Padasalai.Net. XIth Half Yearly Examination 17.

XIth Mathematics Key Answer.

Part B

Q.No: 21.
$$\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$$

all the enteries below the diagonal are zero.

$$(x-1)(x-2)(x-3)=0$$

$x-1=0$ | $x-2=0$ | $x-3=0$

$x=1$ | $x=2$ | $x=3$.

Q.No: 22

unit vector in the direction $\vec{i}+\sqrt{3}\vec{j}$ is

$$\frac{\vec{i}+\sqrt{3}\vec{j}}{\sqrt{1+3}} = \frac{\vec{i}+\sqrt{3}\vec{j}}{2}$$

23) The number of ways in which 10 persons can be arranged in a line

$${}^{10}P_{10} = 10!$$

The Number of ways in which 10 persons can be arranged in a circle

$$(10-1)! = 9!$$

24) $a_n = 2+\frac{1}{n}$

$$a_5 = 2+\frac{1}{5} = \frac{11}{5} \quad a_7 = 2+\frac{1}{7} = \frac{15}{7}.$$

25) The Straight line Parallel to $3x+2y-9=0$ is of the form.

$$3x+2y+k=0$$

$(3,-3)$ sub eqn ① $\quad 9-6+k=0$

$$k=-3$$

the eqn of the required straight line $\boxed{3x+2y-3=0}$

26) $\cos^{-1}(1/2) \quad \cos\theta = 1/2$

$$\theta = \pi/3 \quad (0,\pi)$$

Principal value is $\pi/3$.

27) $A\times B = \{(1,a),(1,b)(2,a)(2,b)\}$

$B\times A = \{(a,1),(a,2),(b,1),(b,2)\}$

28) $\lim_{x\to 3} \frac{x^n-3^n}{x-3} = 108 \quad n3^{n-1} = 108$

$$4\cdot 3^3 = 4\times 27 = 108$$

$$\boxed{n=4}$$

29) $\frac{\tan x}{\cos x} = \int \frac{\tan x}{\cos x} dx = \int \tan x \sec x \, dx$

$= \sec x + c$

30) compulsory

$\cos^4 A - \sin^4 A = (\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)$

$= \cos^2 A - \sin^2 A = 1$

$= 1 - \sin^2 A - \sin^2 A = 1 - 2\sin^2 A$

Part III. 3M/s $7 \times 3 = 21$

31) Let P & Q be the points of trisection AB

A(a) P Q B(b)

$AP = PQ = QB = \lambda$

P divides AB in the ratio 1:2

p.v. of $\bar{P} = \bar{OP} = \frac{\bar{b} + 2\bar{a}}{3}$

Q is midpoint of $\bar{PB} = \frac{\bar{a} + 2\bar{b}}{3}$

32) $(100+1)^3$

$= 100^3 + {}^3C_1 (100)^2 + {}^3C_2 (100) + {}^3C_3 (1)$

$= 1000000 + 30000 + 300 + 1$

$= 1030301$.

33) Let G_1, G_2, G_3, G_4, G_5

$a = 576 \quad b = 9$

$G_1 = 576r$, $G_2 = 576r^2$, $G_3 = 576r^3$, $G_4 = 576r^4$, $G_5 = 576r^5$, $\cancel{96}\ 9 = 576r^6$

$r^6 = \frac{9}{576} = \left(\frac{9}{576}\right)^{1/6} = \left(\frac{1}{64}\right)^{\frac{1}{6}}$

