MATHEMATICS — HALF YEARLY — KEY ANSWER

PART – I

1. d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2. b) $\begin{bmatrix} -6 & 5 \\ 2 & -10 \end{bmatrix}$

3. d) $\sqrt{5}+2$

4. d) $x+iy$

5. c) 1

6. a) one negative and two imaginary Zeros

7. c) $\pi/2 - x$

8. a) $1/\sqrt{2}$

9. c) $(0, 1/8)$

10. c) 2

11. d) $\pi/2$

12. b) 2, 5

13. a) 0

14. b) $(1+xy)e^{xy}$

15. c) 2

16. b) $2/9$

17. a) $\pi/2$

18. a) $y+\sin^{-1}x = c$

19. d) $\cot x$

20. b) (1, 1)

PART – II

21) $i^3 + \frac{1}{i^3} = -i + i = 0$ — (2M)

22) $x^3 + 2x^2 + 3x + 4 = 0$

$\Sigma_1 = -4 \quad \Sigma_2 = 12 \quad \Sigma_3 = -32$ — (1M)

Equation: $x^3 + 4x^2 + 12x + 32 = 0$ — (1M)

23) — (1M)

Domain of $\operatorname{cosec}^{-1}x$ $[-\pi/2, \pi/2] - \{0\}$

$\operatorname{cosec}^{-1}(-1) = \pi/2 \in [-\pi/2, \pi/2] - \{0\}$ — (1M)

24) $4a = 4$

eqn: $(x-h)^2 = 4a(y-k)$

$(x+1)^2 = 4(y+2)$ — (2M)

25) Condition:

$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0$ — (1M)

$16 - 2m + 11 + 9m - 6 = 0$

$m = -3$ — (1M)

26) Given

$\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{v} = \hat{i} + 2\hat{j} + 3\hat{k}$ — (1M)

It cannot write $\vec{u} = \lambda\vec{v}$.

It is not Parallel. — (1M)

27)
f(x) is continuous and differentiable at (-2,2) — (1M)

$f(-2) = f(2) = 5$
$c \in (-2,2)$
$f'(c) = 0 \quad c \in (-2,2)$
$2x = 0$
$2c = 0$ — (1M)
Point (0,1).

28) $dy = \dfrac{2(1-2x)^2(6x-7)}{(3-4x)^2}dx$ — (2M)

29) $x\cos x$ — odd function — (1M)
$\int_{-\pi/2}^{\pi/2} x\cos x\,dx = 0.$ — (1M)

30)
$f(0)=1 \quad f'(0)=-1 \quad f''(0)=1$ — (1M)
$f(x) = e^{-x} = 1 - \dfrac{x}{1!} + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \cdots$ — (1M)

PART – III

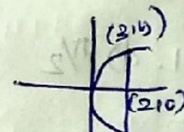
31)
$|F(\alpha)| = 1 \neq 0$ — (1M)
$adj(F(\alpha)) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$
$[F(\alpha)]^{-1} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$ — (1M)
$= F(-\alpha)$ — (1M)

32)
$x - y = -2$
$-x - y = 0$ — (1M)
$x = -1$ — (1M)
$y = 1$ — (1M)

33)
$\alpha = 3+2i$
$\beta = 3-2i$ — (1M)
eqn: $x^2 - 6x + 13 = 0$ — (2M)

34)
$\tan^{-1}\left[\dfrac{\dfrac{x-1}{x-2} + \dfrac{x+1}{x+2}}{1 - \left(\dfrac{x-1}{x-2}\right)\left(\dfrac{x+1}{x+2}\right)}\right] = \pi/4$ — (1M)
$2x^2 - 4 = -3$ — (1M)
$x^2 = 1/2 \Rightarrow x = \pm 1/\sqrt{2}$ — (1M)

35)
$y^2 = 4ax$
$a = 2$
$y^2 = 8x$ — (1)



Parabola on (3,y)
Width $= 4\sqrt{6}$

36)
LHS:-
$(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})$ — (1M)
$= [[\vec{a}\,\vec{b}\,\vec{c}]\vec{b} - 0]\cdot(\vec{c}\times\vec{a})$ — (1M)
$= [\vec{a}\,\vec{b}\,\vec{c}]^2$ — (1M)

