12TH STD - MATHEMATICS

HALF YEARLY EXAMINATION – 2017

ANSWER KEY (**18 - 12- 17)**

ONE MARK QUESTIONS : 40 X 1 = 40	
1	b) 2
2	d) $k^{n-1}(adj\, I)$
3	d) no solution
4	d) has only trivial solution only if rank of the coefficient matrix is equal to the number of unknowns
5	d) $\frac{\pi}{2}$
6	d) $3\sqrt{30}$
7	b) 2
8	b) $x + 9y + 11z = 0$
9	c) skew
10	d) $\sin\theta = \frac{\vec{b}\cdot\vec{n}}{\lvert\vec{b}\rvert\lvert\vec{n}\rvert}$
11	d) collinear
12	c) $-2i\sin(\alpha - \beta + \gamma)$
13	d) (1,0)
14 *	c) $\arg(z_1 + z_2) = \arg(z_1) + \arg(z_2)$ (m.a)
15	d) 90^o
16	a) $\frac{x^2}{144} - \frac{y^2}{432} = 1$
17	c) $\left[\frac{-3}{8}, -24\right]$
18	b) $x^2 + y^2 = a^2$
19	b) $\frac{1}{50\pi}, cm/\min$
20	a) $8cm^2/\sec$
21	d) everywhere concave downward
22	a) vertical tangent $x = x_1$
23	b) $\frac{1}{11}$
24	d) u, u_x, u_y are continuous
25	a) 0
26	d) 9π
27	a) 20π
28	b) $f(2a - x) = -f(x)$
29	b) $\log x$
30	c) $y'' - 2y' + 2y = 0$
31	c) $\frac{2}{3}(x\sqrt{x} + 2)$
32	c) 2, 1
33	c) $(\square\, p) \vee q$
34	b) 3
35	b) 1
36	d) $C - \{0\}$
37	a) all
38	d) 8
39	b) 5
40	c) 80

SIX MARK QUESTIONS : 10 X 6 = 60

41

$|A| = \begin{vmatrix}1 & 2\\ 1 & 1\end{vmatrix} = 1 - 2 = -1 \neq 0 \Rightarrow$ A is invertible.

$|B| = \begin{vmatrix}0 & -1\\ 1 & 2\end{vmatrix} = 1 \neq 0 \Rightarrow$ B is invertible.

AB $= \begin{bmatrix}2 & 3\\ 1 & 1\end{bmatrix}$

$|AB| = -1 \neq 0 \Rightarrow$ AB is invertible.

adj A $= \begin{bmatrix}1 & -2\\ -1 & 1\end{bmatrix}$; $A^{-1} = \begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$

adj B $= \begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$; $B^{-1} = \begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$

adj(AB) $= \begin{bmatrix}1 & -3\\ -1 & 2\end{bmatrix}$; $(AB)^{-1} = \frac{1}{|AB|}$

$[adj(AB)] = \begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix} ---\rightarrow (1)$

$B^{-1}A^{-1} = \begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix} ---- (2)$

From (1) and (2) ,

$(AB)^{-1} = B^{-1}A^{-1}$

42

The corresponding matrix equation is

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}7\\ 18\\ 6\end{bmatrix}$$

Augmented matrix is $[A, B] = \begin{bmatrix}1 & 1 & 1 & 7\\ 1 & 2 & 3 & 18\\ 0 & 1 & 2 & 6\end{bmatrix}$

$$\sim \begin{bmatrix}1 & 1 & 1 & 7\\ 0 & 1 & 2 & 11\\ 0 & 1 & 2 & 6\end{bmatrix} R_2 \rightarrow R_2 - R_1$$

$$\sim \begin{bmatrix}1 & 1 & 1 & 7\\ 0 & 1 & 2 & 11\\ 0 & 0 & 0 & -5\end{bmatrix} R_3 \rightarrow R_3 - R_2$$

It is in echelon form and it has three non-zero rows.