$r = \frac{1}{2}$

$G_1 = 576 \times \frac{1}{2} = 288$, $G_2 = 576 \times \frac{1}{4} = 144$

$G_3 = 576 \times \frac{1}{8} = 72$, $G_4 = 576 \times \frac{1}{16} = 36$

$G_5 = 576 \times \frac{1}{32} = 18$

288, 144, 72, 36, 18 are the required G.m.s b/w 576 & 9

34) i) Since P lies on the line

$7(5t-4) - 4(t+1) + 1 = 0$

$35t - 28 - 4t - 4 + 1 = 0$

$31t - 31 = 0 \quad \boxed{t = 1}$

ii) The Co-ordinates of P

$x = 5t - 4$, $y = t + 1$ $\quad t = 1$

$x = 5 - 4$, $y = 1 + 1$

$x = 1$, $y = 2$ $\quad (1, 2)$

35)

$PT = x^2 + y^2 + 2gx + 2fy + c = 0$

$PT^2 = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

$PT^2 = 4 + 9 - 12 - 24 + 12$

$= -11 < 0$

The point (2,3) lies inside the circle.

36) $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{\frac{1}{2}+\frac{1}{7}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)}$

$\alpha=\frac{1}{2}$ $\beta=\frac{1}{3}$

Q NO wrong

$= \dfrac{\boxed{\frac{9}{14}}}{1-\left(\frac{1}{14}\right)} = \dfrac{\frac{9}{14}}{\frac{13}{14}} = \frac{9}{13}$

$\alpha=\frac{1}{2}$ $\beta=\frac{1}{3}$ $= \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} = 1.$

$\alpha+\beta = \pi/4$

37) $(f\circ g)(x) = f(g(x)) = f(x-1)$

$= (x-1)^2+1 = x^2-2x+2$

$(g\circ f)(x) = g(f(x)) = g(x^2+1)$

$= (x^2+1)-1 = x^2$

$(f\circ g)(x) = x^2-2x+2$

$(g\circ f)(x) = x^2$ $\quad f\circ g \neq g\circ f$

38) $x = a\cos^3 t$ $\quad y = a\sin^3 t$

$\dfrac{dy}{dt} = 3a\sin^2 t\cos t$ $\quad \dfrac{dx}{dt} = -3a\cos^2 t\sin t$

$\dfrac{dy}{dx} = -\dfrac{\sin t}{\cos t} = -\tan t$

39) $\int 5x^4 + 3(2x+3)^4 - 6(4-3x)^5$

$5\int x^4\,dx + 3\int (2x+3)^4 - 6\int (4-3x)^5\,dx$

$\cancel{5}\dfrac{x^5}{\cancel{5}} + 3\,\dfrac{(2x+3)^5}{2\times 5} - 6\,\dfrac{(4-3x)^6}{(-3)(6)}$

$= x^5 + \dfrac{3(2x+5)^5}{10} + \dfrac{(4-3x)^6}{3} + c$

40) $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} = 0$

5 m/3

41) (a) $D = \begin{vmatrix} a(a^2+\lambda) & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix}$

multiply R_1, R_2, R_3 a, b, c respectively

$D = \dfrac{1}{abc}\begin{vmatrix} a(a^2+\lambda) & a^2b & a^2c \\ ab^2 & b(b^2+\lambda) & b^2c \\ ac^2 & bc^2 & c(c^2+\lambda) \end{vmatrix}$

Take a, b, c from C_1, C_2, C_3

$D = \dfrac{abc}{abc}\begin{vmatrix} a^2+\lambda & a^2 & a^2 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$

$= \begin{vmatrix} a^2+b^2+c^2+\lambda & a^2+b^2+c^2+\lambda & a^2+b^2+c^2+\lambda \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$

$(a^2+b^2+c^2+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+\lambda & b^2 \\ c^2 & c^2 & c^2+\lambda \end{vmatrix}$

$(a^2+b^2+c^2+\lambda)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & \lambda & 0 \\ c^2 & 0 & \lambda \end{vmatrix}$ $\quad C_2\to C_2-C_1$, $C_3\to C_3-C_1$

$(a^2+b^2+c^2+\lambda)\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \lambda^2(a^2+b^2+c^2+\lambda)$

42) b) $D = \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix}$ $\quad a=0$

