Mathematics

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HSST Mathematics

Plus One Science (March 2020)

1. i) 0

ii) B, $\{x : x \in \mathbb{R}, 6 < x < 12\}$

iii) C, B

2. $n(U) = 600$ T – tea
$n(T) = 150$ C – coffee
$n(C) = 225$
$n(T \cap C) = 100$
$n(T' \cap C') = ?$

$$n(T' \cap C') = n((T \cup C)')$$
$$= n(U) - n(T \cup C)$$
$$= 600 - \{150 + 225 - 100\}$$
$$= 600 - 275$$
$$= \underline{325}$$

3. Let $\operatorname{cosec} x = -2$

$\therefore \sin x = \frac{-1}{2}$

$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore$ Solution is,

$$n\pi + (-1)^n \frac{7\pi}{6}, \; n \in \mathbb{Z}$$

4. i) $S_{20} = S_{30}$

$$10(2a + 19d) = 15(2a + 29d)$$
$$2(2a + 19d) = 3(2a + 29d)$$
$$4a + 38d = 6a + 87d$$
$$2a + 49d = 0.$$
$$S_{50} = 25(2a + 49d)$$
$$= \underline{0}$$

option C, 0.

ii) $a = -\frac{3}{4}$

$$r = \frac{3}{16} \div \frac{-3}{4} = \frac{-1}{4}$$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{-\frac{3}{4}}{1 + \frac{1}{4}}$$
$$= \underline{\underline{-\frac{3}{5}}}$$

5. $7 + 77 + 777 + \ldots$ up to n terms

$$= \frac{7}{9}\left[9 + 99 + 999 + \ldots \text{ up to n terms}\right]$$
$$= \frac{7}{9}\left[10 - 1 + 10^2 - 1 + 10^3 - 1 + \ldots \text{ up to n terms}\right]$$
$$= \frac{7}{9}\left[10 + 10^2 + 10^3 \ldots \text{ n terms} - n\right]$$
$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$
$$= \underline{\underline{\frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]}}$$

6. i) $P(3, 4, 5)$
$Q(0, 4, 0)$

distance between two points

$$\sqrt{(3-0)^2 + (4-4)^2 + (5-0)^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$
$$= \underline{\underline{\sqrt{34}}}$$

ii) internally in the ratio $m:n$

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$$

$P(3,4,5)$, $Q(0,4,0)$

ratio $2:3$

ie

$$\left(\frac{2\times0+3\times3}{5}, \frac{2\times4+3\times4}{5}, \frac{2\times0+3\times5}{5}\right)$$

$$\left(\frac{9}{5}, \frac{20}{5}, \frac{15}{5}\right)$$

$$\left(\frac{9}{5}, 4, 3\right)$$

$$\left(\frac{9}{5}, 4, 3\right)$$

(7). $f'(x) = \lim_{h\to0} \dfrac{f(x+h)-f(x)}{h}$

$f(x) = \cos x$.

$$f'(x) = \lim_{h\to0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to0} \frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$$

$$= \lim_{h\to0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= \lim_{h\to0} \left(-\sin\left(\frac{2x+h}{2}\right)\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$$

$$= -\lim_{h\to0} \sin\left(x+\frac{h}{2}\right) \lim_{h\to0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= -\lim_{h\to0} \sin\left(x+\frac{h}{2}\right)$$

$$= \underline{-\sin x}$$

(8) i) $f(x) = 1 + x + x^2 + \cdots + x^{50}$

at $x = 1$

$$f'(1) = 0 + 1 + 2\cdot1 + 3\cdot1^2 + \cdots + 50\cdot1^{49}$$

$$= 1+2+3+\cdots 50$$

$$= \frac{n(n+1)}{2}$$

$$= \frac{50(50+1)}{2}$$

$$= 25\times51$$

$$= \underline{1275}$$

Option B, $\underline{1275}$

ii) $f(x) = \begin{cases} \dfrac{|x|}{x} ; & x \neq 0 \\ 0 ; & \text{if } x=0 \end{cases}$

$$= \begin{cases} -1 ; & x<0 \\ 0 ; & x=0 \\ 1 ; & x>0 \end{cases}$$

$LHL = \lim_{x\to0^-} f(x) = \lim_{x\to0} -1 = \underline{-1}$

$RHL = \lim_{x\to0^+} f(x) = \lim_{x\to0} 1 = \underline{1}$

$LHL \neq RHL$

$\therefore$ limit does not exist at $x=0$

(9) a) $\longrightarrow$ ii

b) $\longrightarrow$ iv

c) $\longrightarrow$ iii

d) $\longrightarrow$ i

(10) $P(n): 7^n - 3^n$ is divisible by 4.

