MATHEMATICS SAMPLE PAPER

ANSWER AND SOLUTIONS

SECTION-A

1. Option (1)

$0 \le r < 3$

2. Option (4)

More than 3

3. Option (3)

$\cos\theta \frac{\sqrt{b^2 - a^2}}{b}$

4. Option (2)

$a_n = 3.5$

5. Option (4)

4 : 1

6. Option (3)

Trigonometric ratios of the angles.

7. Option (3)

140°

8. Option (3)

10

9. Option (2)

2.1

10. Option (2)

$\frac{5}{2}$

11. 360 cm^2

12. Median

13. 2 and –2

14. 4

15. 0

OR

–1

16. For equal roots :

$D = 0 \Rightarrow b^2 - 4ac = 0$

$(-3k)^2 - 4 \times 9 \times k = 0$

$\Rightarrow 9k^2 - 36k = 0$

$\Rightarrow 9k(k - 4) = 0 \quad \Rightarrow \quad k = 0$ or $k = 4$

OR

For roots to be real and equal $b^2 - 4ac = 0$

$\Rightarrow (5k)^2 - 4 \times 1 \times 16 = 0$

$\Rightarrow 25k^2 - 64 = 0$

$\Rightarrow k = \pm\frac{8}{5}$

17. Length of diagonal = AB

$= \sqrt{(5-0)^2 + (0-3)^2} = \sqrt{25+9} = \sqrt{34}$

18. $\Delta ABC \sim \Delta QRP$

$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta QRP} = \frac{BC^2}{RP^2} \Rightarrow \frac{9}{4} = \frac{(15)^2}{RP^2}$

$\therefore \frac{3}{2} = \frac{15}{RP}$

$\Rightarrow RP = 10$ cm

19. $\sin^2 A = 2\sin A$.

$\Rightarrow \sin^2 A - 2\sin A = 0 \Rightarrow \sin A(\sin A - 2) = 0$

$\Rightarrow$ either $\sin A = 0$ or $\sin A - 2 = 0$.

$\Rightarrow A = 0°$ [sin A = 2, Not Possible]

$\therefore$ Value of $\angle A = 0°$

20. Let first term is a

$a_7 = 4$

$a + 6d = 4$

$a + 6(-4) = 4$

$a = 4 + 24$

$a = 28$

Thus, first term is 28.

SECTION-B

21. $\frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{1}$

$= \frac{1+3+1}{1}$

$= 5$

22. Here, the total number of possible outcomes = 5.

(i) Since, there is only one queen

$\therefore$ Favourable number of elementary events = 1

$\therefore$ Probability of getting the card of queen $= \frac{1}{5}$.

(ii) Now, the total number of possible outcomes = 4.

Since, there is only one ace

$\therefore$ Favourable number of elementary events = 1

$\therefore$ Probability of getting an ace card $= \frac{1}{4}$.

23. HCF × LCM = Product of two numbers

9 × 360 = 45 × 2nd number

2nd number = 72

OR

Let us assume, to the contrary that $7-\sqrt{5}$ is rational

$7-\sqrt{5} = \frac{p}{q}$, where p & q are co-prime and $q \neq 0$

$\Rightarrow \sqrt{5} = \frac{7q-p}{q}$

$\frac{7q-p}{q}$ is rational = $\sqrt{5}$ is rational which is a contradiction

Hence $7-\sqrt{5}$ is irrational

24. 20^{th} term from the end = ℓ – (n – 1)d

= 253 – 19 × 5

= 158

OR

$7a_7 = 11a_{11}$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$

$\Rightarrow a + 17d = 0$

$\Rightarrow a_{18} = 0$

25. $x = \frac{6-6}{5} = 0$

$y = \frac{-10+15}{5} = 1$

Hence, coordinates of point P(0, 1)

26. Total number of cards = 49

Total number of outcomes = 49

(i) A multiple of 5

Favourable outcomes : 5, 10, 15, 20, 25, 30, 35, 40, 45

Number of favourable outcomes = 9

$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{9}{49}$

(ii) A perfect square

Favourable outcomes : 1, 4, 9, 16, 25, 36, 49

Number of favourable outcomes = 7

$\text{Probability (E)} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{7}{49} = \frac{1}{7}$

