# PRE BOARD EXAMIATION - 2 (2019-20)
# MATHEMATICS (Code 041)

**CLASS: XII** **Time : 3 hrs**

**Max. Marks: 80**

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**General Instructions**

(i) All the questions are compulsory.

(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

**Q1 - Q20 are multiple choice type questions.**

**Select the correct option**

1. Which of the following is not an equivalence relation on Z ( 1 )

a. $aRb \Leftrightarrow a+b$ is an even integer

b. $aRb \Leftrightarrow a-b$ is an even integer

c. $aRb \Leftrightarrow a<b$

d. $aRb \Leftrightarrow a=b$

2. The number of solutions of the equation $\tan^{=1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

a.2 b. 3 c.1 d. none ( 1 )

**OR**

The value of $\sin[\cot^{-1}\{\tan(\cos^{-1} x)\}]$ is

a. x b. $\sqrt{1-x^2}$ c . $\frac{1}{x}$ d. none

3. A square matrix A is invertible if and only if $|A|$ is equal to

a. zero b.1 c. non zero d. none ( 1 )

4. The radius of a sphere is changing at the rate of 0.1 cm/sec. The rate of change of its surface area when the radius is 200 cm is

a. $8\pi$ cm$^2$/sec b.$12\pi$ cm$^2$/ sec c.$160\pi$ cm$^2$ /sec d.$200\pi$ cm$^2$ /sec ( 1 )

5. If $y^2 = ax^2 + b$ then $\frac{d^2y}{dx^2}$ is

a. $\frac{ab}{x^3}$ b. $\frac{y^3}{ab}$ c. $\frac{ab}{y^2}$ d. $\frac{ab}{y^3}$ ( 1 )

6. On evaluating $\int_{-\pi}^{\pi}(sin^{15}x + x^{25})\,dx$ you get

a. 0 b. $\pi$ c. $\frac{\pi}{2}$ d. $-1$ ( 1 )

7. The general solution of the differential equation $\frac{dy}{dx} + y\cot x = \cos ecx$ is

a . x + y sin x = C b. x + y cos x = C c. y + x (sin x + cos x ) = C

d. y sin x =x + C ( 1 )

8. If $\theta$ is the angle between the vectors $2\hat{i} + 2\hat{j} + 4\hat{k}$ and $3\hat{i} + \hat{j} + 2\hat{k}$ then sin$\theta$ is

a. $\frac{2}{3}$ b. $\frac{2}{\sqrt{7}}$ c. $\frac{\sqrt{7}}{2}$ d. $\sqrt{\frac{2}{7}}$ ( 1 )

9. The objective function of Linear Programming Problem is a

a. constraint b. function to be optimized c. relation between the variables d. none ( 1 )

10. If A and B are any two events such that P(A) + P(B) – P(A and B) = P(A), then

a. P(B|A) = 1 b. P(A|B) = 1 c. P(B|A) = 0 d. P(A|B) = 0 ( 1 )

**OR**

Events A and B are independent if

a. $P(A \cap B) = P(B)P(A/B)$ b. $P(A \cap B) = P(A)P(B/A)$
b. $P(A \cap B) = P(A)$ + P(B) d. $P(A \cap B) = P(B)P(A)$

## Fill in the blanks

11. If $f : R \rightarrow R$ defined by $f(x) = (3 - x^3)^{\frac{1}{3}}$ then $fof(x)$ is ------- ( 1 )

12. The value of $\tan^{-1}1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ is ---------- ( 1 )

13. On differentiating $e^{\sin^{-1}x}$ you get ---------------------- ( 1 )

14. If $f(x) = \int_0^x t\sin t\,dt$ then the value of $f'(x)$ is ---------------- ( 1 )

15. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them.

Then $\vec{a} + \vec{b}$ is a unit vector if θ is --------- ( 1 )

**Short Answer Type I Questions**

16. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then find adj.A ( 1 )

17. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ ( 1 )

18. Find the area bounded by $x = 4 - y^2$ and y-axis ( 1 )

19. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. ( 1 )

20. If the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$ then find the equation of the plane. ( 1 )

**Short Answer type II Questions**

21. Prove that $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$ ( 2 )

22. If A = $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ , B = $\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ then ( 2 )

find a matrix D such that CD – AB = O.

23. Find all points of discontinuity of $f$, where $f$ is defined by

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$$ ( 2 )

24. Find the equation of a curve passing through the point (–2, 3), given that the slope of the tangent to the curve at any point $(x, y)$ is $\frac{2x}{y^2}$ ( 2 )

25. Find $|\vec{a} - \vec{b}|$ , if two vectors $\vec{a}$ and $\vec{b}$ are such that

$|\vec{a}| = 2$ , $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$. ( 2 )

26. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl , (ii) at least one is a girl? ( 2 )

**Long Answer type I Questions**

27 . Consider f: $R_+ \to [0, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and find $f^{-1}$. ( 4 )

28. Differentiate $(x\cos x)^x + (x\sin x)^x$ ( 4 )

29. Evaluate as the limit of a sum $\int_0^4 (x+e^{2x})dx$ ( 4 )

**OR**

Find $\int (\sqrt{\cot x} + \sqrt{\tan x})dx$

30. Show that the differential equation $2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$

is homogeneous and find its particular solution, given that, $x = 0$ when $y = 1$. ( 4 )

31. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture. ( 4 )

32. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B? ( 4 )

**OR**

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

**Long Answer type II Questions**

33. Show that $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$ ( 6 )

34. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m. ( 6 )

**OR**

Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

35. Find the area of the region enclosed between the two circles

$x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$. ( 6 )

**OR**

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B (4, 5) and C (6, 3).

36. Find the image of the point having the position vector $\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ in the plane $\vec{r}.\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right)+3=0$. ( 6 )