FIRST YEAR HIGHER SECONDARY MODEL EXAMINATION, FEBRUARY 2020

Part III

MATHEMATICS (SCIENCE)

Maximum; 60 Scores

1. i) $A=\{-2,-1,0,1,2,3,4,5,6\}$

   $B=\{1,2,3,4,5\}$ 1

   ii) $A-B=\{-2,-1,0,6\}$ 1

   $B-A=\phi$

   iii) $LHS=(A-B)\cup(B-A)=\{-2,-1,0,6\}$

   $RHS=A-B=\{-2,-1,0,6\}$

   Hence proved. 1

2. A = Set of students like to play cricket.

   B = Set of students like to play football.

   n(A) = 24, n(B) = 16, $n(A\cup B)$ = 35

   $\therefore$ $n(A\cap B)$ = n(A) + n(B) – $n(A\cup B)$

   = 24 + 16 – 35 = 5

   No. of students like to play both cricket and football = 5. 3

3. i) $\sin 315=-\frac{1}{\sqrt{2}}$ 1

   ii) $\operatorname{cosec}\frac{7\pi}{6}=\operatorname{cosec}\left(\pi+\frac{\pi}{6}\right)$

   $=-\operatorname{cosec}\frac{\pi}{6}=-2$

   LHS $=2\times\left(\frac{1}{2}\right)^2+(-2)^2\left(\frac{1}{2}\right)^2$

   $=2\times\frac{1}{4}+4\times\frac{1}{4}=\frac{1}{2}+1$

   $=\frac{3}{2}=RHS$ 2

4. $\frac{n}{2}\left[2(2)+(n-1)3\right]=\frac{n}{2}\left[2(57)+(n-1)2\right]$

$4+3n-3=114+2n-2$

$3n-2n=112-1 \Rightarrow n=111$ 3

5. i) Let the terms be $\frac{a}{r}, a, ar$

$\frac{a}{r}+a+ar=\frac{13}{12}$..........(1)

$\frac{a}{r}.a.ar=-1$

$a^3=-1$

$\therefore a=-1$

$in(1),\ \frac{-1}{r}+-1+(-1)r=\frac{13}{12}$

$-12-12r-12r^2=13r$

$12r^2+25r+12=0$

$12r^2+16r+9r+12=0$

$4r(3r+4)+3(3r+4)=0$

$(3r+4)(4r+3)=0$

$3r=-4\ (or)\ 4r=-3$

$r=\frac{-4}{3}\ (or)\ r=\frac{-3}{4}$ 2

ii) when $a=-1\ and\ r=\frac{-4}{3}$

$t_1=\frac{a}{r}=\frac{-1}{\frac{-4}{3}}=-1\times\frac{3}{-4}=\frac{3}{4}$

$t_2=a=-1$

$t_3=ar=(-1)\times\frac{-4}{3}=\frac{4}{3}$

$when\, a=-1\, and\ r=\frac{-3}{4}$

$t_1=\frac{4}{3}$

$t_2=-1$

$t_3=\frac{3}{4}$

$\therefore GP\ is\ \frac{3}{4},-1,\frac{4}{3}(or)\frac{4}{3},-1,\frac{3}{4}$ 1

6. i) Since the XY plane divides the line joining (2,4,-3) and (-3,5,4) its

$$z = 0$$
$$\therefore m:n = -z_1 : z_2$$
$$= -(-3):4$$
$$= 3:4$$ 2

ii) Co-ordinates of the point $= \left(\frac{3(-3)+4(2)}{3+4}, \frac{3(5)+4(4)}{3+4}, 0\right)$

$$= \left(\frac{-9+8}{7}, \frac{15+16}{7}, 0\right)$$
$$= \left(-\frac{1}{7}, \frac{31}{7}, 0\right)$$ 1

7. $f(x) = \cos x$

$$f(x+h) = \cos(x+h)$$
$$f(x+h) - f(x) = \cos(x+h) - \cos x = -2\sin\left(\frac{2x+h}{2}\right).\sin\left(\frac{h}{2}\right)$$
$$f'(x) = \lim_{h\to 0}\left[-\sin\left(\frac{2x+h}{2}\right)\times\frac{\sin(h/2)}{\left(\frac{h}{2}\right)}\right]$$
$$= -\sin x \times 1 = -\sin x$$ 3

8. $f(x) = \frac{\cos x}{1+\sin x}$

$$f'(x) = \frac{(1+\sin x)(-\sin x) - \cos x(0+\cos x)}{(1+\sin x)^2}$$
$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^2} = -\frac{1}{1+\sin x}$$ 3

9. i) fig. 2 1

ii) fig. 1 1

iii) domain = R

range = {–1,0,1} 2

10. $P(n)=7^n-3^n$ is divisible by 4

$P(1): 7'-3'=4$ is divisible by 4

$P(1)$ is true.