SECTION-C

27. LHS = sinθ(1 + tanθ) + cosθ(1 + cotθ)

$= \sin\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta + \cos\theta\frac{\cos\theta}{\sin\theta}$

$= (\sin\theta + \cos\theta) + \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta}$

$= (\sin\theta + \cos\theta) + \frac{\sin^2\theta + \cos^3\theta}{\sin\theta\cos\theta}$

$= (\sin\theta + \cos\theta)\left[1+\dfrac{\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$= (\sin\theta + \cos\theta)\left[1+\dfrac{1}{\sin\theta\cos\theta}-1\right]$

$= \sin\theta + \cos\theta \times \dfrac{1}{\sin\theta\cos\theta}$

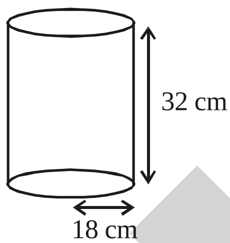
$= \dfrac{1}{\cos\theta}+\dfrac{1}{\sin\theta}$

$= \sec\theta + \text{cosec}\theta$

= RHS

Hence proved

28. Volume of cylindrical bucket = Volume of conical heap of sand.



$\pi r^2 h = \dfrac{1}{3}\pi R^2 \times 24$

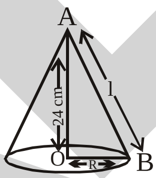
$\pi \times 18 \times 18 \times 32$

$= \dfrac{1}{3}\pi R^2 \times 24$

$R^2 = \dfrac{18\times18\times32\times3}{24} = \dfrac{18\times18\times32\times3}{24}$

R = 36 cm

In the ΔAOB of conical heap.



$AB^2 = AO^2 + OB^2$

$\ell^2 = 24^2 + 36^2$

$\ell = \sqrt{576+1296}$

$= \sqrt{1872}$

ℓ = 43.27 cm = 43.3 cm

OR

Number of balls $= \dfrac{\text{Volume of solid sphere}}{\text{Volume of 1 spherical ball}}$

$= \dfrac{\frac{4}{3}\times\pi\times3\times3\times3}{\frac{4}{3}\times\pi\times0.3\times0.3\times0.3}$

= 1000

29. We know that an odd positive integer n is of the form (4q + 1) or (4q + 3) for some integer q.

Case-I When n = (4q + 1)

In this case $n^2 - 1 = (4q + 1)^2 - 1$

$= 16q^2 + 8q = 8q(2q + 1)$

which is clearly divisible by 8.

Case-II When n = (4q + 3)

$n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8$

$= 8(2q^2 + 3q + 1)$

which is clearly divsible by 8.

Hence, it n is an odd positive integer then $(n^2 - 1)$ is divisible by 8.

30. Since two zeros are $\sqrt{\dfrac{5}{3}}$ and $-\sqrt{\dfrac{5}{3}}$,

so $\left(x-\sqrt{\dfrac{5}{3}}\right)\left(x+\sqrt{\dfrac{5}{3}}\right) = x^2 - \dfrac{5}{3}$ is a factor of the given polynomial.

Now, we divide the given polynomial by $\left(x^2-\dfrac{5}{3}\right)$ to obtain other zeros.

$$\begin{array}{r|l} & 3x^2+6x+3 \\ \hline x^2-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\ & 3x^4 \quad\quad -5x^2 \\ & - \quad\quad\quad + \\ \hline & 6x^3+3x^2-10x \\ & 6x^3 \quad\quad -10x \\ & - \quad\quad\quad + \\ \hline & 3x^2-5 \\ & 3x^2-5 \\ & - \quad + \\ \hline & 0 \end{array}$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2-\dfrac{5}{3}\right)$

$(3x^2 + 6x + 3)$

Now, $3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x + 1)^2$

$= 3(x + 1)(x + 1)$

So its zeros are –1, –1,

Thus, all the zeros of given polynomial are

$\sqrt{5/3}, -\sqrt{5/3}$, –1 and –1.

