SECOND PRE BOARD EXAMINATION (2019–20)
CLASS: XII

Subject: MATHEMATICS **Date: 16 .01.2020**

Time Allowed: 3 Hours Maximum Marks: 80

General instructions:

(1) **All** questions are **compulsory**.
(2) Please check that this question paper contains 8 printed pages only.
(3) Please check that this question paper contains 36 questions.
(4) There are four sections, Section A, B, C and D.
(5) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of r marks each. Section D comprises of 4 questions of 6 marks each.
(6) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(7) Use of calculators is not permitted.

SECTION –A (20 × 1 = 20 Marks)

Q 1. If a relation R is defined on the set Z of integers as follows :

$(a, b) \in R \Leftrightarrow a^2 + b^2 = 25$. Then, domain (R) is

(a) {3, 4, 5} (b) {0, 3, 4, 5} (c) {0, ±3, ±4, ±5} (d) None of these

Q 2. The value of $\tan\left\{\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right\}$ is

(a) $\frac{\sqrt{29}}{3}$ (b) $\frac{29}{3}$ (c) $\frac{\sqrt{3}}{29}$ (d) $\frac{3}{29}$

Q 3. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

(a) 27 (b) 18 (c) 81 (d) 512

Q 4. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A (b) I – A (c) I (d) 3A

Q 5. If $A_r = \begin{vmatrix} 1 & r & 2^r \\ 2 & n & n^2 \\ n & \frac{n(n+1)}{2} & 2^{n+1} \end{vmatrix}$, then the value of $\sum_{r=1}^{n} A_r$ is

(a) n (b) 2n (c) – 2n (d) n^2

Q 6. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$, then

(a) $\Delta_1 + \Delta_2 = 0$ (b) $\Delta_1 + 2\Delta_2 = 0$ (c) $\Delta_1 = \Delta_2$ (d) none of these

Q 7. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$

If f(x) is continuous and differentiable at any point, then

(a) $a = \frac{1}{2}, b = -\frac{3}{2}$ (b) $a = -\frac{1}{2}, b = \frac{3}{2}$ (c) a = 1, b = - 1 (d) none of these

Q 8. The equation of motion of a particle is $s = 2t^2 + \sin 2t$, where s is in metres and t is in seconds. The velocity of the particle when its acceleration is 2 m/sec^2, is

(a) $\pi + \sqrt{3}$ m/sec (b) $\frac{\pi}{3} + \sqrt{3}$ m/sec (c) $\frac{2\pi}{3} + \sqrt{3}$ m/sec (d) $\frac{\pi}{3} + \frac{1}{\sqrt{3}}$ m/sec

Q 9. $\int \sqrt{\frac{x}{1-x}} dx$ is equal to

(a) $\sin^{-1}\sqrt{x} + C$

(b) $\sin^{-1}\{\sqrt{x} - \sqrt{x(1-x)}\} + C$

(c) $\sin^{-1}\{\sqrt{x(1-x)}\} + C$

(d) $\sin^{-1}\sqrt{x} - \sqrt{x(1-x)} + C$

Q 10. $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx =$

(a) $2\log_e \cos(xe^x) + C$ (b) $\sec(xe^x) + C$ (c) $\tan(xe^x) + C$ (d) $\tan(x + e^x) + C$

Q 11. Evaluate $\int \frac{x+3}{(x+4)^2}e^x dx =$

OR

Evaluate : $\int_{\pi/4}^{\pi/2} \sqrt{1-\sin 2x}\, dx$

Q 12. Vectors $\vec{a}$ and $\vec{b}$ are inclined at angle θ = 120°.

If $|\vec{a}|=1, |\vec{b}|=2$, then find value of $[(\vec{a}+3\vec{b})x(3\vec{a}-\vec{b})]^2$.

Q 13. If the vectors 4i + 11j + mk, 7i + 2j + 6k and i + 5j + 4k are coplanar, then find the value of m.

Q 14. Find the shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and, $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

OR

Evaluate $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)}d\theta$

Q 15. If a plane passes through the point (1, 1, 1) and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ then find its perpendicular distance from the origin .

Q 16. Find the point at which the maximum value of x + y, subject to the constraints $x + 2y \le 70, 2x + y \le 95, x, y \ge 0$.

OR

Find the projection of the vector 7i + j – 4k on 2i + 6j + 3k.

Q 17. A speaks truth in 75% cases and B speaks truth in 80% cases then find the probability that they contradict each other.

Q 18. Two natural numbers are chosen at random from the first 20 natural numbers. Find the probability that their product is even.

Q 19. Find the solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$.

Q 20. Evaluate $\int \frac{1}{1 - \cos x - \sin x} dx =$

SECTION –B(6 × 2 = 12 Marks)

Q 21. What is the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?

Q 22. If y = $\log\sqrt{\tan x}$, write $\frac{dy}{dx}$.

Q 23. Write the angle made by the tangent to the curve $x = e^t \cos t$, $y = e^t \sin t$ at $t = \frac{\pi}{4}$ with the x-axis.

Q 24. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2i + 6j + 3k$.

OR

Find the area of the triangle whose vertices are A(3, -1, 2), B(1, -1, -3) and C(4, -3, 1).

Q 25. If a line makes angles α, β and γ with the coordinate axes, find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

Q 26. In a competition A, B and C are participating. The probability that A wins is twice that of B, the probability that B wins is twice that of C. Find the probability that A losses.

OR

If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

SECTION –C(6 × 4 = 24 Marks)

Q 27. Evaluate : $\int_0^1 \cot^{-1}(1-x+x^2)dx$

Q 28. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it, is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it.

OR

A company has two plants to manufacture scooters. Plant I manufactures 70% of the scooters and Plant II manufactures 30%. At Plant I, 80% of the scooters are rated as of standard quality and at Plant II, 90% of the scooters are rated as of standard quality. A scooter is chosen at random and is found to be of standard quality. What is the probability that it has come from Plant II?

Q 29. In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours. In how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present.

Q 30. A house wife wishes to mix together two kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg of food is given below:

	Vitamin A	Vitamin B	Vitamin C
Food X:	1	2	3
Food Y:	2	2	1

One kg of food X costs Rs 6 and one kg of food Y costs Rs 10. Find the least cost of the mixture which will produce the diet.

Q 31. Show that f : [-1, 1] → R, given by $f(x) = \frac{x}{x+2}$ is one-one. Find the inverse of the function

f : [-1, 1] → Range (f).

Q 32. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if } x < 0 \\ a, & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{if } x > 0 \end{cases}$

Determine the value of a so that f(x) is continuous at x = 0.

OR

The function $f(x) = \begin{cases} x^2/a, & \text{if } 0 \le x < 1 \\ a, & \text{if } 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^2}, & \text{if } \sqrt{2} \le x < \infty \end{cases}$

is continuous on $[0, \infty)$, then find the most suitable values of a and b.

SECTION – D(4 × 6 = 24 Marks)

Q 33. Show that:

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$$

OR

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformations.

34. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

OR

A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum ? Also, find this maximum volume.

Q 35. Using integration, find the area of the triangle ABC coordinates of whose vertices are A(2,5), B(4, 7) and C (6,2).

Q 36. Show the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the plane containing these lines.

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