Class : 12

Register Number | | | | | | |

Higher Secondary Half Yearly Examination 2017-18

Time Allowed : 3 Hours] **MATHEMATICS** [Max. Marks : 200

INSTRUCTION : 1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

Part - A

Note : (i) Answer All the questions. $40\times1=40$

(ii) Choose the correct answer and write the option code & the corresponding answer.

1. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then λ is
   (a) 1 (b) 2 (c) 3 (d) any real number

2. If I is the unit matrix of order n, where $k \neq 0$ is a constant, then adj(k I) is :
   (a) k^n (adj I) (b) k (adj I) (c) k^2 (adj I) (d) k^{n-1}(adj I)

3. In a system of 3 linear non homogeneous equation with three unknowns, if $\Delta = 0$, and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$ then the system has
   (a) unique solution (b) two solutions (c) infinitely many solutions (d) no solution

4. Which of the following statement is correct regarding homogeneous system ?
   (a) always inconsistent (b) has only trivial solution (c) has only non trivial solutions
   (d) has only trivial solution only if rank of the coefficient matrix is equal to the number of unknowns

5. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$, then the angle between $\vec{a}$ and $\vec{b}$ is
   (a) $\frac{\pi}{6}$ (b) $\frac{2\pi}{3}$ (c) $\frac{5\pi}{3}$ (d) $\frac{\pi}{2}$

6. The area of the parallelogram having a diagonal $3\vec{i} + \vec{j} - \vec{k}$ and a side $\vec{i} - 3\vec{j} + 4\vec{k}$ is
   (a) $10\sqrt{3}$ (b) $6\sqrt{30}$ (c) $\frac{3}{2}\sqrt{30}$ (d) $3\sqrt{30}$

7. If the magnitude of moment about the point $\vec{j} + \vec{k}$ of a force $\vec{i} + a\vec{j} - \vec{k}$ acting through the point $\vec{i} + \vec{j}$ is $\sqrt{8}$ then the value of a is
   (a) 1 (b) 2 (c) 3 (d) 4

8. The equation of the plane passing through the point (2, 1, -1) and the line of intersection of the planes $\vec{r}.(\vec{i} + 3\vec{j} - \vec{k}) = 0$ $\vec{r}.(\vec{j} + 2\vec{k}) = 0$ is :
   (a) x+4y-z=0 (b) x+9y+11z=0 (c) 2x+y-z+5=0 (d) 2x-y+z=0

9. The following two lines are $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-1}{2}$
   (a) parallel (b) intersecting (c) skew (d) perpendicular

10. The angle between the line $\vec{r} = \vec{a} + t\vec{b}$ and the plane $\vec{r}.\vec{n} = q$ is connected by the relation
    (a) $\cos\theta = \frac{\vec{a}.\vec{n}}{q}$ (b) $\cos\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}|\,|\vec{n}|}$ (c) $\sin\theta = \frac{\vec{a}.\vec{b}}{|\vec{n}|}$ (d) $\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}|\,|\vec{n}|}$

11. If a = 3+i and z = 2-3 i then the points on the Argand diagram representing az, 3az, -az are
    (a) Vertices of a right angled triangle (b) Vertices of an equilateral triangle
    (c) Vertices of an isosceles triangle (d) Collinear

12. If a = cos α - i sin α, b = cos β - i sin β, c = cos γ - i sin γ then $(a^2c^2-b^2)/abc$ is
    (a) cos 2(α-β+γ)+i sin 2(α-β+γ) (b) -2 cos(α-β+γ) (c) -2i sin(α-β+γ) (d) 2 cos(α-β+γ)

13. If $\frac{1-i}{1+i}$ is a root of the equation $ax^2 + bx + 1 = 0$ where a, b, are real then (a,b) is
    (a) (1, 1) (b) (1, -1) (c) (0, 1) (d) (1, 0)

