Code: 041

PRE BOARD EXAMINATION-2 (JANUARY 2020)

CLASS: XII **MATHEMATICS** **Time: 3 hours.**

MAX. MARKS: 80

General Instructions:

*(i) **All** questions are compulsory.*
*(ii) This question paper consists of **36** questions divided into four sections A, B, C and D. Section A comprises of **20** questions of **one mark** each, Section B comprises of **6** questions of **two marks** each, Section C comprises of **6** questions of **four marks** each and Section D comprises of **4** questions of **six marks** each.*
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choice. However, internal choice has been provided in 3 questions of Section A, 2 questions of Section B, 2 questions of Section C and 2 questions of Section D. You have to attempt only one of the alternatives in all such questions.
*(v) Use of calculators is **not** permitted.*

SECTION A

Q1 - Q10 are multiple choice type questions. Select the correct option :

1. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$ (1)

(a) 3 (b) 1 (c) -1 (d) 9

2. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 1 \end{bmatrix}$ is skew – symmetric, find the values of ‘a’ and ‘b’ (1)

(a) $a = -2, b = 3$
(b) $a = 2, b = 3$
(c) $a = -2, b = -3$
(d) $a = 2, b = -3$

3. Write the projection of $\hat{\imath} - \hat{\jmath}$ on the vector $\hat{\imath} + \hat{\jmath}$ (1)

(a) 0 (b) 1 (c) -1 (d) 2

4. If P (A) $= \frac{1}{2}$, P(B) $= 0$, then P (A|B) is (1)

(a) 0 (b) $\frac{1}{2}$ (c) not defined (d) 1

5. Let $f = 4x + 6y$ be the objective function of the Linear programming problem. If the corner points are (0, 2), (3, 0), (6, 8) and (0, 5), then the difference between the maximum and minimum of 'f' is __________. (1)

(a) 48 (b) 42 (c) 60 (d) 18

6. The value of $x + y + xy$ if $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ is (1)

(a) 3 (b) -1 (c) 1 (d) 0

7. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is (1)

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{36}$

8. Write the value of $\int \frac{3x}{3x-1} dx$ (1)

(a) $x + \frac{1}{3}\log|3x-1| + C$

(b) $\frac{1}{3}\log|3x-1| + C$

(c) $\log|3x-1| + C$

(d) $x - 3\log|3x-1| + C$

9. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of the z-axis. (1)

(a) $\frac{\pi}{2}, \frac{5\pi}{6}$ (b) $\frac{\pi}{3}, \frac{5\pi}{6}$ (c) $\frac{\pi}{6}, \frac{5\pi}{6}$ (d) $\frac{\pi}{2}, \frac{\pi}{3}$

10. If the planes $2x - y + \lambda z = 5$ and $3x + 2y + 2z = 4$ are perpendicular, find λ (1)

(a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(Q11 - Q15) Fill in the blanks :

11. The integrating factor of the differential equation $x\frac{dy}{dx} + y = 2x^2$ is ________. (1)

12. The value of k so that the function $f(x) = \begin{Bmatrix} \frac{k\cos x}{\pi - 2x}, & if\ x \neq \frac{\pi}{2} \\ 3, & if\ x = \frac{\pi}{2} \end{Bmatrix}$ is continuous at $x = \frac{\pi}{2}$ is ________. (1)

13. If $\begin{bmatrix} x & x-y \\ 2x+y & 7 \end{bmatrix}=\begin{bmatrix} 3 & 1 \\ 8 & 7 \end{bmatrix}$, then the value of y is _______. (1)

14. The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is ___________. (1)

OR

The point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$ is ______.

15. The value of 'a' if the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear is _________. (1)

OR

The direction cosines of the vector $-2\hat{\imath}+\hat{\jmath} - 5\hat{k}$ are __________.

(Q16 - Q20) Answer the following questions :

16. Using property of determinants, prove that $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$ (1)

17. Evaluate: $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$ (1)

18. Find: $\int \frac{(x+1)(x+\log x)^2}{x} dx$ (1)

OR

Find: $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$

19. Evaluate: $\int \frac{sin^2 x}{1+\cos x} dx$ (1)

20. Determine whether the following relation is reflexive, symmetric and transitive: (1)
R = {(1,3), (3, 1), (1, 1), (3, 2)} on set A = {1, 2, 3}

SECTION B

21. If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, then find the value of $\cot^{-1} x + \cot^{-1} y$ (2)

OR

Let $A = \{1,2,3,4,5,6,7,8,9\}$ and R be the relation in $A \times A$ defined by $(a,b)R(c,d) if$ $a + d = b + c$ for $(a,b),(c,d)$ in $A \times A$.Prove that R is transitive. Also obtain the equivalence class [(2, 5)].

22. If $y = \sqrt{1 + x^2}$, then show that $\frac{dy}{dx} = \frac{xy}{1+x^2}$ (2)

23. The sides of an equilateral triangle are increasing at the rate of $2cm/sec$. Find the rate at which the area increases, when side is 10cm. (2)

24. Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5). (2)

OR

The x- coordinate of a point on the line joining the point $P(2,2,1) and\ Q(5,1,-2)$ is 4. Find its z- coordinate?

25. Find the value of λ so that the lines $\frac{1-x}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{-7}$ are perpendicular to each other. (2)

26. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that both balls are red? (2)

SECTION C

27. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertble with the inverse (f^{-1}) of f given by $f^{-1}(y) = \sqrt{y-4}$ where $R_+$is the set of all non-negative real numbers. (4)

28. Differentiate $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ w.r.t x (4)

OR

If $y = Ae^{mx} + Be^{nx}$,show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

29. Find the general solution of the differential equation $y\ log\ y\ dx - x\ dy = 0$. (4)

30. Find $\int \frac{x+3}{x^2-2x-5}\,dx$ (4)

31. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X. (4)

OR

The members of a consulting firm rent cars from three rental agencies: 50% from agency X, 30% from agency Y and 20% from agency Z. From past experience it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of the cars from agency Y need a service and tuning before renting and 10% of the cars from agency Z need a service and tuning before renting. If the rental car delivered to the firm needs service and tuning, find the probability that agency Z is not to be blamed.

32. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit. (4)

SECTION D

33. If $a+b+c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then prove that $a = b = c$. (6)

OR

If A= $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1}.Using A^{-1}, solve the system of linear equations $x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7.$

34. Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12.$ (6)

35. Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of the volume of the sphere. (6)

OR

The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

36. Find the equation of the plane pasing through the point $(-1,3,2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$. (6)

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