$\rho[A, B] = 3$ and $\rho(A) = 2$.

$\rho[A, B] \neq \rho(A)$

43

Cartesian form : $\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z+3}{-1} = \lambda(say)$

Any point : $(2\lambda + 2, -\lambda + 1, -\lambda - 3)$

This point lie on the plane $x - 2y + 3z + 7 = 0$

$(2\lambda + 2) - 2(-\lambda + 1) + 3(-\lambda - 3) + 7 = 0$

$2\lambda + 2 + 2\lambda - 2 - 3\lambda - 9 + 7 = 0 \Rightarrow \lambda = 2$

$\therefore$ The required point is $(6, -1, -5)$

44

(i)

$(\vec{x}\cdot\vec{a}) = 0 \Rightarrow \vec{x} \perp \vec{a}$, $(\vec{x}\cdot\vec{b}) = 0 \Rightarrow \vec{x} \perp \vec{b}$

$\vec{x}$ is $\perp$ to both $\vec{a}$ *and* $\vec{b}$

$\vec{x}$ is $\parallel$ to $\vec{a}\times\vec{b} \Rightarrow \vec{x} = \lambda(\vec{a}\times\vec{b})$

$(\vec{x}\cdot\vec{c}) = 0$

$\Rightarrow \lambda(\vec{a}\times\vec{b})\cdot\vec{c} = 0$

$\Rightarrow [\vec{a}, \vec{b}, \vec{c}] = 0$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar.

(ii)

The angle between the line and plane is $sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$

$\vec{b}\cdot\vec{n} = 6 - 2 + 12 = 16$

$|\vec{b}| = 3;\ |\vec{n}| = 7$

$$\theta = sin^{-1}\left[\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right] = sin^{-1}\left[\frac{16}{21}\right]$$

45

Let $z_1 = r_1(cos\theta_1 + isin\theta_1)$

$z_2 = r_2(cos\theta_2 + isin\theta_2)$.

$|z_1| = r_1,\ arg(z_1) = \theta_1;$

$|z_2| = r_2,\ arg(z_2) = \theta_2$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

$\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$ $\quad \arg\frac{z_1}{z_2} = \theta_1 - \theta_2 = \arg z_1 - \arg z_2$

46

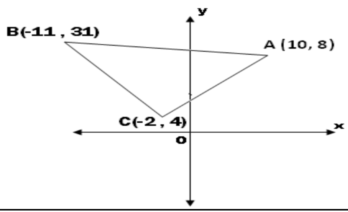
AB= $|-21+23i| = \sqrt{970}$

BC= $|9-27i| = \sqrt{810}$

CA = $|12+4i| = \sqrt{144+16} = \sqrt{160}$

$AB^2 = BC^2 + CA^2 = 970$

Hence the given points represent a right angled triangle on the Argand plane.



47 (i)

$$y = \frac{1}{x} \; as \; x \to \infty, y \to 0$$

$$\lim_{x\to+\infty} \frac{sin\frac{2}{x}}{\frac{1}{x}} = \lim_{y\to 0} \frac{\sin 2y}{y} \left(\frac{0}{0} form\right)$$

$$= \lim_{y\to 0}\left[\frac{2\cos 2y}{1}\right] = 2(1) = 2$$

(ii) Continuous on [0,2]

Not differential on (0,2)

Rolls theorem cannot applied to the given function.

