FIRST PRE BOARD EXAMINATION (2019–20)
CLASS: XII

Subject: MATHEMATICS **Date: 15 .12.2019**
Time Allowed: 3 Hours Maximum Marks: 80

General instructions:

(1) **All** questions are **compulsory**.
(2) Please check that this question paper contains 7 printed pages only.
(3) Please check that this question paper contains 36 questions.
(4) There are four sections, Section A, B, C and D.
(5) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of r marks each. Section D comprises of 4 questions of 6 marks each.
(6) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(7) Use of calculators is not permitted.

Section- A
[20X1= 20 Marks]

1.Given $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, which of the following result is true?

(a) $A^2 = I$ (B)$A^2 = -I$ (C) $A^2 = 2I$ (D) None of these

2. If A is 2X3 matrix , B is a matrix such that $A'B$ *and* BA' are both defined then B is of the type

(a) 2X3 (b) 2X2 (c) 3X3 (d) 3X2

3. If $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$ then $|\vec{b}|$ will be

(a) 3 (b) 4 (c) 2 (d) 9

4. If A and B are two independent events such that P(A)= 0.2, P(B) = 0.4 then P(AUB) is

(a) 0.72 (b) 0.62 (c) 1 (d) 0.52

5. $Cos^{-1}\left(Cos\frac{7\pi}{6}\right) + Sin^{-1}(Sin\frac{2\pi}{3})$ is equal to

(a) $-\frac{\pi}{6}$ b) $\frac{7\pi}{6}$ (c) $-\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

6. The point which lie in the half plane $x - 2y \geq 6$

(a) (8,1) (b) (4,2) (c) (2,3) (d) (4,5)

7. Probability of solving specific problem by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, the probability the problem will be solved is

(a) 2/3 (b) 1/3 (c) ¾ (d) none of these

8. Value of $\int Cotx.logSinx\, dx$ is

(a) $logSinx + c$ $(b)(logSinx)^2 + c$ (c) $\frac{(logSinx)^2}{2}$ +c (d) none of these

9. Distance between the two planes 3x+5y+7z=3 and 9x+15y+21z=27 ?

(a) 0 (b) 3 (c) $6/\sqrt{83}$ (d) 6

10. Equation of the line $\frac{x-1}{3} = \frac{y-5}{1}, z = 2$ in vector form is

(a) $\vec{r} = (3\hat{\imath} + \hat{\jmath}) + \mu(\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$ (b) $\vec{r} = (\hat{\imath} + 5\hat{\jmath} + 2\hat{k}) + \mu(\widehat{3\imath} + \hat{\jmath})$

(c) $\vec{r} = (\hat{\imath} + 5\hat{\jmath}) + \mu(3\hat{\imath} + \hat{\jmath} + 2\hat{k})$ (d) none of these

11. If $f = \{(1, a), (2, b), (3, c)\}$ *and* g are two functions such that

$fog = gof = I$, then value of function g *is* ……...

12. If $f(x) = \begin{cases} \frac{k\ Cosx}{\pi - 2x} & if\ x \neq \frac{\pi}{2} \\ 3 & if\ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then value of k is…

13. If $\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, then value of x will be…….

14. If the cost function C(x)=$x^2 + 3x + 2$ and revenue function R(x)= $15x + 7$. What value of x will make Marginal revenue and marginal cost equal?

OR

Find the interval in which $10 - 6x - 2x^2$ is strictly decreasing.

15. $The\ magnitude\ of\ projection\ of\ (\hat{\imath} - \widehat{2\jmath} + \hat{k})\ on\ (\widehat{\imath + \jmath + 3k})\ is$ ......

OR

If $(\hat{\imath} + \widehat{2\jmath} + 5\hat{k}) \times (p\hat{\imath} - q\hat{\jmath} + 5\hat{k}) = \vec{0}$, then value of 'q' is…

16. Is $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$? Justify your answer.

17. Evaluate $\int_0^1 \frac{2x}{1+x^2} dx$

18. $\int x^6 Sin(5x^7)dx = \frac{k}{5} Cos(5x^7), x \neq 0$ then the value of k if constant of integration is zero.

OR

Find $\int \frac{dx}{Secx+tanx}$

19. Find $\int \frac{dx}{Sin^2 x.Cos^2 x}$

20. Find the general solution of the differential equation $\frac{dy}{dx} = y.tanx.$

Section- B

[6X2= 12 Marks]

21. Write the value of $Cot^{-1}(\sqrt{1+x^2} - x)$ in the simplified form.

OR

Check whether the relation R on real numbers defined by

$R = \{(x, y): x \leq y^3\}$ is transitive.

22. If $y = Sin^{-1}x$ then prove that $(1 - x^2)y_2 - xy_1 = 0$

23. Find the approximate change in the surface area of a cube of side x

meters caused by decreasing the side by 1%.

24. Find μ if the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \mu\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$

are coplanar.

OR

Prove that $[\vec{a}, \vec{b}, \vec{c} + \vec{d}] = [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}]$

25. Find the angle between the lines

$\frac{2-x}{2} = \frac{y-1}{2} = \frac{z+3}{-3}$ *and* $\frac{x+2}{-1} = \frac{y-4}{8}, z = 5$

26. A die is thrown. If E is the event 'the number appearing is a multiple of

3' and F be the event 'the number appearing is even' then find whether

E and F are independent?

Section- C
[6X4= 24 Marks]

27. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 5$. Show that

$f: N \rightarrow s$ where S is the range of f is invertible. Find the inverse of f.

28. Prove that $-1 \leq \frac{2^{x+1}}{1+4^x} \leq 1$ *hence di*fferentiate the following w.r.t. x,

$y = Sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.

OR

If $y = e^{aCos^{-1}x}, -1 \leq x \leq 1. Show\ that\ (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

29. Show that the differential equation $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$ is

homogenous hence find its particular solution at y=2 when x=1.

30. Evaluate $\int_0^{\pi} \frac{xdx}{a^2Cos^2x+b^2Sin^2x}$

31. Two numbers are selected at random (without replacement) from first 7

natural numbers. If X denotes smaller of two numbers obtained. Find the probability distribution of X. Also, find mean of the distribution.

OR

Assume that the chance of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

32. Minimise $Z = x + 2y$ subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$.

Section- D

[4X6= 24 Marks]

33. Using properties of determinants, prove that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

OR

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, find\ A^{-1}$. Using A^{-1} solve the system of equations

2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3.

34. Using integration, find the area of the region bounded by

$\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$

35. A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi circular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of semi circular ends is $\pi: \pi + 2$

OR

Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

36. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ *and* $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.

Also find the equation of the plane containing them.