14. If z_1 and z_2 are any two complex numbers then which one of the following is false ?
    (a) $Re(z_1+z_2) = Re(z_1) + Re(z_2)$ (b) $Im(z_1+z_2) = Im(z_1) + Im(z_2)$
    (c) $arg(z_1+z_2) = arg(z_1) + arg(z_1)$ (d) $|z_1z_2| = |z_1||z_2|$

15. The angle between the two tangents drawn from the point (-4, 4) to $y^2 = 16x$ is
    (a) 45^0 (b) 30^0 (c) 60^0 (d) 90^0

16. The difference between the focal distances of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 24 and the eccentricity is 2. 2. Then the equation of the hyperbola is

(a) $\frac{x^2}{144}-\frac{y^2}{432}=1$ (b) $\frac{x^2}{432}-\frac{y^2}{144}=1$ (c) $\frac{x^2}{12}-\frac{y^2}{12\sqrt{3}}=1$ (d) $\frac{x^2}{12\sqrt{3}}-\frac{y^2}{12}=1$

17. The normal to the rectangular hyperbola $xy=9$ at $\left[6,\frac{3}{2}\right]$ meets the curve again at

a) $\left[\frac{3}{8}, 24\right]$ (b) $\left[-24, \frac{-3}{8}\right]$ (c) $\left[\frac{-3}{8}, -24\right]$ (d) $\left[24, \frac{3}{8}\right]$

18. The locus of the foot of perpendicular from the foucs on any tangent to the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) $x^2+y^2=a^2-b^2$ (b) $x^2+y^2=a^2$ (c) $x^2+y^2=a^2+b^2$ (d) $x=0$

19. A spherical snow ball is melting in such a way that its volume is decreasing at a rate of 1 cm^3/min. The rate at which the diameter is decreasing when the diameter is 10 cms, is

a) $-\frac{1}{50\pi}$ cm/min (b) $\frac{1}{50\pi}$ cm/min (c) $\frac{-11}{75\pi}$ cm/min (d) $\frac{-2}{75\pi}$ cm/min

20. If the volume of an expanding cube is increasing at the rate of 4 cm^3/sec then the rate of change of surface area when the volume of the cube is 8 cubic cm is

(a) 8 cm^2/sec (b) 16 cm^2/sec (c) 2 cm^2/sec (d) 4 cm^2/sec

21. The curve $y=-e^{-x}$ is

(a) concave upward for $x>0$ (b) concave downward for $x>0$
(c) everywhere concave upward (d) everywhere concave downward

22. A continuous graph $y=f(x)$ is such that $f'(x)\to\infty$ as $x\to x_1$ at (x_1, y_1). Then $y=f(x)$ has a :

(a) vertical tangent $y=x_1$ (b) horizontal tangent $x=x_1$
(c) vertical tangent $x=x_1$ (d) horizontal tangent $y=y_1$

23. The percentage error in the 11[th] root of the number 28 is approximately ________ times the percentage error in 28.

a) $\frac{1}{28}$ (b) $\frac{1}{11}$ (c) 11 (d) 28

24. If $u=f(x, y)$ then with usual notations $u_{xy}=u_{yx}$ if

(a) u is continuous (b) u_x is continuous (c) u_y is continuous (d) u, u_x, u_y are continuous