48

$f^{'}(x) = \sec^2 x - \operatorname{cosec}^2 x$

$f^{'}\left(\frac{\pi}{2}\right) > 0, \quad f^{'}\left(\frac{\pi}{6}\right) < 0$

f is not monotonic on $\left(0, \frac{\pi}{2}\right)$

49

Let $y = f(x) = x^{1/2}$

$dy = f^{'}(x)dx = \frac{1}{2}(x)^{-1/2}dx$

Let $x = 36,\ dx = \Delta x = 0.1,\ \ f(36) = 6$

$$dy = \frac{1}{2}(36)^{-1/2} \times 0.1 = \frac{0.1}{12} = 0.0083$$

$$f(x+\Delta x) \approx y + dy$$

$\therefore\ \sqrt{36.1} \approx f(36) + 0.008 = 6.008$

$\sqrt{36.1} \approx 6.01$ (two decimal)

50

$C.F = (Ax+B)e^{-x}$

$P.I_1 = 7 - 4x + x^2$, $P.I_2 = x - 2$, $P.I_3 = 1$

General Solution :

$y = C.F + P.I = (Ax+B)e^{-x} + x^2 - 3x + 6$

51

p	q	p ∧ q	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\sim p$	$\sim q$	$(\sim p)\vee(\sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

The last columns are identical.

$\therefore\ \ \sim(p\wedge q) \equiv (\sim p)\vee(\sim q)$

52

STATEMENT: Let G be a group. $\forall\, a, b, c \in G$, Then

(i) $a*b = a*c \Rightarrow b = c$ (left cancellation law)

(ii) $b*a = c*a \Rightarrow b = c$ (Right cancellation law)

Proof : (i) $a*b = a*c$

$$\Rightarrow a^{-1}*(a*b) = a^{-1}*(a*c)$$

$$\Rightarrow (a^{-1}*a)*b = (a^{-1}*a)*c$$

$$\Rightarrow e*b = e*c$$

$$\Rightarrow b = c$$

(ii) $b*a = c*a$

$$\Rightarrow (b*a)*a^{-1} = (c*a)*a^{-1}$$

$$\Rightarrow b*(a*a^{-1}) = c*(a*a^{-1})$$

$$\Rightarrow b*e = c*e$$

$$\Rightarrow b = c$$

53

$P(X<40) = 0.1\ ;\ P(X>90) = 0.1$

$\therefore P(40 < X < 90)$

$= 1 - [\,P(X<40) + P(X>90)\,]$

$= 1 - 0.1 - 0.1 = 0.8$

Out of 800 Students, number of students scored between 40 and 90 is

$800 \times 0.8 = 640$ students

54 (i)

p = 9/10, q= 1/10

Mean = 450, variance = 45

S.D = $3\sqrt{5}$

(ii) p = 2/100, n = 200

$\lambda = 4$

$P(X > 3) = 0.5669$

55 (a)

Equation of tangent at any point $'t'$ $on\ \ xy = c^2\ is\ x + yt^2 = 2ct$

$\frac{x}{2ct} + \frac{y}{2c/t} = 1$

$\therefore$ Intercept on the axes are $a = 2ct$, $b = \frac{2c}{t}$

Equation of normal at 't' on $xy = c^2\ is$

$$y - xt^2 = \frac{c}{t} - ct^3$$

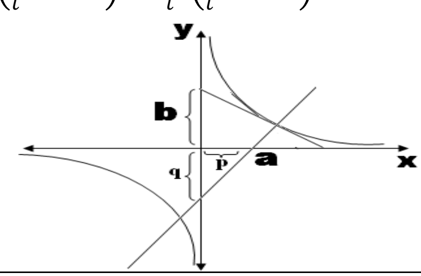
$$\frac{x}{\left(\frac{\frac{c}{t}-ct^3}{-t^2}\right)} + \frac{y}{\left(\frac{c}{t}-ct^3\right)} = 1$$

$\therefore$ Intercept on axes are

$$p = \frac{-1}{t^2}\left(\frac{c}{t} - ct^3\right);\quad q = \frac{c}{t} - ct^3$$

$\therefore ap + bq = 2ct\left(\frac{-1}{t^2}\right)\left(\frac{c}{t} - ct^3\right) + \frac{2c}{t}\left(\frac{c}{t} - ct^3\right)$

