SET-1

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 question of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted

Section - A

Q1-Q10 are multiple choice type questions. Select the correct option.

1. Let A be a square matrix of order 3 × 3 then |KA| equal to which of the following?

(A) 3K|A| (B) $K^3|A|$ (C) K|A| (D) $K^2|A|$

2. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then $A^2 - 5A + 7I$ is :-

(A) Diagonal matrix (B) Identity matrix (C) Zero matrix (D) None of these

3. If $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$, then the magnitude of vector $3\vec{a} - 2\vec{b}$ is :

(A) 18 (B) 17 (C) $\sqrt{326}$ (D) 19

4. Three persons A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2, respectively. The probability of two hits is

(A) 0.024 (B) 0.188 (C) 0.336 (D) 0.452

5. Consider the following LPP :

Maximise : Z = 12 x + 10 y

Subject to : $4x + 3y \le 480$

$2x + 3y \le 360$

Value of Z will be maximum at

(A) (120, 0) (B) (60, 80) (C) (100, 80) (D) (60, 100)

6. The value of $\cot^{-1}[2\tan(\tan^{-1}x + \tan^{-1}x^3)]$ is

(A) $\frac{\pi}{2} + 2\tan^{-1} x$ (B) $\frac{\pi}{2} - \tan^{-1} x$ (C) $\frac{\pi}{2} + \tan^{-1} x$ (D) $\frac{\pi}{2} - 2\tan^{-1} x$

7. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is :

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

8. $\int \frac{x+3}{(x+4)^2} e^x \, dx$ is equal to

(A) $e^x\left(\frac{1}{x+4}\right)+C$ (B) $e^{-x}\left(\frac{1}{x+4}\right)+C$ (C) $e^{-x}\left(\frac{1}{x-4}\right)+C$ (D) $e^{2x}\left(\frac{1}{x-4}\right)+C$

9. Two lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x}{5}=\frac{y}{10}=\frac{z}{15}$ are mutually :

(A) Perpendicular (B) Skew (C) Intersect (D) Parallel

10. The equation of plane whose intercepts on the co-ordinate axes are –4, 2, 3 is :

(A) $\frac{x}{4}+\frac{y}{2}+\frac{z}{3}=1$ (B) $3x-6y-4z=-12$ (C) $-\frac{x}{4}-\frac{y}{2}+\frac{z}{3}=1$ (D) $-\frac{x}{4}+\frac{y}{2}+\frac{z}{3}+1=0$

(Q11-Q15) Fill in the blanks.

11. If f and g are two invertible functions such that their composite function gof be defined, then

$(gof)^{-1}$ = _____

12. Let $f(x)=\begin{cases} \frac{1-\cos 4x}{x^2} & , x<0 \\ a & , x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , x>0 \end{cases}$ then a = …….. for which 'f' continuous at x = 0 ?

13. If A is a skew-symmetric matrix, then A^2 is a …….. matrix.

14. The equation of normal to the curve y = tan x at (0, 0) is .....

OR

The radius of circular plate is increasing at the rate of 0.2 cm/s. When r = 10, then the rate of change of the area of plate is ……..

15. The direction cosines of the vector $6\hat{i}+2\hat{j}-3\hat{k}$ is ……..

OR

Direction ratios of two parallel lines will be ……...

(Q16-Q20) Answer the following questions.

16. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

17. If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$, find the value of a.

18. Evaluate $\int \sqrt{x}.\log x\,dx$

OR

Find : $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\,dx$

19. Evaluate $\int \frac{\cos x - \sin x}{\sqrt{1+\sin 2x}}dx$

20. Find the differential equation of family of concentric circles with centre (1,2).

Section - B

21. Solve the following equation : $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

OR

If the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 15} given by R = {(a,b) : a,b ∈ Z, |a – b| is multiple of 5} is an equivalence relation, then find the equivalence class [2].

22. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

23. Find the angle of intersection between the curve $x^2 = 32y$ and $y^2 = 4x$ at point (16, 8)

24. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

OR

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

25. Find the value of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other . Also find the equations of line passing through a point (3,2 –4) and parallel to line l_1.

26. A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calcualte the probability that there are exactly 2 girls on the committee

Section - C

27. Consider $f : R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

28. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

OR

If $x = a(\cos 2t + 2t \sin 2t)$ and $y = a(\sin 2t - 2t \cos 2t)$, then find the $\frac{d^2y}{dx^2}$.

29. Solve the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

30. Evaluate : $\int_0^{\pi/4}\left(\frac{\sin x + \cos x}{3 + \sin 2x}\right)dx$

31. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

OR

In a bulb factory, machines. A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulb produced respectively by A, B and C are found to the defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A.

32. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically.

Section - D

33. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = O$ find k.

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Hence using A^{-1} solve the system of equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

34. Using integration, find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$.

35. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find its dimensions in order that the area may be maximum.

OR

An Open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

36. From the point P(1, 2, 4), perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the equation, the length and the coordinates of the foot of the perpendicular.