Code: 041

PRE-BOARD EXAMINATION-1 (NOVEMBER 2019)

CLASS: XII **MATHEMATICS** **Time: 3 hours.**

MAX. MARKS: 80

General Instructions:

*(i) **All** questions are compulsory.*
*(ii) This question paper consists of **36** questions divided into four sections A, B, C and D. Section A comprises of **20** questions of **one mark** each, Section B comprises of **6** questions of **two marks** each, Section C comprises of **6** questions of **four marks** each and Section D comprises of **4** questions of **six marks** each.*
(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(iv) There is no overall choice. However, internal choice has been provided in 3 questions of Section A, 2 questions of Section B, 2 questions of Section C and 2 questions of Section D. You have to attempt only one of the alternatives in all such questions.
*(v) Use of calculators is **not** permitted.*

SECTION A

Q1 - Q10 are multiple choice type questions. Select the correct option

1. If A is a square matrix and |A| = 2, then find the value of $|AA'|$, where $|A'|$ is the transpose of matrix A. (1)

   (a) 0 (b) 1 (c) 2 (d) 4

2. Write the order of product matrix $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}$ (1)

   (a) 1×3 (b) 3×3 (c) 1×1 (d) 3×1

3. For what value of 'a' the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear. (1)

   (a) 25 (b) 4 (c) -4 (d) -25

4. If A and B are two events such that P (A)$= 0.2$, P (B) $= 0.4$ and P (A $\cup$ B) $= 0.5$, then value of P (A|B) is ?

   (a) 0.1 (b) 0.25 (c) 0.5 (d) 0.08 (1)

5. The corner points of the feasible region are (0, 10), (5, 5), (15, 15), (0, 20). The objective function is $z = px + qy$ where $p, q > 0$. The condition p and q such that the maximum of z occurs at both the points (15, 15) and (0, 20) is ________. (1)

(a) $p = 3q$
(b) $p = 2q$
(c) $2p = q$
(d) $3p = q$

6. The value of x if $\tan^{-1} 4 + \cot^{-1} x = \frac{\pi}{2}$ is (1)

(a) -4 (b) 4 (c) 14 (d) 0

7. If A and B are events such that P(A|B) = P(B|A), then (1)

(a) $A \subset B$ but $A \neq B$
(b) A = B
(c) $A \cap B = \emptyset$
(d) P(A) = P(B)

8. Write the value of $\int \frac{dx}{x^2+16}$ (1)

(a) $\tan^{-1} x + C$

(b) $\frac{1}{4} \tan^{-1} \frac{x}{4} + C$

(c) $\frac{1}{2} \tan^{-1} \frac{x}{2} + C$

(d) $\frac{1}{16} \tan^{-1} \frac{x}{16} + C$

9. Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes. (1)

(a) $x + y + z = 5\sqrt{3}$
(b) $x + y + z = 5$
(c) $x + y + z = 15$
(d) $x + y + z = 1$

10. Find the sum of the intercepts cut off by the plane 2x + y – z = 5 on the coordinate axes (1)
(a) $\frac{5}{2}$
(b) 5
(c) $\frac{2}{5}$
(d) 0

(Q11 - Q15) Fill in the blanks :

11. If $f(x) = |x|$ and $g(x) = |5x - 2|$, $fog(x) =$ ________ (1)

12. The function $f(x) = \begin{Bmatrix} \frac{x^2-x-6}{x+2}, x \neq -2 \\ k \quad\quad , x = -2. \end{Bmatrix}$ Is continuous at $x = -2$, then $k =$ ____ (1)

13. If $\begin{bmatrix} 3x & -1 \\ y-x & 4 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -3 & 4 \end{bmatrix}$.The value of y is _________ (1)

14. The slope of the tangent is 6, the point on the curve $y = x^2$ is _______ (1)

OR

The rate of change of the area of a circle with respect to its radius r at $r = 6cm$ is ____

15. $(\hat{\imath} + \hat{\jmath} + \hat{k})p$ a unit vector. Then the value of p is __________ (1)

OR

The value of $(\hat{\imath} \times \hat{\jmath}).\hat{k} + \hat{\imath}.\hat{\jmath}$ is ___________

(Q16 - Q20) Answer the following questions :

16. Using property of determinants, Write the value of the determinant: (1)

$$\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$$

17. Evaluate: $\int_2^3 \frac{1}{x} dx$ (1)

18. Evaluate: $\int \frac{sec^2 x}{3+\tan x} dx$ (1)

OR

If $\int_0^1 (3x^2 + 2x + k\,)dx = 0$, then find the value of k.

19. If $\int \left(\frac{x-1}{x^2}\right) e^x dx = f(x)e^x + C$, then write the value of $f(x)$ (1)

20. Find the differential equation representing the curve $y = cx + c^2$ (1)

SECTION B

21. Prove that : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$, $x \in [\frac{-1}{2}, \frac{1}{2}]$ (2)

OR

Show that the relation R on the set R of real numbers, defined as
$R = \{(a,b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

22. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis. (2)

23. A radius of a spherical air bubble is increasing at the rate of 0.5cm/sec. At what rate is the volume of the bubble increasing when its radius is 1cm? (2)

24. If $\vec{a}, \vec{b}$ *and* $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ (2)

OR

Find the area of the parallelogram whose diagonals are represented by the vectors
$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

25. Find the direction cosines of the line passing through two points $(-2,4,-5)$ and $(1,2,3)$ (2)

26. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, ‘the number is even,’ and B be the event, ‘the number is red’. Are A and B independent? (2)

SECTION C

27. Let A = R−{3}, B = R−{1}, $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall x \in A$. Is f one-one and onto? Is f invertible. If yes, find the inverse? (4)

28. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 17$ is (4)

(i) increasing (ii) is decreasing

OR

If $y = (cot^{-1}x)^2$,then show that $(x^2+1)^2.\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$

29. Solve the differential equation $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$ (4)

30. Find $\int \frac{x^2}{(x^2+4)(x^2+9)}\, dx$ (4)

31. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III. (4)

OR

Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution.

32. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹ 4 per unit and F_2 costs ₹ 6 per unit. One unit of food F_1 contains 3 units of Vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of Vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements. (4)

SECTION D

33. By using properties of determinant, show that (6)

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

OR

Using matrices, solve the following system of equations:

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \leq 8, \ x^2 \leq 2y\}$ (6)

35. A cuboidal shaped godown with square base is to be constructed. Three times as much cost per square meter is incurred for constructing the roof as compared to the walls. Find the dimensions of the godown if it is to enclose a given volume and minimize the cost of constructing the roof and the walls. (6)

OR

Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to the diameter of its base.

36. Find the equation of the plane through the line $\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ and parallel to the line $\frac{x+1}{2} = \frac{1-y}{4} = \frac{z+2}{1}$. Hence find the shortest distance between the lines. (6)