$= -\frac{2c}{t}\left(\frac{c}{t} - ct^3\right) + \frac{2c}{t}\left(\frac{c}{t} - ct^3\right) = 0$



55 (b)

$$\int_0^{\frac{\pi}{2}} \sin^4 x\,(1 - \sin^2 x)\ dx$$

$$= \int_0^{\frac{\pi}{2}} \sin^4 x\ dx + \int_0^{\frac{\pi}{2}} \sin^6 x\ dx$$

$$= \frac{\pi}{32}$$

TEN MARK QUESTIONS : 10 X 10 = 100

56

<u>SOLUTION:</u>

Let $x, y\ and\ z$ be the number of coins in each category Re.1 , Rs.2 and Rs.5 respectively.

$\therefore x + y + z = 30$

$x + 2y + 5z = 100$

Put $z = k, k \in R$.

$x + y = 30 - k \qquad x + 2y = 100 - 5k,\ k \in R$

$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$

$\Delta_x = 3k - 40; \qquad \Delta_y = 70 - 4k$

By Cramer's rule ,

$x = \frac{\Delta_x}{\Delta} = 3k - 40\ ; \qquad y = \frac{\Delta_y}{\Delta} = 70 - 4k$

The Solution set $(x, y, z) = (\ 3k - 40,\ \ 70 - 4k,\ \ k)$, $14 \le k \le 17$

possible solutions are (2,14,14) , (5,10,15) , (8,6,16) and (11,2,17)

57

$$\vec{a} \times \vec{b} = \vec{i} + \vec{j} - 2\vec{k}$$
$$\vec{c} \times \vec{d} = \vec{i} - 3\vec{j} + \vec{k}$$
$$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = -5\vec{i} - 3\vec{j} - 4\vec{k} \longrightarrow (1)$$
$$[\vec{a} \quad \vec{b} \quad \vec{d}] = -2$$
$$[\vec{a} \quad \vec{b} \quad \vec{d}]\vec{c} = -4\vec{i} - 2\vec{j} - 2\vec{k}$$
$$[\vec{a} \quad \vec{b} \quad \vec{c}] = 1$$
$$[\vec{a} \quad \vec{b} \quad \vec{c}]\vec{d} = \vec{i} + \vec{j} + 2\vec{k}$$
$$[\vec{a} \quad \vec{b} \quad \vec{d}]\vec{c} - [\vec{a} \quad \vec{b} \quad \vec{c}]\vec{d}$$
$$= -5\vec{i} - 3\vec{j} - 4\vec{k} - - \longrightarrow (2)$$

From (1) and (2),

$$\boldsymbol{(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \quad \vec{b} \quad \vec{d}]\vec{c} - [\vec{a} \quad \vec{b} \quad \vec{c}]\vec{d}}$$

58

VECTOR FORM:

$\vec{a} = 2\vec{i} + 2\vec{j} + \vec{k}$, $\vec{u} = 2\vec{i} + 3\vec{j} + 3\vec{k}$ and $\vec{v} = 3\vec{i} + 2\vec{j} + \vec{k}$

The equation of the plane is

$$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$$
$$\boldsymbol{\vec{r} = (2\vec{i} + 2\vec{j} + \vec{k}) + s(2\vec{i} + 3\vec{j} + 3\vec{k}) + t(3\vec{i} + 2\vec{j} + \vec{k})}$$

CARTESIAN FORM:

$(x_1, y_1, z_1) = (2, 2, 1), (l_1, m_1, n_1) = (2, 3, 3)$ and $(l_2, m_2, n_2) = (3, 2, 1)$

The equation of the plane is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$
$$\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 2 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$
$$\boldsymbol{3x - 7y + 5z + 3 = 0}$$

59

The roots of the equation are $p \pm iq$

Let $\alpha = p + iq$ and $\beta = p - iq$;