For $n = 1$

$P(1) = 7^1 - 3^1 = 4$

which is divisible by 4.

$\therefore P(1)$ is true.

Assume that $P(k)$ is true.

$P(k): 7^k - 3^k$ is divisible by 4

$\therefore 7^k - 3^k = 4d.$

Now $P(k+1) = 7^{k+1} - 3^{k+1}$

$= 7^k \cdot 7 - 3^k \cdot 3$

$= 7(4d + 3^k) - 3 \cdot 3^k$

$= 28d + 7 \cdot 3^k - 3 \cdot 3^k$

$= 28d + 4 \cdot 3^k$

$= 4(7d + 3^k)$

which is divisible by 4.

$\therefore P(k+1)$ is true.

$\therefore$ The result is true by induction.

(11) i) $|z| = 2 \qquad \theta = \frac{\pi}{3}$

$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 1 + i\sqrt{3}$

ii) $a + ib = \sqrt{1 + i\sqrt{3}}$

$(a+ib)^2 = 1 + i\sqrt{3}$

$a^2 - b^2 + 2abi = 1 + i\sqrt{3}$

$a^2 - b^2 = 1$

$2ab = \sqrt{3}$

$\pm\sqrt{\frac{|z|+a}{2}} \pm i\sqrt{\frac{|z|-a}{2}}$

ie $\pm\sqrt{\frac{2+1}{2}} \pm i\sqrt{\frac{2-1}{2}}$

ie $\pm\sqrt{\frac{3}{2}} \pm i\sqrt{\frac{1}{2}}$

Product is positive.

$\therefore$ The roots are

$\sqrt{\frac{3}{2}} + i\cdot\frac{1}{\sqrt{2}}, \quad -\frac{\sqrt{3}}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

(12) $2x + y = 4$

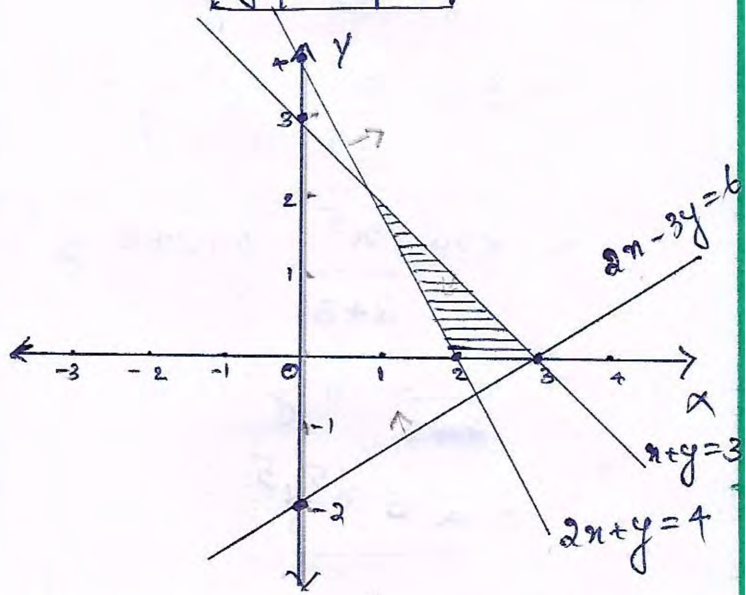
x	0	2
y	4	0

$x + y = 3$

x	0	3
y	3	0

$2x - 3y = 6$

x	0	3
y	-2	0



Solution region is the shaded region in the first quadrant.