25. The Value of $\int_{-\pi/2}^{\pi/2}\left(\frac{\sin x}{2+\cos x}\right)dx$ is

(a) 0 (b) 2 (c) log 2 (d) log4

26. The volume generated by rotating the triangle with vertices (0, 0) (3, 0) and (3, 3) about x-axis is

(a) 18π (b) 2π (c) 36π (d) 9π

27. The curved surface area of a shhere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is

(a) 20π (b) 40π (c) 10π (d) 30π

28. $\int_0^{2a} f(x)\,dx=0$ if

(a) $f(2a-x)=f(x)$ (b) $f(2a-x)=-f(x)$ (c) $f(x)=-f(x)$ (d) $f(-x)=f(x)$

29. The integrating factor $\frac{dy}{dx}+\frac{1}{x\log x}y=\frac{2}{x^2}$ is

(a) e^x (b) $\log x$ (c) $1/x$ (d) e^{-x}

30. The differential equation formed by eliminating A and B from the relation $y=e^x[A\cos x+B\sin x]$ is :

(a) $y''+y'=0$ (b) $y''-y'=0$ (c) $y''-2y'+2y=0$ (d) $y''-2y'-2y=0$

31. If $f'(x)=\sqrt{x}$ and $f(1)=2$ then $f(x)$ is

a) $-\frac{2}{3}(x\sqrt{x}+2)$ (b) $-\frac{3}{2}(x\sqrt{x}+2)$ (c) $\frac{2}{3}(x\sqrt{x}+2)$ (d) $\frac{2}{3}x(\sqrt{x}+2)$

32. The order and degree of the differential equation $y'+(y'')^2=(x+y'')^2$ are

(a) 1, 1 (b) 1, 2 (c) 2, 1 (d) 2, 2

33. The conditional statement $p \rightarrow q$ is equivalent to :
(a) $p \vee q$ (b) $p \vee (\sim q)$ (c) $(\sim p) \vee q$ (d) $p \wedge q$

34. With usual notations, in the multiplicative group of cube root of unity, the order of ω^2 is
(a) 4 (b) 3 (c) 2 (d) 1

35. In the set of integers under the operation * defined by $a*b = a+b-1$, the identity element is
(a) 0 (b) 1 (c) a (d) b

36. $\div$ is a binary operation on :
(a) N (b) R (c) Z (d) C-{0}

37. Which of the following are correct ?
(i) $E(aX+b)=a\,E(X)+b$ (ii) $\mu_2{}' = \mu_2{}' - (\mu_1{}')^2$ (iii) μ_2=Variance (iv) $Var(aX+b)=a^2 VarX$
(a) all (b) (i), (ii), (iii) only (c) (ii), (iii) only (d) (i), (iv) only

38. Variance of a random variable X is 4. Its mean is 2. then $E(X^2)$ is
(a) 2 (b) 4 (c) 6 (d) 8

39. If a random variable X follows a poison distribution such that $E(X^2)=30$ then the variance of the distribution is
(a) 6 (b) 5 (c) 30 (d) 25

40. The marks secured by 400 students in a mathematics test were normally distributed with mean 65. If 120 students got more marks above 85, the number of students securing marks between 45 and 65 is
(a) 120 (b) 20 (c) 80 (d) 160

Part - B

Note : (i) Answer any 10 questions. **10x6=60**
(ii) Question No. 55 is compulsory and choose any 9 questions from the remaining.

41. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1} A^{-1}$

42. Examine the consistency of the system $x+y+z=7$; $x+2y+3z=18$; $y+2z=6$, by rank method. If it is consistent then solve the system.

43. Find the meeting point of the line $\vec{r} = (2\vec{i}+\vec{j}-3\vec{k}) + t(2\vec{i}-\vec{j}-\vec{k})$ and the plane $x-2y+3z+7=0$.

44. (i) If $\vec{x}.\vec{a} = 0$, $\vec{x}.\vec{b} = 0$, $\vec{x}.\vec{c} = 0$ and $\vec{x} \neq 0$ then show that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.
(ii) Find the angle between the line $\vec{r} = (\vec{i}+2\vec{j}-\vec{k}) + \mu(2\vec{i}+\vec{j}+2\vec{k})$ and the plane $\vec{r}.(3\vec{i}-2\vec{j}+6\vec{k}) = 0$.

45. For any two complex numbers $z_1, z_2 \neq 0$ Prove $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, and $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

46. Prove that, the points representing the complex numbers $10+8i, -2+4i, -11+31i$ on the argand plane form a right angled triangle.