37)

$$\lim_{x\to 0^+}\left(\frac{e^x - x - 1}{x(e^x - 1)}\right) = \frac{0}{0} \text{ form} \quad \text{— (1M)}$$

$$\lim_{x\to 0^+} \frac{e^x - 1}{x e^x + e^x - 1} = \frac{0}{0} \quad \text{— (1M)}$$

$$= \lim_{x\to 0} \frac{e^x}{x e^x + 2e^x} = \frac{1}{2} \quad \text{— (1M)}$$

38) $F(x) = \log\left(\frac{3-x}{3+x}\right)$

$f(-x) = -f(x)$ } — (2M)

$f(x)$ is odd function

$$\int_{-1}^{1} \log\left(\frac{3-x}{3+x}\right) dx = 0 \quad \text{— (1M)}$$

39) $y e^{-3x} = C\cos 2x + D\sin 2x$

$e^{-3x}(-3y + y') = -2C\sin 2x + 2D\cos 2x$ — (1M)

$e^{-3x}(y'' - 3y' - 3y' + 9y) = -4(C\cos 2x + D\sin 2x)$ — (1M)

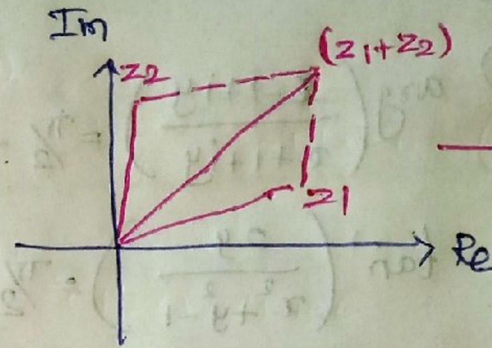
$y'' - 6y' + 13y = 0$ — (1M)

40)

let $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$ } — (2M)



— (1M)

PART – IV

41

(a)

$\Delta = -22$

$\Delta x = -44$

$\Delta y = -66$

$\Delta z = -88$

} — (3M)

by cramer's rule

$x = \frac{\Delta x}{\Delta}$, $y = \frac{\Delta y}{\Delta}$, $z = \frac{\Delta z}{\Delta}$ — (1M)

$(x, y, z) = (2, 3, 4)$ — (1M)

(b)

$$[A|B] = \left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{array}\right] \quad \text{— (1M)}$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{— (1M)}$$

$\rho(A) = \rho[A|B] = 1$ — (1M)

Infinetly no of solutions — (1M)

$(x, y, z) = \left(\frac{1}{2}(2 + s - t), s, t\right)$ — (1M)

$s, t \in R$

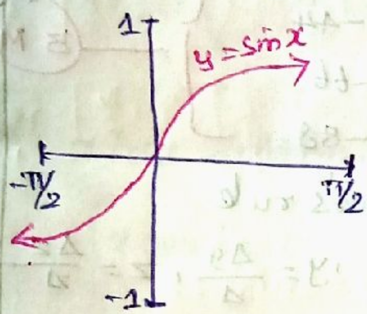
42

(a) $\arg\left(\frac{x-1+iy}{x+1+iy}\right)=\pi/2$ — (1M)

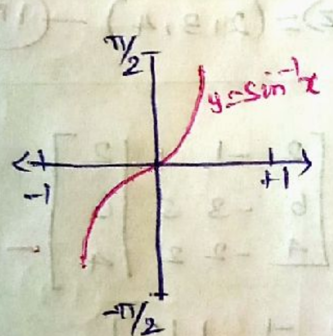
$\tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right)=\pi/2$ — (2M)

$x^2+y^2-1=0$
$x^2+y^2=1$ — (2M)

42 (b)



— $2\frac{1}{2}$ M



— $2\frac{1}{2}$ M

43

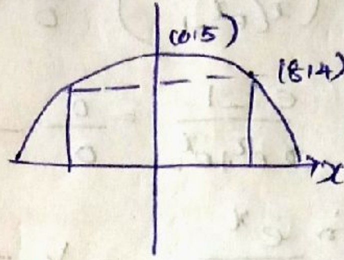
(a) $(x-2)^2+(y-2)^2=\frac{1}{4}(x-7)^2$
$3x^2+4y^2-2x-24y+3=0$ } — (1M)

$\frac{(x-1/3)^2}{100/9}+\frac{(y-3)^2}{100/12}=1$ — (2M)

Length of Major and Minor axes } $\frac{20}{3}, \frac{10}{\sqrt{3}}$ — (2M)

43

(b)



— (1M)

$\frac{x^2}{a^2}+\frac{y^2}{5^2}=1$ — (1M)

(8,4) lies on ellipse — (1M)