$D = \begin{vmatrix} b+c & -c & -b \\ b-c & c & b \\ c-b & c & b \end{vmatrix} = 0$

a is a fact

$= k\,abc$ $\quad a=1\; b=1\; c=1$

$D = 8abc$

42 a) $\overline{AB} = \bar{i} + \bar{j} - 2\bar{k}$ $|AB| = \sqrt{6}$

$\overline{BC} = \bar{i} - 2\bar{j} + \bar{k}$ $|\overline{BC}| = \sqrt{6}$

$\overline{CA} = -2\bar{i} + \bar{j} + \bar{k}$ $|\overline{CA}| = \sqrt{6}$.

$|AB| = |BC| = |\overline{CA}|$

42 b) $\frac{7x-1}{6-5x+x^2} = \frac{7x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

$7x-1 = A(x-2) + B(x-3)$

Put $x=2$ $13 = A(-1)$ $= A = -13$

put $x=3$ $20 = B(1)$ $B = 20$

$= \frac{-13}{x-2} + \frac{20}{x-3}$

43 a) $n=1$ $P(1) = 1 = 1$. $P(1)$ is true

$n=k$ $P(k)$ be true

$1^2+2^2+3^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$

To prove $P(k+1)$ is true

$1^2+2^2+3^2+\dots+k^2\,(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$ is true

$(1^2+2^2+3^2\dots+k^2) + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{(k+1)(k+2)(2k+3)}{6}$ $p(k+1)$ is true

43 b) $T_n = \frac{1}{a+(n-1)d}$ $T_5 = \frac{1}{a+4d} = 12$

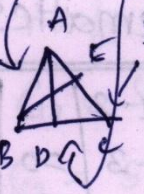
$T_{12} = \frac{1}{a+11d} = 5$ $d = \frac{1}{60}$ $a = \frac{1}{60}$

$T_{15} = \frac{1}{a+14d} = \frac{60}{15} = 4$.

44 a) $x-y-5=0$ — (1) (1)(3) Solve $(2,-3)$

$2x-y-8=0$ — (2)

$3x-y-9=0$ — (3)



$x+2y+k=0$

$(2,-3) \Rightarrow k=4$

$x+2y=-4$ — (5)

Solve (1),(2) $(3,-2)$

$3x-y-9=0$ (perpen)

$x+3y+k=0$

$(3,-2) = x+3y=-3$ — (6)

Sol (5)(6) $O = (-6,1)$

44 b) $x^2+y^2+2gx+2fy+c=0$ — (1)

Sub $(1,0)$ $(0,-1)$ $(0,1)$ in (1)

we get $2g+c=-1$ — (1)

$-2f+c=-1$ — (2)

$2f+c=-1$ — (3)

(2)+(3) $= 2c=-2$ $c=-1$

(1) $f=0$ $g=0$ $x^2+y^2-1=0$

45 a) $A+B=45^\circ$ $\tan(A+B) = \tan 45^\circ$

$\frac{\tan A + \tan B}{1-\tan A \tan B} = 1$

$\tan A + \tan B + \tan A \tan B = 1$

$\div \tan A \tan B$ $= \frac{1}{\tan A \tan B}$

$1 = \cot A \cot B - \cot A - \cot B$

add (1) Both side $A = B = 22\frac{1}{2}$

$\cot 22\frac{1}{2} - 1 = \sqrt{2}$

$= \sqrt{2}+1$

45 b) $\sin 18^\circ$

$\theta = 18^\circ \quad 5\theta = 90^\circ \quad 2\theta = 90^\circ - 3\theta$

$\sin 2\theta = \sin(90^\circ - 3\theta) = \sin 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta = 4\cos^2\theta - 3$

$2\sin\theta = 1 - 4\sin^2\theta$

$4\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = \dfrac{-2 \pm \sqrt{4+16}}{8} = \dfrac{-1+\sqrt{5}}{4}$