31. Let the numerator be x and denominator be y.

$\therefore$ Fraction = $\frac{x}{y}$

Now, according to question,

$\frac{x-1}{y}=\frac{1}{3}$ $\Rightarrow$ 3x – 3 = y

$\therefore$ 3x – y = 3 ....(i)

and $\frac{x}{y+8}=\frac{1}{4}$ $\Rightarrow$ 4x = y + 8

$\therefore$ 4x – y = 8 ....(ii)

Now, subtracting equation (ii) from (i), we have

$$3x - y = 3$$

$$\begin{array}{r} 4x - y = 8 \\ - \quad + \quad - \\ \hline - x = -5 \end{array}$$

x = 5

Putting the value of x in equation (i), we have

3 × 5 – y = 3 $\Rightarrow$ 15 – y = 3 $\Rightarrow$ 15 – 3 = y

$\therefore$ y = 12

Hence, the required fraction is $\frac{5}{12}$.

OR

Let the speed of car at A be x km/h

And the speed of car at B be y km/h

Case 1 8x – 8y = 80

x – y = 10

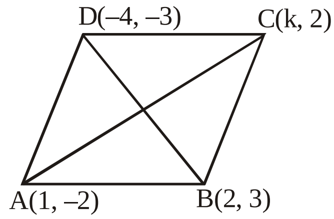
Case 2 $\frac{4}{3}$x + $\frac{4}{3}$y = 80

x + y = 60

On solving x = 35 and y = 25

Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively

32.



Diagonals of parallelogram bisect each other

$\Rightarrow$ midpoint of AC = midpoint of BD

$$\Rightarrow \left(\frac{1+k}{2}, \frac{-2+2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$$

$$\Rightarrow \frac{1+k}{2} = \frac{-2}{2}$$

$\Rightarrow$ k = –3

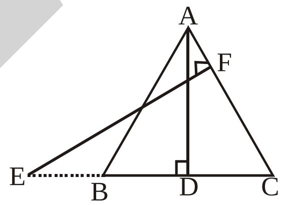
33. 200 – 250 is the modal class

$$\text{Mode} = \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 200 + \frac{12-5}{24-5-2} \times 50$$

= 200 + 20.59 = `220.59

34.



In ΔABD and ΔCEF

AB = AC (Given)

$\Rightarrow \angle$ABC = $\angle$ACB

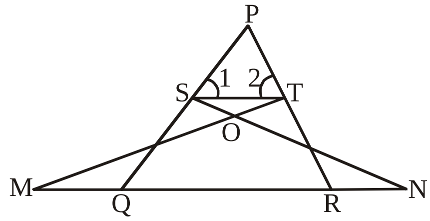
(Equal sides have equal oppposite angles)

$\angle$ABD = $\angle$ECF

$\angle$ADB = $\angle$EFC [Each 90°]

So, ΔABD ~ ΔCEF (AA – Similarity)

OR



$\angle$1 = $\angle$2

$\Rightarrow$ PT = PS ... (1)

$\Delta NSQ \cong \Delta MTR$

$\Rightarrow \angle NQS = \angle MRT$

$\Rightarrow \angle PQR = \angle PRQ$

$\Rightarrow PR = PQ$ ... (2)

From (1) and (2)

$$\frac{PT}{PR} = \frac{PS}{PQ}$$

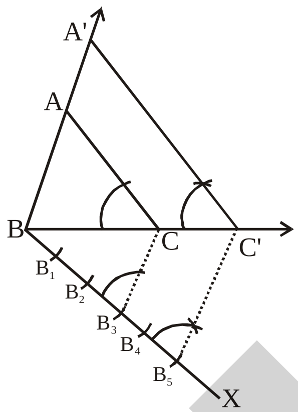
Also, $\angle TPS = \angle RPQ$ (common)

$\Rightarrow \Delta PTS \sim \Delta PRQ$ (by SAS similarity criteria)

SECTION-D

35. Steps of Construction :

Step I : Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.



Step II : From B cut off 5 arcs B_1, B_2, B_3, B_4 and B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

Step III : Join B_3 to C and draw a line through B_5 parallel to B_3C, intersecting the extended line segment BC at C'.

