PRE BOARD -1 Examination - 2019-20
Class – XII
Mathematics - 041

Max. Marks: 80

General Instructions: Time: 3 hrs

1. All questions are compulsory.
2. The question paper consists of 36 questions divided into four sections A, B, C and D.
3. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
4. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions on 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted.

SECTION-A

Questions 1 to 20 carry one mark each.

Q1 - Q10 are multiple choice type questions. Select the correct option

1. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then the value of k is [1]

(A) 0 (B) A (C) 2 (D) -2

2. The number of all possible matrices of order 2×2 with each entry 1, 2 or 3 is [1]

(A) 9 (B) 81 (C) 4 (D) 12

3. A vector equally inclined to axes is [1]

(A) $\hat{\imath} + \hat{\jmath} + \hat{k}$ (B) $\hat{\imath} - \hat{\jmath} + \hat{k}$ (C) $\hat{\imath} - \hat{\jmath} - \hat{k}$ (D) $-\hat{\imath} + \hat{\jmath} - \hat{k}$

4. Let A and B be two given events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A'/B')$ is [1]

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{3}{8}$ (D) $\frac{6}{7}$

5. Distance of plane $\vec{r}\,.\left(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) + 2 = 0,$ from origin is [1]

(A) 2 (B) 14 (C) $\frac{2}{7}$ (D) $-\frac{2}{7}$

6. If $3\sin^{-1}x + \cos^{-1}x = \pi,$ then x is equal to [1]

(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) -1 (D) $\frac{1}{2}$

7. Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is [1]

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$ (C) $\frac{1}{5}$ (D) $\frac{2}{5}$

8. If $\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{3}\sin^{-1}(ax) + C$, then the value of $'a'$ is [1]

(A) 2 (B) 4 (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

OR

$\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\,dx, \quad \frac{3\pi}{4} < x < \frac{7\pi}{4}$ is equal to

(A) $\log|\sin x + \cos x|$ (B) x (C) $\log|x|$ (D) $-x$

9. A line makes angle α, β, γ with $x - axis, y - axis$ and $z - axis$ respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to [1]

(A) 2 (B) 1 (C) -2 (D) -1

10. Distance between planes $\vec{r}.(2\hat{\imath} + \hat{\jmath} - 2\hat{k}) + 5 = 0$ and $\vec{r}.(6\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 2 = 0$ is [1]

(A) $\frac{9}{13}$ (B) $\frac{15}{4}$ (C) $\frac{13}{9}$ (D) $\frac{1}{13}$

(Q11 - Q15) Fill in the blanks

11. If f is an invertible function defined as $f(x) = \frac{3x-4}{5}$, then the $f^{-1}(x)$ = ____________ [1]

12. If $y = 2^{\sqrt{x}}$, then $\frac{dy}{dx}$ is ___________. [1]

OR

If $y = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\right)$, then $\frac{dy}{dx}$ is equal to _____________.

13. If $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ then A^{16} is ___________ matrix. [1]

14. The absolute maximum value of $y = x^3 - 3x + 2$ in $0 \le x \le 2$ is __________. [1]

OR

The slope of the tangent to the cure $x = 3t^2 + 1$, $y = t^3 - 1$ at $t = 1$ is __________

15. If $(2\hat{\imath} + 6\hat{\jmath} + 14\hat{k}) \times (\hat{\imath} - \beta\hat{\jmath} + 7\hat{k}) = \vec{0}$, the value of β is ________. [1]

(Q16 - Q20) Answer the following questions

16. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix ? [1]

17. Evaluate : $\int_{e}^{e^2} \frac{dx}{x \log x}$. [1]

18. Evaluate: $\int \frac{\log|\sin x|}{\tan x} dx$ [1]

19. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^5 x\, dx.$ [1]

20. Write the integrating factor of the following differential equation. [1]

$(1 + y^2) + (2xy - \cot y)\ \frac{dy}{dx} = 0.$

SECTION-B

Questions **21** to **26** carry two marks each.

21. If $f: R \to R$ is defined as $f(x) = 10x + 7.$ Find the function $g: R \to R,$ [2]

such that $fog = gof = I_R$.

OR

If $\tan^{-1}\left(\frac{1}{1+1\times2}\right) + \tan^{-1}\left(\frac{1}{1+2\times3}\right) + \tan^{-1}\left(\frac{1}{1+3\times4}\right) + \cdots + \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) = \tan^{-1}\theta$, find the value of $\theta.$

22. Find the value of $k,$ so that the function [2]

$$f(x) = \begin{cases} \left(\frac{1-\cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k\ , & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0.$

23. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing, when the side of triangle is 10 cm? [2]

OR

Using differentials, find approximate value of $\sqrt{49.5}$.

24. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{c} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, then find a unit vector perpendicular to both of the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$. [2]

25. Find the vector equation of the line passing through the point $A\ (1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z.$ [2]

26. A die, whose faces are marked 1,2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Check whether A and B are independent events or not. [2]

SECTION-C

Questions **27** to **32** carry four marks each.

27. If $A = R - \{2\}$, $B = R - \{1\}$ and $f: A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that f is one-one and onto. Hence find f^{-1}. [4]

28. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, where $x \neq y$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$. [4]

OR

If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ then find $\frac{d^2y}{dx^2}$.

29. Solve the differential equation : $e^x \tan y \; dx + (1 - e^x) sec^2 y \; dy = 0$ [4]

30. Evaluate $\int_0^{\pi} \frac{x \; dx}{1 + \sin\alpha \sin x}$ [4]

OR

Evaluate $\int_0^4 \left(|x| + |x-2| + |x-4| \right) \, dx$

31. A and B throw a pair of dice alternately. A wins the game, if he gets a total of 7 and B wins the game, if he gets a total of 10. If A starts the game, then find the probability that B wins. [4]

32. One kind of cake requires 200 g of flour and 25 g of cheese, another kind of cake requires 100 g of flour and 50 g of cheese. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of cheese, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an LPP and solve it graphically. [4]

SECTION-D

Questions **33** to **36** carry six marks each.

33. Using properties of determinants, show the that [6]

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (c+a)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

OR

Find A^{-1} , where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence, solve the system of equations,

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

and

$$3x - 3y - 4z = 11$$

34. Using integration find the area of the region bounded by the curves $y = \sqrt{4 - x^2}$, $x^2 + y^2 - 4x = 0$ and the X axis. [6]

35. Show that the right circular cylinder, open at the top and of given surface area and maximum volume is such that its height is equal to the radius of the base. [6]

OR

If the length of three sides of a trapezium other than the base are each equal to 10 cm, then find the area of the trapezium, when it is maximum.

36. Find the equation of the plane through the line of intersection of planes $x + y + z = 1$ and $2x + 3y + 4z = 5,$ which is perpendicular to the plane $x - y + z = 0.$ Also, find the distance of the plane obtained above, from the origin. [6]

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