$\alpha - \beta = 2iq$

$$y = q \cot\theta - p$$
$$y + \alpha = q(\cot\theta + i)$$
$$= q\left[\frac{\cos\theta}{\sin\theta} + i\right] = \frac{q}{\sin\theta}[\cos\theta + i\sin\theta]$$
$$(y + \alpha)^n = \frac{q^n}{\sin^n\theta}(\cos\theta + i\sin\theta)^n$$
$$(y + \alpha)^n = \frac{q^n}{\sin^n\theta}(\cos n\theta + i\sin n\theta)$$

Similarly,

$$(y + \beta)^n = \frac{q^n}{\sin^n\theta}(\cos n\theta - i\sin n\theta)$$
$$\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{q^n}{(2iq)\sin^n\theta}(2i\sin n\theta)$$
$$\boldsymbol{\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{q^{n-1}\sin n\theta}{\sin^n\theta}}$$

60

Consider the parabola is open downward.

Equation of the parabola is $x^2 = -4ay$.

It passes through the point B(3, -2.5)

$3^2 = -4a(-2.5)$

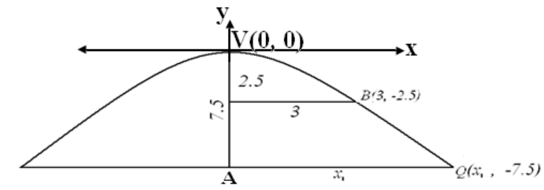
$a = \frac{9}{10}$

$x^2 = -4\frac{9}{10}y - - - - - - - - \langle 1 \rangle$

The point $(x_1, -7.5)$ lies on $\langle 1 \rangle$

$x_1^2 = -4\frac{9}{10}(-7.5) = 27 \Rightarrow x_1 = 3\sqrt{3}$

The water strikes the ground $3\sqrt{3}$ m beyond this vertical line.



61

Let F_1 be the position of the Sun.

Given e = 0.206 and a = 36 million miles.

The shortest distance

$F_1A = CA - CF_1 = a - ae = a(1 - e)$

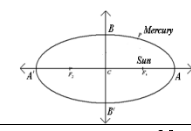
$= 36(1 - 0.206) = 28.584$ million miles

The longest distance $F_1A' = CF_1 + CA'$

$= a + ae = a(1 + e) = 36(1 + 0.206)$

$= 36 \times 1.206$

$= 43.416$ million miles



62

Center of ellipse = centre of hyperbola

Length of LR = 9/2

$\frac{2b^2}{a} = \frac{9}{2}$ ie., $a = 4$

$e = \frac{5}{4}$ $b = 3$

Conjugate axis x =1

Conjugate axis parallel to Y axis

Transverse axis is X axis

$$\frac{(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$$
$$x^2 + y^2 = a^2 - b^2$$
$$(x-1)^2 + (y+1)^2 = 16 - 9$$
$$x^2 + y^2 - 2x + 2y - 5 = 0$$

63

$x = a\cos^3\theta$, $y = a\sin^3\theta$

$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$; $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$;

$\frac{dy}{dx} = -\frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta}$

Slope of the tangent at 'θ'is $-\frac{\sin\theta}{\cos\theta}$

Slope of the normal at 'θ'is $\frac{\cos\theta}{\sin\theta}$

Equation of the normal at 'θ'is

$$y - a\sin^3\theta = \frac{\cos\theta}{\sin\theta}(x - a\cos^3\theta)$$
$$y\sin\theta - a\sin^4\theta = x\cos\theta - a\cos^4\theta$$
$$x\cos\theta - y\sin\theta = a(\cos^4\theta - \sin^4\theta)$$
$$x\cos\theta - y\sin = a(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$$
$$\boldsymbol{x\cos\theta - y\sin\theta = a\cos 2\theta}$$

64

Let x and y be the length and breadth of the printed area. $xy = 384 \Longrightarrow y = \frac{384}{x}$

Dimensions of the poster area are $x + 8$ and $y + 12$.