Step IV : Draw a line through C' parallel to CA intersecting the extended line segment BA at A' (see figure). Then A'BC' is the required triangle.

36. $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2a + 29d] \Rightarrow S_{30} = 30a + 435d$ ...(i)

$\Rightarrow S_{20} = \frac{20}{2}[2a + 19d] \Rightarrow S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \Rightarrow S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30}$

[From (i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

OR

Sum of first seven terms,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2a + (7-1)d] = \frac{7}{2}[2a + 6d]$

$\Rightarrow 63 = 7a + 21d$

$\Rightarrow a = \frac{63 - 21d}{7}$ ... (1)

$\Rightarrow S_{14} = \frac{14}{2}[2a + 13d]$

$\Rightarrow S_{14} = 7[2a + 13d] = 14a + 91d$

But ATQ,

$S_{1-7} + S_{8-14} = S_{14}$

$63 + 161 = 14a + 91d$

$\Rightarrow 224 = 14a + 91d$

$2a + 13d = 32$

$2\left(\frac{63 - 21d}{7}\right) + 13d = 32$ (from 1)

$\Rightarrow 126 - 42d + 91d = 224$

$\Rightarrow 49d = 98$

$\Rightarrow d = 2$

$\Rightarrow a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = 3$

$\Rightarrow a_{28} = a + 27d = 3 + 27 \times 2$

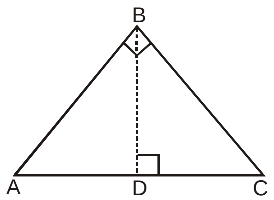
$\Rightarrow a_{28} = 3 + 54 = 57$

37. In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given : A ΔABC in which $\angle B = 90°$.

To prove : $AC^2 = AB^2 + BC^2$.

Construction : From B, Draw $BD \perp AC$.



Proof :

In ΔADB and ΔABC, we have :

$\angle BAD = \angle CAB = \angle A$ (Common)

$\angle ADB = \angle ABC$ (Each = 90°)

$\therefore \Delta ADB \sim \Delta ABC$ (By AA axiom of similarity)

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ (Corr. sides of similar Δs are proportional)

$\Rightarrow AB^2 = AD \times AC$ .....(1)

In ΔCDB and ΔCBA, we have :

$\angle CDB = \angle CBA$ (Each = 90°)

$\angle BCD = \angle ACB = \angle C$ (Common)

$\therefore \Delta CDB \sim \Delta CBA$ (By AA axiom of similarity)

$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ (Corr. sides of similar Δs are proportional)

$\Rightarrow BC^2 = DC \times AC$ .....(2)

Adding (1) and (2), we get

$AB^2 + BC^2 = AD \times AC + DC \times AC$

$= (AD + DC) \times AC = AC^2$ ($\because AD + DC = AC$)

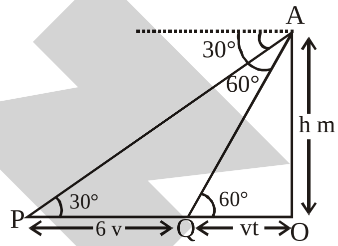
Hence, $AB^2 + BC^2 = AC^2$.

38. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.

After 6 seconds, the car reaches to Q such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

$PQ = 6v$ ($\because$ Distance = speed × time)

and let the car take t seconds to reach the tower OA from Q (Figure). Then OQ = vt metres.



Now, in ΔAQO we have

$\tan 60° = \frac{OA}{QO}$

$\Rightarrow \sqrt{3} = \frac{h}{vt} \Rightarrow h = \sqrt{3}\ vt$ ....(i)

Now, in ΔAPO, we have

$\tan 30° = \frac{OA}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \Rightarrow \sqrt{3}h = 6v + vt$ ....(ii)

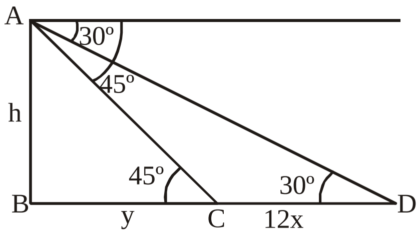
Now, substituting the value of h from (i) and into (ii), we have

$\sqrt{3} \times \sqrt{3}\ vt = 6v + vt$

$\Rightarrow 3vt = 6v + vt \Rightarrow 2vt = 6v \Rightarrow t = \frac{6v}{2v} = 3$

Hence, the car will reach the tower from Q in 3 seconds.