47. (i) Evaluate : $\lim_{x \to \infty} \dfrac{\sin\left(\frac{2}{x}\right)}{\left(\frac{1}{x}\right)}$
(ii) Verify Roll's theorem for the function $f(x) = |x-1|, 0 \leq x \leq 2$

48. Prove that the function $f(x) = \tan x + \cot x$ is not monotonic in the interval $(0, \pi/2)$

49. Find the approximate value of $\sqrt{36.1}$ to two decimal places using differential.

50. Solve the differential equation $(D^2+2D+1)y = x^2+2x+1$.

51. Show that $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

52. State and prove cancellation on groups.

53. Marks is an aptitude test given to 800 students of a school was found to be normally distributed. 10% of the students scored below 40 marks and 10% of the students scored above 90 marks. Find the number of students scored between 40 and 90.

54. (i) If on an average 1 ship out of 10 do not arrive safely to ports, find the mean and standard deviation of ships returning safely out of a total of 500 ships.

(ii) If the probability of a defective fuse from a manufacturing unit is 2% in a box of 200 fuses find the probability that more than 3 fuses are defective. [$e^{-4}=0.0183$]

55. (a) The tangent at any point of the rectangular hyperbola $xy=c^2$ makes intercepts a, b and the normal at the point makes intercepts p, q on the axes. Prove that $ap+bq=0$. **(OR)**

(b) Evaluate : $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$.

Part - C

Note : (i) Answer any 10 questions. **10x10=100**

(ii) Question No. 70 is compulsory and choose any 9 questions from the remaining.

56. A bag contains 3 types of coins namely ₹1, ₹2 and ₹5. There are 30 coins amounting to ₹100 in total. Find the number of coins in each category.

57. Verify $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$ for $\vec{a} = \vec{i}+\vec{j}+\vec{k};\ \vec{b} = 2\vec{i}+\vec{k};\ \vec{c} = 2\vec{i}+\vec{j}+\vec{k};\ \vec{d} = \vec{i}+\vec{j}+2\vec{k}$

58. Find the vector and CArtesian equation fo the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$

59. If α and β are the roots of the equation $x^2-2px+(p^2+q^2)=0$ and $\tan\theta = \frac{q}{y+p}$ show that $\frac{(y+\alpha)^n-(y+\beta)^n}{\alpha-\beta} = q^{n-1}\frac{\sin n\theta}{\sin^n\theta}$

60. Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

61. The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi major axis is of length 36 million miles and the eccentricityof the orbit is 0.206. Find (i) how close the mercury gets to sun ? (ii) the greatest possible distance between mercury and sun.

62. Find the equation of the director circle of the hyperbola if:

(i) The centre of the hyperbola is same as the centre of the ellipse $\frac{(x-1)^2}{9}+\frac{(y+1)^2}{16}=1$.

(ii) The length of the latus rectum is $\frac{9}{2}$ and the eccentricity $\frac{5}{4}$

(iii) The equation of the conjugate axis is $x=1$

63. Show that the equation of the normal to the curve $x=a\cos^3\theta$, $y=a\sin^3\theta$, at θ is $x\cos\theta - y\sin\theta = a\cos 2\theta$.

64. The top and bottom margins of a poster are each 6 cms and the side margins are each 4 cms. If the area of the printed material on the poster is fixed at 384 cm², find the dimension of the poster with the smallest area.

65. $u = \tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \sin 2u$.

66. Find the surface area of the solid generated by revolving the cycloid $x=a(t+\sin t), y=a(1+\cos t)$ about its base (x-axis).

67. Find the common area enclosed by the parabolas $4y^2=9x$ and $3x^2=16y$.

68. A cup of coffee at temperature 100°C is placed in a room whose temperature is 15°Cand it cools to 60°C in 5 minutes. Find its temperature after a further interval of 5 minutes.

69. An urn contains 4 white and 3 red balls. Find the probability distribution of number of red balls in three draws one by one from the urn. (i) with replacement (ii) without replacement

70. (a) Find the cubic polynomial in x which attains its maximum value 4 and minimum value 0 at $x=-1$ and 1 respectively.

(Or)

(b) Show that the set M of complex numbers Z with the condition $|z|=1$ forms a group with respect to the operation of multiplication of complex numbers.