46) a) $y = \dfrac{x^2-2x+4}{x^2+2x+4}$

$yx^2 + 2xy + 4y - x^2 + 2x - 4 = 0$

$x^2(y-1) + x(2y+2) + (4y-4) = 0$

x is real

Discrimient ≥ 0 $\quad B^2 - 4AC \geq 0$

$(2y+2)^2 - 4(y-1)(4y-4) \geq 0.$

+ $\quad$ (−) $\quad$ (+)

$-\infty \quad 1/3 \quad 3 \quad \infty$

$-12y^2 + 40y - 12 \geq 0$

$\div \quad 3y^2 - 10y + 3 \leq 0$

$(3y-1)(y-3) \leq 0$

$(3y-1)(y-1)$ is negative

$[\frac{1}{3}, 3] \quad (y-\frac{1}{3})(y-3) \leq 0$

b) i) $I = \int 5x^4 e^{x^5} dx$

$x^5 = u \quad 5x^4 dx = du.$

$I = \int (e^{x^5})(5x^4 dx)$

$\int e^u du = e^u + c = e^{x^5} + c$

ii) $I = \int \dfrac{e^x}{5+e^x} dx \qquad 5+e^x = u$

$e^x dx = du$

$I = \int \dfrac{1}{5+e^x}(e^x dx)$

$\int \frac{1}{u} du$

$I = \log u + c = \log(5+e^x) + c.$

47 a) $y = x^3 - 1 \quad y_1 = 3x^2 \quad y_2 = 6x$

$y_3 = 6$

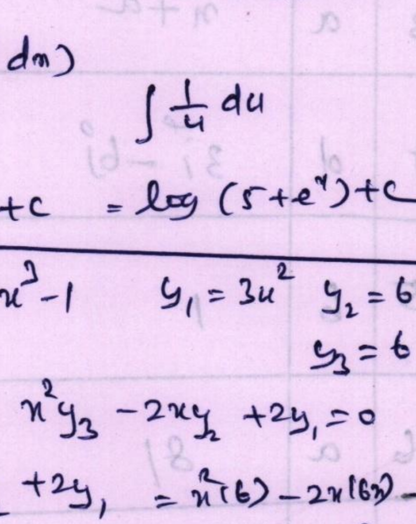
To prove $x^2 y_3 - 2xy_2 + 2y_1 = 0$

$x^2 y_3 - 2xy_2 + 2y_1 = x^2(6) - 2x(6x) + 2(3x^2)$

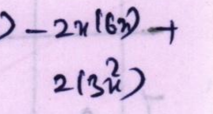
$= 6x^2 - 12x^2 + 6x^2 = 0$ RHS

47 b)

$\theta = 0.$



$y = \dfrac{\sin\theta}{\theta}$



$\dfrac{\sin(-\theta)}{-\theta} = \dfrac{\sin\theta}{\theta} \quad OA = OB = 1$

$\cos\theta = OC \quad \theta = \frac{1}{2}$ on $AB \quad \angle OAD = 90^\circ$

OAD, $AD = \tan\theta \qquad AB = 2\theta$

$AB = 2\sin\theta$

Sum of two tangents $AD = BD = 2\tan$

$\lim_{\theta=0} \cos\theta = 1 \quad \lim_{x\to 0} \dfrac{\sin\theta}{\theta} = 1.$

Part I. Choose.

1	d	3×2	11	b	GP
2	a	$x = 1$	12	d	$(1, 1)$
3	a	$x + a$	13	d	IV
4	d	$3\hat{i} - 6\hat{j}$	14	b	Δ^2
5	b	1	15	c	$[0, \infty)$
6	a	81	16	b	$f: R - [1, \infty) : f(x) = x^2 +$
7	a	6	17	a	$x = 0$
8	a	x	18	a	0
9	c	$2y = 4x + 1$	19	b	$\sin x$
10	a	1	20	b	$\frac{-1}{e^x} + c$

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