Assume that $P(k)$ be true

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$P(k): 7^k-3^k$ is divisible by 4

$\Rightarrow 7^k-3^k=4d$

$\therefore 7^k=4d+3^k$

To prove that $P(k+1)$ is ture

$P(k+1): 7^{k+1}-3^{k+1}$ is divisible by 4

$$\Rightarrow 7^k.7-3^k.3=7\left[4d+3^k\right]-3.3^k$$

$$=7(4d)+7.3^k-3.3^k=4(7d)+4.3^k$$

$$=4\left[7d+3^k\right], \textit{ is divisible by } 4$$

$\therefore P(k+1)$ is true

$\therefore P(n)$ is true for all $n\in N$

$\therefore P(k+1)$ is true

$\therefore P(n)$ is true for all $n\in N$ 4

11. i) $r=\sqrt{2}$

$$\arg(z)=\frac{3\pi}{4}$$

$$\therefore z=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$ 1

$$\cos\frac{3\pi}{4}=\cos\left(\pi-\frac{\pi}{4}\right)=-\cos\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$$

$$\sin\frac{3\pi}{4}=\sin\left(\pi-\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

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ii) $z=\sqrt{2}\left(-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)=-1+i$

13. $\left(3-\frac{x^3}{6}\right)^8$

$a=3,\ b=-\frac{x^3}{6},\ n=8\,(even)$

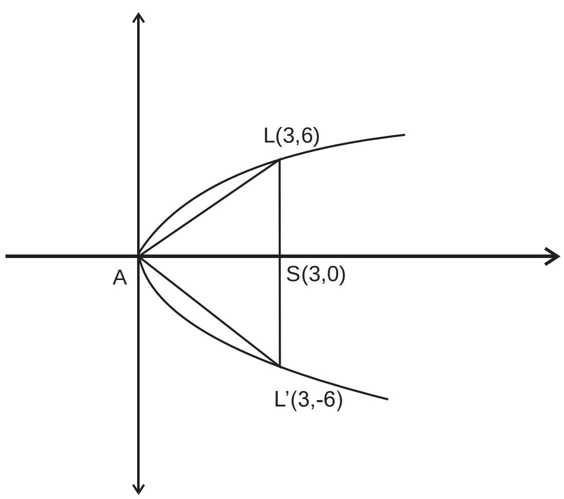
Middle term $=\left(\frac{8}{2}+1\right)^{th}$ termc = 5^{th} term.

$t_{r+1}={}^nC_r\,a^{n-r}.b^r$

$t_{4+1}={}^8C_4\times 3^{8-4}\times\left(\frac{-x^3}{6}\right)^4$ 4

$t_5\ =70\times 3^4\times\frac{x^{12}}{6^4}=70\times 3^4\times\frac{x^{12}}{3^4\times 2^4}=\frac{70}{16}x^{12}=\frac{35}{8}x^{12}$

14. i)



$y^2=12x$

$x=3\Rightarrow y^2=12\times 3=36\Rightarrow y=\pm 6$

$ar\left(\Delta ALL'\right)=\frac{1}{2}\times LL'\times AS=\frac{1}{2}\times 12\times 3=18\,sq.units$ 2

ii) centre (3,0)

$d=LL'=12cm\Rightarrow r=6cm$

$Eqn.\ is\ \left(x-3\right)^2+\left(y-0\right)^2=6^2$

$x^2-6x+9+y^2-36=0$

$x^2+y^2-6x-27=0$

2

15. i) All triangles are equilateral triangle. 1

ii) Let us assume that $\sqrt{7}$ is a rational number.

$\therefore \sqrt{7} = \frac{a}{b}$, where a and b are co-prime. i.e., a and b have no common factors, which implies that

$7b^2 = a^2 \Rightarrow 7$ divides a.

$\therefore$ there exists an integer 'k' such that $a = 7k$

$\therefore 7b^2 = 49k^2 \Rightarrow b^2 = 7k^2 \Rightarrow 7k^2 = b^2 \Rightarrow 7$ divides b.

i.e., 7 divides both a and b, which is contradiction to our assumption that a and b have no other common factors except 1.

$\therefore$ our supposition is wrong.