(13) i) $\left(x + \frac{1}{x}\right)^6$

$= {}^6C_0\, x^6\left(\frac{1}{x}\right)^0 + {}^6C_1\, x^5\left(\frac{1}{x}\right) + {}^6C_2\, x^4\left(\frac{1}{x}\right)^2$

$+ {}^6C_3\, x^3\left(\frac{1}{x}\right)^3 + {}^6C_4\, x^2\left(\frac{1}{x}\right)^4 + {}^6C_5\, x\left(\frac{1}{x}\right)^5$

$+ {}^6C_6\left(\frac{1}{x}\right)^6$

$= 1 \cdot x^6 + 6 \cdot x^5 \cdot \frac{1}{x} + 15 x^4 \cdot \frac{1}{x^2}$
$+ 20 x^3 \cdot \frac{1}{x^3} + 15 x^2 \frac{1}{x^4} + 6 \cdot x \cdot \frac{1}{x^5}$
$+ 1 \cdot \frac{1}{x^6}$

$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

ii) $\left(\frac{x}{3} + 9y\right)^{10}$

$T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{10-r} (9y)^r$

Middle term is, $\left(\frac{n}{2}+1\right)^{th}$ term

ie 6^{th} term

$T_{5+1} = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5$

$= 252 \frac{x^5}{3^5} \cdot 9^5 \cdot y^5$

$= 252 \frac{x^5}{243} \cdot 59049 \, y^5$

$= \frac{\cancel{252} \times 243}{\cancel{243}} x^5 y^5$

$= 61236 \, x^5 y^5$

(14) The path is a circle.

i) Centre is $A(1,2)$ and

radius $= \sqrt{(3-1)^2 + (5-2)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

Equation of a circle with centre $(1,2)$ and radius $\sqrt{13}$ is

$(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-2)^2 = (\sqrt{13})^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 13$

$x^2 + y^2 - 2x - 4y - 8 = 0$

ii) a) $a = 3 \quad b = 2$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\therefore \frac{x^2}{4} + \frac{y^2}{9} = 1.$

b) $a^2 = b^2 + c^2$

$c = \sqrt{a^2 - b^2}$

$= \sqrt{9-4}$

$= \sqrt{5}$

$\therefore$ focii are, $(0, \sqrt{5})$ and $(0, -\sqrt{5})$

(15) i) contrapositive is,

If a number is not ~~not~~ divisible by 3 then it is not divisible by 9.

ii) p : $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ is rational then there exist two positive integers such that

$\sqrt{5} = \frac{a}{b}$ — (1)

where a and b have no common factors

Squaring $5 = \frac{a^2}{b^2}$

$5b^2 = a^2$ —— (2)

$\Rightarrow$ 5 divides a.

$\therefore$ there exist a number c such that

$a = 5c$

Squaring,

$a^2 = 25c^2$ —— (3)

from (2) and (3)

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow$ 5 divides b.

which contradicts our assumption that a and b have no common factors. $\therefore$ ~~By~~ contradiction $\sqrt{5}$ ~~is~~ irrational.

(18) i) a) P(E or F)

$= P(E \cup F)$

$= P(E) + P(F) - P(E \text{ and } F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$

$= \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

b) P(not E and not F)

$= P(E' \cap F')$

$= P((E \cup F)')$

$= 1 - P(E \cup F)$

$= 1 - \frac{5}{8}$

$= \frac{3}{8}$

ii) a) ~~P(one ma~~

P(one man).

one man out of 2 can be selected in ${}^2C_1$ ways and one woman out of 2 can be selected in ${}^2C_1$ ways.

a) $P(\text{one man}) = \frac{{}^2C_1 \times {}^2C_1}{{}^4C_2} = \frac{2}{3}$

b) Two men can be selected in ${}^2C_2$ ways.

$\therefore P(\text{Two men}) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$

(17)

i) $\tan x = \frac{-5}{12}$ —— (1)

$\cot x = \frac{-12}{5}$ —— (2)

$1 + \tan^2 x = \sec^2 x$

$\sec x = \sqrt{1 + \tan^2 x}$

$= \sqrt{1 + \frac{25}{144}}$

$= \sqrt{\frac{169}{144}}$

$= \frac{13}{12}$.

$\sec x = -\frac{13}{12}$ —— (3)

$\cos x = \frac{-12}{13}$ —— (4)

$\tan x = \frac{\sin x}{\cos x}$

$\sin x = \tan x \cdot \cos x$.