Area of the poster $A = (x + 8)(y + 12)$

$$A(x) = 12x + 8\left(\frac{384}{x}\right) + 480$$
$$A'(x) = 12 - 8\left(\frac{384}{x^2}\right)$$
$$A''(x) = 16\left(\frac{384}{x^3}\right)$$

For max/min, $A'(x) = 0 \Longrightarrow x = \pm 16$

$\therefore x = 16$ [$\because x$ cannot be $-ve$]

When $x = 16$, $A''(x) = \frac{3}{2} > 0$

When $x = 16$, A is minimum

When $x = 16$, $y = 24$.

Dimensions of the poster are 24 cm and 36 cm.

65

R.H.S. is not a homogeneous function.

Define $f = tanu = \frac{x^3+y^3}{x-y}$

f is a homogeneous function of degree 2.

By Euler's theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f$$

$$x\frac{\partial (tanu)}{\partial x} + y\frac{\partial (tanu)}{\partial y} = 2(tanu)$$

$$xsec^2u\frac{\partial u}{\partial x} + ysec^2u\frac{\partial u}{\partial y} = 2\frac{sinu}{cosu}$$

$$\boldsymbol{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u}$$

66

$x = a(t + sint);\quad y = a(1 + cost)$

Put $y = 0 \Rightarrow 1 + cost = 0 \Rightarrow cost = -1 \Rightarrow t = -\pi,\ \pi$

$\frac{dx}{dt} = a(1 + cost);\qquad \frac{dy}{dt} = -asint$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 2acos\frac{t}{2}$

Surface area $= \int_a^b 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$$= \int_{-\pi}^{\pi} 2\pi a(1 + cost)2acos\frac{t}{2}dt$$

$$= 16\pi a^2\int_0^{\pi} cos^3\frac{t}{2}dt$$

$= 16\pi a^2\int_0^{\frac{\pi}{2}} 2cos^3u\,du$

Put $\frac{t}{2} = u, dt = 2du$

$$= 32\pi a^2\int_0^{\frac{\pi}{2}} 2cos^3u\,du$$

$= 32\pi a^2\left(\frac{2}{3}\right)$ ∵ by reduction formula.

$$= \boldsymbol{\frac{64\pi a^2}{3}\ square\ units}.$$

67

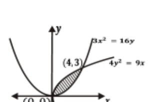
The point of intersection of $4y^2 = 9x$ and $3x^2 = 16y$ are (0,0) and (4, 3).

Required area can be solved about x-axis.

Required area $=\int_0^4 (y_1 - y_2)dx$

y_1 means y from $4y^2 = 9x$

y_2 means y from $3x^2 = 16y$



R.A.$= \int_0^4 \left[\frac{3}{2}\sqrt{x} - \frac{3}{16}x^2\right]dx = \left[x^{\frac{3}{2}} - \frac{x^3}{16}\right]_0^4$

$= 8 - 4 = 4$ square units.

68

Let T be the temperature of the coffee at any time t.

t	T	S
0	100	15
5	60	15
10	?	15

By Newton's law of cooling,

$\frac{dT}{dt} \propto (T - 15)$ since S = 15°C ⇒

$\frac{dT}{dt} = k(T - 15) \Rightarrow T - 15 = ce^{kt}$

When $t = 0,\ T = 100 \Rightarrow 100 - 15 = ce^0 \Rightarrow c = 85$

$\therefore T - 15 = 85e^{kt}$

When t = 5, T = 60

$\Rightarrow 60 - 15 = 85e^{5k} \Rightarrow 45 = 85e^{5k}$

$\Rightarrow e^{5k} = \frac{45}{85}$

When t = 10, $T - 15 = 85e^{10k}$

$T = 15 + 85\left(e^{5k}\right)^2$

$= 15 + 23.82$°C $= 38.82$°C

$T = 38.82$°C

The required temperature after a further interval of 5 minutes is 38.82°C

69

$P(X < 40) = 0.1\ ;\ P(X > 90) = 0.1$

$\therefore P(40 < X < 90)$

$= 1 - [\,P(X < 40) + P(X > 90)\,]$

$= 1 - 0.1 - 0.1 = 0.8$

Out of 800 Students, number of students scored between 40 and 90 is

$800 \times 0.8 = 640$ students

(i) **WITH REPLACEMENT:**

X can take the values as 0, 1, 2, 3.