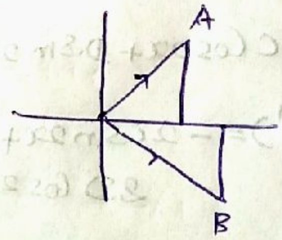
$a=40/3$

$2a=80/3=26.67$

$2a=26.67$ — (2M)

44

(a)



— (1M)

$\vec{a}=\cos\alpha\hat{i}+\sin\alpha\hat{j}$
$\vec{b}=\cos\beta\hat{i}-\sin\alpha\hat{j}$ } — (2M)

$\hat{b}\times\hat{a}=\hat{k}(\sin\alpha\cos\beta+\cos\alpha\sin\beta)$

$\hat{b}\times\hat{a}=\hat{k}(\sin(\alpha+\beta))$ } — (2M)

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

44

(b)

$\vec{a} = 2\hat{i}+2\hat{j}+\hat{k}$
$\vec{b} = 2\hat{i}+3\hat{j}+3\hat{k}$
$\vec{c} = 3\hat{i}+2\hat{j}+\hat{k}$ — (1M)

Vector equation:

$\vec{r} = (2\hat{i}+2\hat{j}+\hat{k}) + s(2\hat{i}+3\hat{j}+3\hat{k}) + t(3\hat{i}+2\hat{j}+\hat{k})$ — (2M)

Cartesian equation:

$3x - 7y + 5z + 3 = 0$ — (2M)

45

(a)

$r = \frac{5h}{12}$

— (1M)

$V = \frac{25\pi h^3}{3 \times 144}$ — (1M)

$\frac{dv}{dt} = \frac{25\pi}{3\times144} 3h^2 \frac{dh}{dt}$ — (1M)

$\boxed{\frac{dh}{dt} = \frac{9}{10\pi}}$ — (2M)

45

(b)

$xy = A$

$y = A/x$

$L = 2x+2y$

$= 2x + \frac{2A}{x}$ — (1M)

$L' = 2 - \frac{2A}{x^2}$

$L' = 0 \Rightarrow$ — (1M)

$\frac{2A}{x^2} = 2$

$\boxed{x = \sqrt{A}}$ — (1M)

$L'' = \frac{22A}{x^3} > 0$

L is minimum

$x = y = \sqrt{A}$, Square only — (2M)

46

(a)

$u(\lambda x, \lambda y) = \lambda^{3/2}(u(x,y))$ — (1M)

a) u is Homogeneous of degree $n = 3/2$ — (2M)

By Euler's Theorem

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3/2\, u$ — (2M)

46

(b)

$I.F = e^{\int 2\cot x\,dx}$ — (1M)

$I.F = \sin^2 x$ — (1M)

Solution

$y\sin^2 x = \int 3x^2 dx + C$ — (1M)

$y\sin^2 x = x^3 + C$ — (2M)

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PG ASST IN MATHEMATICS

PATTUKKOTTAI

47

(a)

$\frac{dA}{dt} \propto A$ — (1 M)

$\frac{dA}{dt} = kA$

$A = Ce^{kt}$ — (1M)

(i) $t=0$, $C = 1,30,000$

$A = 1,30,000e^{kt}$

(ii) $t = 30$, $A = 1,60,000$

$\frac{16}{13} = e^{30k}$ — (1M)

I) $t = 90$

$A = 1,30,000\, e^{90k} = 1,30,000 (e^{30k})^3$

$= 1,30,000 \left(\frac{16}{13}\right)^3$ → (1M)

$= 1,30,000\ (1.86)$

$A = 2,41,800$ — (1M)

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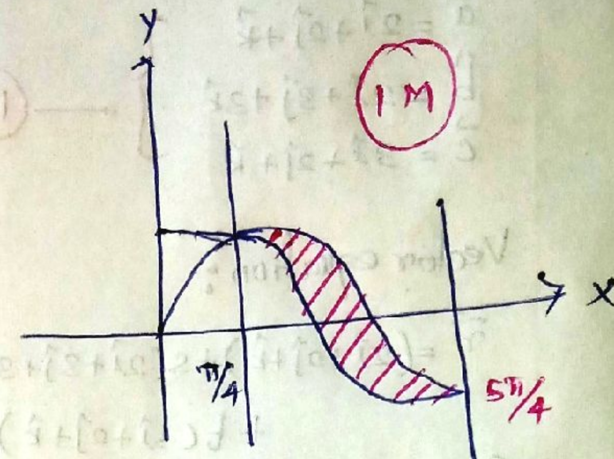
47)

b)



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx$$ — (2M)

$A = 2\sqrt{2}$ — (2M)

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