$\therefore \sqrt{7}$ is an irrational number. 3

16. i) P(all kings) $\frac{{}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \frac{1}{7735}$ 2

ii) P(3 kings) $\frac{{}^4C_3 \times {}^{48}C_4}{{}^{52}C_7} = \frac{9}{1547}$ 2

17. i) $\sin 2x - \sin 4x + \sin 6x = 0$

$$2\sin\left(\frac{6x+2x}{2}\right).\cos\left(\frac{6x-2x}{2}\right) - \sin 4x = 0$$

$$2\sin 4x.\cos 2x - \sin 4x = 0$$

$$\sin 4x\left(2\cos 2x - 1\right) = 0$$

$$If\ \sin 4x = 0 \Rightarrow 4x = n\pi \Rightarrow x = \frac{n\pi}{4}, n \in Z$$

$$If\ \ 2\cos 2x - 1 = 0 \Rightarrow 2\cos 2x = 1$$

$$\therefore \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore 2x = 2n\pi \pm \frac{\pi}{3}$$

$$\therefore x = n\pi \pm \frac{\pi}{6}, n \in Z$$

4

ii) $\frac{a+b}{c} = \frac{2R\sin A + 2R\sin B}{2R\sin C}$

$$=\frac{\sin A+\sin B}{\sin C}=\frac{2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)}{2\sin\frac{C}{2}\cos\frac{C}{2}}$$

$$=\frac{\sin\left(90-\frac{C}{2}\right).\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}.\cos\frac{C}{2}}$$ 2

$$=\frac{\cos\frac{C}{2}.\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}.\cos\frac{C}{2}}=\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$$

18. i) Ans: (a) 1

ii) a) The word PERMUTATION has 12 letters.

No.of permutations if the word starts with 'P' and ends in "S" (P and S are fixed)

$$=\frac{10!}{2!}=1814400$$ 1

b) If vowels are all together;

AEIOU - 1 (5 different letters)

P - 1

R - 1

M - 1

T - 2

N - 1

S - 1

Total - 8

No. of permutations $=\frac{8!}{2!}\times 5!=2419200$ 1

iii) The word INVOLUTE has 4 vowels (I,O,E,U) and 4 consonants (N,V,L and T).

The number of ways of selecting 3 vowels from 4 vowels $={}^4C_3=4$

The number of ways of selecting 2 consonants from 4 consonants $={}^4C_2=6$

Of these $4\times 6=24$ combinations 5 letters (AEIOU) can be arranged in 5! = 120 ways.

$\therefore$ Total no. of words $=4\times 6\times 120=2880$ 3

19. i) Equation of a line $\perp r$ to $x+3y-7=0$ is $3x-y+k=0 ....(1)$

(1) passes through (3,8)

$3(3)-8+k=0$

$1+k=0$

$k=-1$

$in\ (1) \Rightarrow 3x-y-1=0$ is the required line. 2

ii) Foot M of the line from (3,8):

$x+3y-7=0$ .... (1)

$3x-y-1=0$ .... (2)

$(1)+(2)\times 3 \Rightarrow$

$x+3y-7=0$

$9x-3y-3=0$

....

$10x \quad -10=0$

$10x=10$

$\therefore\ x=1$

$in(1) \Rightarrow 1+3y-7=0 \Rightarrow 3y-6=0 \Rightarrow 3y=6 \Rightarrow y=2$

$\therefore$ *Foot M is* $(1,2)$

$Now\ \frac{x+3}{2}=1 \quad \frac{y+8}{2}=2$

$x+3=3 \quad y+8=4$

$x=2-3=-1 \quad y=4-8=-4$

$\therefore$ *image is* $(-1,-4)$ 3

iii) $\perp r$ distance from (3,8) to the line $x+3y-7=0$ is

$$\left|\frac{3+3\times 8-7}{\sqrt{1^2+3^2}}\right| = \left|\frac{3+24-7}{\sqrt{1+9}}\right| = \left|\frac{20}{\sqrt{10}}\right| = \frac{20}{\sqrt{10}}$$

$$= \frac{20}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{20\sqrt{10}}{10} = 2\sqrt{10}\ units.$$ 2

20.

xi	fi	$fixi$	xi^2	$fixi^2$
0 1 2 3 4	1 9 7 5 3	0 9 14 15 12	0 1 4 9 16	0 9 28 45 48
	$N=25$	$\sum fixi$ $=50$		$\sum fixi^2$ $=130$

i) Mean, $\bar{x}=\frac{\sum fixi}{N}=\frac{50}{25}=2$ 1

ii) SD = $\sqrt{\frac{\sum fixi^2}{N}-\left(\frac{\sum fixi}{N}\right)^2}$

$=\sqrt{\frac{130}{25}-\left(\frac{50}{25}\right)^2}$

$=\sqrt{5.2-4}$

$=\sqrt{1.2}=1.09=1.1(approx.)$ 3

iii) For Team A, C.V = $\frac{\sigma}{\bar{x}}\times 100=\frac{1.1}{2}\times 100=55$

For Team B, C.V = $\frac{\sigma}{\bar{x}}\times 100=\frac{1.25}{2}\times 100=62.5$

Since CV of Team A is less than that of Team B, Team A is more consistent. 2

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