P(success)$=P(S) = \frac{3}{7}$; P(Failure) $= P(F) = \frac{4}{7}$

$P(X = 0) = \frac{64}{343}\qquad P(X = 1) = \frac{144}{343}$

$P(X = 2) = \frac{108}{343}\qquad P(X = 3) = \frac{27}{343}$

The required probability distribution is

X	0	1	2	3
$P(X = x)$	$\frac{64}{343}$	$\frac{144}{343}$	$\frac{108}{343}$	$\frac{27}{343}$

$$\sum p_i = 1,\ \forall\, p_i \geq 0$$

(ii) **WITHOUT REPLACEMENT**:

$$P(X = 0) = \frac{3C_0 \cdot 4C_3}{7C_3} = \frac{1 \cdot 4}{35} = \frac{4}{35}$$

$$P(X = 1) = \frac{3C_1 \cdot 4C_2}{7C_3} = \frac{3 \cdot 6}{35} = \frac{18}{35}$$

$$P(X = 2) = \frac{3C_2 \cdot 4C_1}{7C_3} = \frac{3 \cdot 4}{35} = \frac{12}{35}$$

$$P(X = 3) = \frac{3C_3 \cdot 4C_0}{7C_3} = \frac{1 \cdot 1}{35} = \frac{1}{35}$$

70 (a)

Let the cubic polynomial be $y = f(x)$.

Since it attains maximum at $x = -1$ and a minimum at $x = 1$.

$\frac{dy}{dx} = 0$ at $x = -1$ and $x = +1$.

$$\frac{dy}{dx} = k(x + 1)(x - 1)$$

$$dy = k(x^2 - 1)dx$$

$\Rightarrow \int dy = k\int(x^2 - 1)\,dx \quad \Rightarrow y = k\left(\frac{x^3}{3} - x\right) + c$

When x= -1 and y = 4 ⇒ $2k + 3c = 12$ ----(1)

When x= +1 and y = 0 ⇒ $-2k + 3c = 0$ -----(2)

Solving (1) and (2) we get, k=3 and c=2

The cubic polynomial is $\boldsymbol{y = x^3 - 3x + 2}$.

70 (b)	Let $M = \{z \in C \ / \ \lvert z\rvert = 1\}$ **(i) Closure axiom:** Let $z_1 , \ z_2 \in M$ $\lvert z_1 z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert = 1 \cdot 1 = 1, \ \forall z_1 , \ z_2 \in M$ $\therefore \ z_1 \ z_2 \in M$ ∴ Closure axiom is true. **(ii) Associative axiom:** Complex multiplication is always associative . ∴ Associative axiom is true. **(iii) Identity axiom:** Let $z \in M$ and $\lvert 1\rvert = 1$ such that $1 \cdot z = z \cdot 1 = 1 \Rightarrow 1 \in M$. ∴ The identity element $1 \in M$. ∴ The identity axiom is true. **(iv) Inverse axiom:** Let z $\in M$, $where \ \lvert z\rvert = 1$. Also $\left\lvert\frac{1}{z}\right\rvert = \frac{1}{\lvert z\rvert} = \frac{1}{1} = 1 \Rightarrow \frac{1}{z} \in M$ such that z$\cdot \frac{1}{z} = \frac{1}{z} \cdot z = 1$ ∴ The inverse element of z is $\frac{1}{z} \in M$ ∴ The inverse axiom is true. ∴ $(\boldsymbol{M},\cdot)$ **is a group.**

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