$= \frac{-5}{12} \cdot \frac{-12}{13}$

$\sin x = \frac{5}{13}$ —— (5)

$\operatorname{Cosec} x = \frac{13}{5}$ —— (6)

ii) $\dfrac{\sin x + \cos x}{\sin x - \cos x}$

dividing by ~~tan x~~.

$$\dfrac{\dfrac{\sin x + \cos x}{\cos x}}{\dfrac{\sin x - \cos x}{\cos x}} = \dfrac{\tan x + 1}{\tan x - 1}$$

But $\tan x = \dfrac{3}{4}$

$$\therefore \; = \dfrac{\frac{3}{4}+1}{\frac{3}{4}-1} = \dfrac{\frac{7}{4}}{\frac{-1}{4}} = \underline{\underline{-7}}$$

$$= \dfrac{1}{\cancel{\frac{1}{4}}} = \cancel{2}$$

iii) $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$$= \dfrac{2\sin\left(\frac{5x+3x}{2}\right)\cancel{\cos\left(\frac{5x-3x}{2}\right)}}{2\cos\left(\frac{5x+3x}{2}\right)\cancel{\cos\left(\frac{5x-3x}{2}\right)}}$$

$$= \dfrac{\sin 4x}{\cos 4x} = \underline{\underline{\tan 4x}}$$

18. i) $\boxed{AUE}$ DGHTR

No. of permutations, when all the vowels are occur together

$= 6! \times 3!$

$= 4320.$

ii) a) Four cards are of the same suits.

$\cancel{4 \times {}^{13}C_4}$ $4 \times {}^{13}C_4 = 2860$

b) Four cards belongs to different suits.

$${}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$$

$$= 13^4$$

c) Two are red cards and two are black cards

$${}^{26}C_2 \times {}^{26}C_2 = 105625$$

19. i) midpoint of AB.

$$\left(\frac{2+4}{2}, \frac{3+1}{2}\right)$$

ie $(3, 2)$.

Slope of $AB = \dfrac{1-3}{4-2} = \dfrac{-2}{2} = -1$

Slope of the line $\perp^r$ to AB

$$= \frac{-1}{m} = \underline{\underline{1}}$$

$\therefore$ Equation of the line with slope 1 and passing through $(3, 2)$ is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 1(x-3)$$
$$y - 2 = x - 3$$
$$x - y - 1 = 0.$$

ii) $x - y - 1 = 0$ is the equation of a line passing through the midpoint of AB and $\perp^r$ to AB.

$\therefore$ ~~Put~~ $y = 0$.

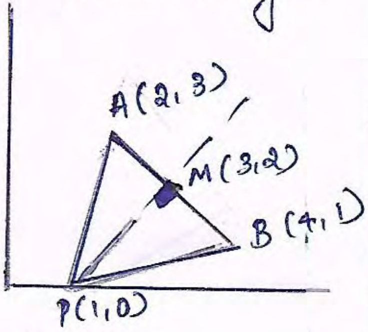
then $x = 1$

$\therefore$ the point C is $(1, 0)$.

iii) Area of ΔABC

It is an isosceles triangle.

$\therefore PA = PB$



Area of ΔPMB

$= \frac{1}{2} bh$

$\therefore$ Area of $\Delta ABC = bh$

$= (MB) \times (PM)$

$PB = \sqrt{(1-4)^2+(0-1)^2} = \sqrt{9+1} = \sqrt{10}$

$PM = \sqrt{(3-1)^2+(2-0)^2} = \sqrt{4+4} = \sqrt{8}$

$MB = \sqrt{(4-3)^2+(2-1)^2} = \sqrt{2}$.

$\therefore$ Area of $\Delta ABC = bh$

$= \sqrt{2} \times \sqrt{8}$

$= \sqrt{16}$

$= 4$ sq. units

(20)

class	f_i	x_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
30-40	3	35	105	-27	729	2187
40-50	7	45	315	-17	289	2023
50-60	12	55	660	-7	49	588
60-70	15	65	975	3	9	135
70-80	8	75	600	13	169	1352
80-90	3	85	255	23	529	1587
90-100	2	95	190	33	1089	2178
	50		3100			10050

i) $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \frac{3100}{50} = 62$

ii) Variance, $\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{N}$

$= \frac{10050}{50}$

$= 201$

iii) coefficient of variation

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{14.177}{62} \times 100$

$= 22.866$

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