OR



Let the speed of car be x m/ minutes

In ΔABC

$\frac{h}{y} = \tan 45^\circ$

$\Rightarrow h = y$

In ΔABD

$\frac{h}{y+12x} = \tan 30^\circ$

$\Rightarrow h\sqrt{3} = y + 12x$

$\Rightarrow y\sqrt{3} - y = 12x$

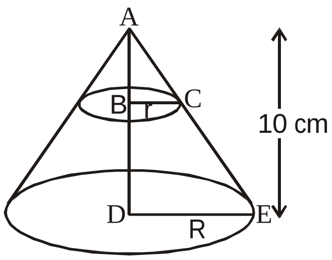
$\Rightarrow y = \frac{12x}{\sqrt{3}-1} = \frac{12x(\sqrt{3}+1)}{2}$

$\Rightarrow y = 6x(\sqrt{3} + 1)$

Time taken from C to B = $6(\sqrt{3} + 1)$ minutes

39. Let BC = r cm , DE = R cm and height of cone h = 10 cm

Also, $\Delta ABC \sim \Delta ADE$



$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE} = \frac{1}{2}$

i.e., $BC = \frac{1}{2}DE = \frac{1}{2} \times R$ or $r = \frac{R}{2}$

Now, $\frac{\text{Volume of cone}}{\text{Volume of the frustum}}$

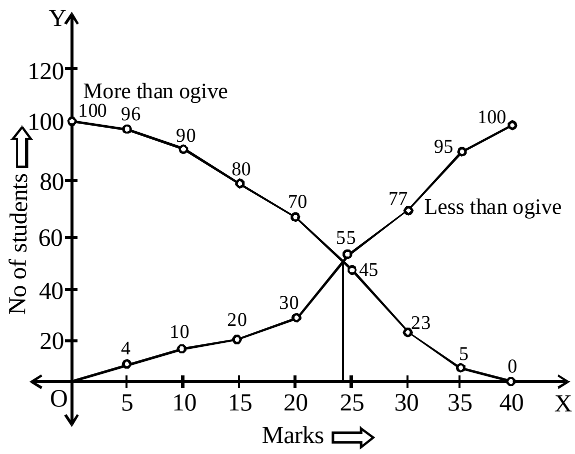
$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi \frac{h}{2}\left[R^2 + r^2 + rR\right]} = \frac{R^2}{4\left[R^2 + \frac{R^2}{4} + \frac{R^2}{2}\right]}$$

$$= \frac{1}{4.\frac{7}{4}} = \frac{1}{7}$$

$\therefore$ The requried ratio = 1 : 7

40.

Marks	Cumulative Frequency	Marks	Cumulative Frequency
Less than 5	4	More than 0	100
Less than 10	10	More than 5	96
Less than 15	20	More than 10	90
Less than 20	30	More than 15	80
Less than 25	55	More than 20	70
Less than 30	77	More than 25	45
Less than 35	95	More than 30	23
Less than 40	100	More than 35	5



Hence, median marks = 24

OR

Class Interval	Frequency	cf
0 – 100	2	2
100 – 200	5	7
200 – 300	x	7 + x
300 – 400	12	19 + x
400 – 500	17	36 + x
500 – 600	20	56 + x
600 – 700	y	56 + x + y
700 – 800	9	65 + x + y
800 – 900	7	72 + x + y
900 – 1000	4	76 + x + y

N = 100

$\Rightarrow 76 + x + y = 100$

$\Rightarrow x + y = 24$ ...(i)

Median = 525

$\Rightarrow$ 500 – 600 is median class

$$\text{Median} = \ell + \frac{\frac{n}{2} - cf}{f} \times h$$

$$\Rightarrow 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100 = 525$$

$\Rightarrow (14 - x) \times 5 = 25$

$\Rightarrow x = 9$

$\Rightarrow$ from (1), y = 15