A Venila - Kumbakonam - Thanjavur

Reg.No. | | | | | | |

COMMON SECOND REVISION TEST - 2020

STANDARD - XII

Time : 3.00 hrs **Mathematics** **Marks: 90**

Part - I **20 x 1 = 20**

Note: i) All questions are compulsory. ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer:

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$

   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

2. If $x^a y^b = e^m, x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively.

   a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$ c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$

   d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

3. If $|z-2+i| \leq 2$, then the greatest value of $|z|$ is

   a) 1 b) 2 c) 3 d) 5

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   a) $cis\frac{2\pi}{3}$ b) $cis\frac{4\pi}{3}$ c) $-cis\frac{2\pi}{3}$ d) $-cis\frac{4\pi}{3}$

5. The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r(-1)^r x^r$ is

   a) 0 b) n c) $\angle n$ d) r

6. A zero of $x^3 + 64$ is

   a) 0 b) 4 c) 4i d) -4

7. $Sin^{-1}\left[\tan\frac{\pi}{4}\right] - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation.

   a) $x^2 - x - 6 = 0$ b) $x^2 - x - 12 = 0$ c) $x^2 + x - 12 = 0$ d) $x^2 + x - 6 = 0$

8. $Sin(\tan^{-1}x), |x|<1$ is equal to

   a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

9. The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is

a) (4, 7) b) 7, 4) (9, 4) d) (4, 9

10. If the coordinates at one and of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11, 2), the coordinates of the other end are

a) (-5, 2) b) (2, -5) c) (5, -2) d) (-2, 5)

11. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$ then the value $\lambda + \mu$ is

a) 0 b) 1 c) 6 d) 3

12. If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$, $\lambda > 0$ is 1/5, then the value of λ is

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 6 d) 9

13. The minimum value of the function $|3-x| + 9$ is

a) 0 b) 3 c) 6 d) 9

14. The point of inflection of the curve $y = (x - 1)^3$ is

a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)

15. The approximate change in the volume V of a cube of side x metre's caused by increasing the side by 1% is

a) 0.3 xdx m^3 b) 0.03xm^3 c) 0.03x^2 m^3 d) 0.03$x^3$$m^3$

16. If $\int_0^x f(t)\, dt = x + \int_x^1 t f(t)\, dt$, then the value of f(1) is

a) 1/2 b) 2 c) 1 d) 3/4

17. For any value of $n \in Z$ $\int_0^{\pi} e^{\cos^2 x} \cos^3 [(2n+1)x] dx$ is

a) $\pi/2$ b) π c) 0 d) 2

18. Let x have a Bernoulli distribution with mean 0.4, then the variance of (2x - 3) is

a) 0.24 b) 0.48 c) 0.6 d) 0.96

19. If P(x=0) = 1 - P(x = 1). If E(X) = 3 Var (X), then P (x = 0) is

a) 2/3 b) 2/5 c) 1/5 d) 1/3

20. Which one of the following is a binary operation on N?

a) subtraction b) multiplication c) division d) all the above

Part - B

II. Answer any 7 questions. Question No. 30 is compulsory. **7 x 2 = 14**

21. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

22. Simplify $(\sin \pi/6 + i\cos \pi/6)^{18}$

23. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

24. Find the domain of the following functions $\tan^{-1}\left(\sqrt{9-x^2}\right)$

25. Find the equation of the hyperbola in each of the cases given below:

    i) foci $(\pm 2, 0)$, eccentricity = 3/2

26. Find the acute angle between the following lines

    $\vec{r} = (4\hat{i} - \hat{j}) + t(\hat{i} + 2\hat{j} - 2\hat{k})$, $\vec{r} = (\hat{i} - 2\hat{j} + 4\hat{k}) + s(-\hat{i} - 2\hat{j} + 2\hat{k})$

27. Suppose f(x) is a differentiable function for all x with $f'(x) \leq 29$ and f(x) = 17, what is the maximum value of f(7)?

28. Let $U(x,y,z) = xyz$, $x = e^{-t}$, $y = e^{-1}\cos t$, $z = \sin t$, $t \in R$. Find $\frac{du}{dt}$

29. Evaluate $\int_0^1 x^3 e^{-2x} dx$

30. Establilsh the equivalence property: $p \to q \equiv \neg p \vee q$

Part - C

III. Answer any seven questions. Question No.40 is compulsory **7 x 3 = 21**

31. Solve 6x - 7y = 16. 9x - 5y = 35 by using Cramer's rule.

32. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

33. If p and q are the roots of the equation $x^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

34. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}.\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}.\sin\frac{\pi}{9}\right)$

35. Show that the points (2, 3, 4) (-1, 4, 5) and (8, 1, 2) are collinear.

36. Find two positive number whose sum is 12 and their product is maximum.

37. Evaluate $\int_1^2 \frac{x}{(x+1)(x+2)} dx$

38. The mean and variance of a binomial variate X are respectively 2 and 1.5. Find i) P (X = 0) ii) P (X = 1)

39. Verify that $(p \bar{\vee} q) \wedge (p \bar{\vee} \neg q)$ iscontradiction.

40. If w (x, y, z) = $x^3 + y^3 + z^3 + 3xyz$ show that $\frac{\partial^2\omega}{\partial y.\partial z} = \frac{\partial^2\omega}{\partial z.\partial y}$

Part - D

IV. Answer all the questions: **7 x 5 = 35**

41. a) Test for consistency of the following system of linear equations and if possible solve:

$x + 2y - z = 3$; $3x - y + 2z = 1$; $x - 2y + 3z = 3$; $x - y + z + 1 = 0$ **(OR)**

b) Prove that $2\tan^{-1}\left(\frac{1}{5}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

42. a) If z = $(\cos\theta + i\sin\theta)$ show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i \sin n\theta$. **(OR)**

b) Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

43. a) Solve the equation $x^4 - 14x^2 + 45 = 0$ **(OR)**

b) State and prove Appollonius's theorem.

44. a) Sketch the curve $y = f(x) = x^3 - 6x - 9$ **(OR)**

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

45. a) A watermelon has an ellipsoid shape which can be obained by revolving an ellipse with major-axis 20cm and minor - axis 10cm about its major-axis. Find its volume using integration.

(OR)

b) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% annum compounded continuously. How much money will be in his bank account 18 months later?

46. a) A random variable x has the following probability mass function.

X	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find P(x<x<6) ii) $P(2 \leq x<5)$ iii) $P(x \leq 4)$ iv) P(3<x)

(OR)

b) Using truth table, prove DeMorgan's law.

47. a) Find the parametric vector, non-parametric vector and cartesian form of the equations of the plane passing through the points (3, 6, -2), (-1, -2, 6) and (6, -4, -2) **(OR)**

b) Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
