma dy}{N\Sigma dy^2 - (\Sigma dy)^2}$$

Regression Equation y on x

$$Y - \bar{Y} = b_{yx}(X - \bar{X})$$

$$b_{yx} = \frac{N\Sigma dx\,dy - \Sigma dx\,\Sigma dy}{N\Sigma dx^2 - (\Sigma dx)^2}$$

30.) $C = 10 + 4x^5 + 3x^6$

(i) Average cost function $= \frac{C(x)}{x} = \frac{10}{x} - 4x^4 + 3x^5$

(ii) Marginal cost function $= \frac{d}{dx}[C(x)]$

$$= \frac{d}{dx}[10 - 4x^5 + 3x^6]$$

$$= 0 - 4(5x^4) + 3(6x^5)$$

$$= -20x^4 + 18x^5$$

$$= 18x^5 - 20x^4$$

PART- III

31.) PART A : out of 10 questions 8 can be selected in $10C_8$

PART B : out of 10 questions 5 can be selected in $10C_5$

The total no. of selection $= 10C_8 \times 10C_5$

$$= 10C_2 \times \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{10 \times 9}{2 \times 1} \times 18 \times 7$$

$= 90 \times 126$

$= 11340$ //

32.) Given equations are $x^2+y^2-64=0 \Rightarrow$ radius $a^2 = 64$

Given lines are $3x+4y-k=0 \Rightarrow$ compare $y = mx+c$

$m = -\frac{3}{4}$, $c = \frac{k}{4}$

Condition for the tangency is $c^2 = a^2(1+m^2)$

$$\frac{k^2}{16} = 64\left(1+\frac{9}{16}\right)$$

$$\frac{k^2}{16} = 64\left(\frac{25}{16}\right)$$

$k = \pm(8 \times 5)$

$k = \pm 40$

33.) $LHS = \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 1$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = \frac{1+1-4}{4} = -\frac{2}{4} = -\frac{1}{2} = RHS$$

Hence verified

34) let $y = x^2 \sin x$

$d(uv) = u\,dv + v\,du$

$$\frac{dy}{dx} = x^2.(\cos x.1) + \sin x.(2x.1)$$

$$\frac{dy}{dx} = x(x\cos x + 2\sin x)$$

35.) Let $y = \sin^2 x$

$$\frac{dy}{dx} = 2\sin x.(\cos x.1)$$

$$\frac{dy}{dx} = \sin 2x$$

36.) Given Demand $x = 100 - 2p$ & Supply $x = 3p - 50$

Given Equilibrium $100 - 2p = 3p - 50$

$150 = 5p$

Equilibrium price $\boxed{30 = p}$

Equilibrium quantity $x = 3(30) - 50$

$x = 90 - 50$

$\boxed{x = 40}$

37.) Given $a = ₹\ 5000$ payable at the end of each year

$n = 4$ years

$i = 10\% = \frac{10}{100} = 0.1$

Amount of annuity $A = \frac{a}{i}[(1+i)^n - 1]$

$= \frac{5000}{0.1}[(1+0.1)^4 - 1]$

$= 50000[(1.1)^4 - 1]$

$= 50000[1.4641 - 1]$

$= 50000[0.4641]$

$= 23205.0000$

$\boxed{A = ₹\ 23,205}$

$S = \{5W, 3B\}$ $n(S) = 8$

38.) Let A, B be the events of getting a black ball in the first and second draw

probability of drawing a black ball in the first attempt is $P(A) = \frac{3}{5+3} = \frac{3}{8}$

probability of drawing a second black ball given that the first ball drawn is black.

$P(B/A) = \frac{2}{5+2} = \frac{2}{7}$

The probability that both balls drawn are black $P(A \cap B) = P(A) \cdot P(B/A) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

39.) Let x_1 be a product of type A
x_2 be a product of type B

Given Maximize $z = 30x_1 + 40x_2$

subject to the constraints

raw material $60x_1 + 120x_2 \leq 12000$

Machining $8x_1 + 5x_2 \leq 600$

Assembling $3x_1 + 4x_2 \leq 500$

and $x_1, x_2 \geq 0$

40.) Given two lines of regression are

$3x - 2y = -1 \rightarrow (1)$

$2x - y = 2 \rightarrow (2)$

Solve (1) & (2)

(1) ⇒ $3x - 2y = -1$

(2) × 2 ⇒ $4x - 2y = 4$

(−) (+) (−)

$-x = -5$

$x = 5$ sub (1)

$2(5) - y = 2$

$y = 10 - 2$

$y = 8$

Hence $X = 5, Y = 8$

$\bar{X} = 5$, $\bar{Y} = 8$

PART - IV

41.) a) Let x, y and z be a three numbers.

Given $x + y + z = 20 \rightarrow (1)$

$2x + y - z = 23 \rightarrow (2)$

$3x + y + z = 46 \rightarrow (3)$

The given system of equation can be written in Matrix form

$$AX = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$$

$X = A^{-1}B$

To find: $A^{-1} = \frac{1}{|A|} \text{adj} A$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{vmatrix} = 1(1+1) - 1(2+3) + 1(2-3)$$

$|A| = 2 - 5 - 1 = -4 \neq 0$

$$A_c = \begin{bmatrix} +(1+1) & -(2+3) & +(2-3) \\ -(1-1) & +(1-3) & -(1-3) \\ +(-1-1) & -(-1-2) & +(1-2) \end{bmatrix}$$

$$A_c = \begin{bmatrix} 2 & -5 & -1 \\ 0 & -2 & 2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$\text{adj} A = (A_c)^T = \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \frac{1}{-4} \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} 40 + 0 - 92 \\ -100 - 46 + 138 \\ -20 + 46 - 46 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -52 \\ -8 \\ -20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 2 \\ 5 \end{bmatrix}$$

(or)

b) condition for pair of st. lines $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ → ①

Given st. lines are $4x^2 - 12xy + 9y^2 + 18x - 27y + 8 = 0$

compare $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

here $a = 4$, $2h = -12$, $b = 9$, $2g = 18$, $2f = -27$

$h = -6$, $g = 9$, $f = -\frac{27}{2}$ $c = 8$

sub ①

$LHS = (4)(9)(8) + \cancel{2}\left(\frac{-27}{\cancel{2}}\right)(9)(-6) - 4\left(\frac{-27}{2}\right)^2 - 9(9)^2 - 8(-6$

$= \cancel{288} + 1458 - 729 - 729 - \cancel{288}$

$= \cancel{1458} - \cancel{1458}$

$= 0 = RHS$

Hence given lines are pair of st. lines.
Hence proved.

Condition for pair of parallel st lines. $h^2 - ab = 0$

$36 - 36 = 0$

Hence the given equation represents a pair of parallel st lines.

Now $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$

$= (2x - 3y)^2$

Let separate equations are $2x - 3y + m = 0$ & $2x - 3y + n = 0$

Now $4x^2 - 12xy + 9y^2 + 18x - 27y + 8 = (2x - 3y + m)(2x - 3y + n)$

Equate coeff. of x, y & constant term

$18 = 2n + 2m \rightarrow (1)$ | $8 = mn \rightarrow (3)$

$-27 = -3n - 3m \rightarrow (2)$

from (1) $n + m = 9$

$\boxed{n = 9 - m}$ sub (2)

from (2) $+9 = n + m$

Cannot get m & n value

So, we assume any one can take 1 & 8

Satisfied above 3 equations

So, we take $m = 1$, $n = 8$

So, we separate equations are $2x - 3y + 1 = 0$
& $2x - 3y + 8 = 0$