Pre Board -1 Examination – December 2019

Roll No. | | | | | | | |

Series SSR / 1 **Code No. 041/ 1 / 3**

- Please check that this question paper contains 5 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 36 questions.
- Please write down the serial number of the question before attempting it.

Mathematics

Class : XII Time allowed : 3 hrs.
Date : 14-12-2019 Max marks : 80

General Instructions:

(i) All the questions are compulsory.

(ii) The question paper consists of 36 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each , and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION A

1. If A and B are matrices of same order, then AB'-BA' is a

a) Skew-symmetric matrix b) null matrix

c) symmetric matrix d) unit matrix

2. If A=$\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, B=$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, where i=$\sqrt{-1}$ then the correct relation is

a) A+B=0 b) $A^2 = B^2$ c)A-B=0 d) $A^2 + B^2 = 0$

3. If θ is the angle between any two vectors $\vec{a}$and $\vec{b}$ then $|\vec{a}\,.\,\vec{b}| = |\vec{a} \times \vec{b}|$ where θ is equal to

a) 0 b)$\pi/4$ c) $\pi/2$ d) π

4. A and B are two events such that P(B) = $\frac{3}{5}$, P(A/B)=$\frac{1}{2}$ and P(A∪B)=$\frac{4}{5}$ then P(A) equals

a) $\frac{3}{10}$ b) $\frac{1}{5}$ c) $\frac{1}{2}$ d) $\frac{3}{5}$

5. Objective function of a LPP is

a) constraint b) a function to be optimized

c) a relation between the variables d) none of these

6. If $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1} a$ then a is equal to

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) 1

7. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement then the probability of getting exactly one red ball is

$a)\frac{15}{196}$ b) $\frac{131}{392}$ c) $\frac{15}{56}$ d$\frac{15}{29}$

8. $\int e^x (\cos x$ - $\sin x)$dx is equal to

a) $e^x \cos x$ + C b) $e^x \sin x$ + C c) -$e^x \cos x$ + C d) -$e^x \sin x$ + C

9. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$,find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

$a)\ \frac{3}{2}$ b) $\frac{-3}{2}$ c) $\frac{1}{2}$ d) $\frac{-1}{2}$

10. Distance between the two planes 2x+3y+4z=4 and 4x+6y+8z=12 is

a) 2 units b) 4 units c) 8 units d) $\frac{2}{\sqrt{29}}$ units

Fill in the blanks

11. If f be the greatest integer function defined as f(x) = [x] and g be the modulus function defined as g(x)=$|x|$,then the value of (g of) ($\frac{-5}{4}$) is________

12. The value of C in Rolle's Theorem for the function f(x) = $e^x \sin x$,x∈[0,π] is ___

13. $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix for x= ______

14. For the curve y=3$x^2 + 4x$, find the slope of the tangent to the curve at the point whose x coordinate is -2.

OR

The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases when side is 10cm is ______

15. The magnitude of projection of the vector $\vec{i} + 3\vec{j} + \vec{k}$ on the vector $7\vec{i} - \vec{j} + 8\vec{k}$ is ______

OR

The vector from origin to the points A and B are $\vec{a} = 2\vec{i} - 3\vec{j} + 2\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + \vec{k}$ respectively then the area of the triangle OAB is ______

Answer the following

16. Without expanding Show that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ vanishes.

17. Evaluate $\int_e^{e^2} \frac{dx}{x \log x}$

18. Evaluate $\int \frac{x^3 - x^2 + x - 1}{x-1} dx$

OR

Find $\int (cos^2 2x - sin^2 2x] dx$

19. Find $\int \frac{e^{\tan^{-1} x}}{1+x^2}$ dx

20. Form the differential equation of the family of curves y=a $\cos(x+b)$ where a and b are arbitrary constants.

SECTION B

21. Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ where $-\frac{\pi}{4} < x < \frac{\pi}{4}$ in the simplest form.

OR

If $\sin^{-1} x + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then find x.

22. Solve the differential equation:

$$x\cos y dy = (xe^x \log x + e^x)dx.$$

23. Show that the function given by f(x)=$\tan^{-1}(\sin x + \cos x)$ is decreasing for all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

24. If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, prove that $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar.

OR

If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|$=3, $|\vec{b}|=\frac{2}{3}$ and $\vec{a}$ x $\vec{b}$ is a unit vector then write the angle between $\vec{a}$ and $\vec{b}$.

25. Find the angle between the pair of lines given by $\vec{r} = 3\hat{i}+2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i}-2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

26. The probability of two students A and B coming to school on time are $\frac{2}{7}$ and $\frac{4}{7}$ respectively. Assuming that the events "A coming on time" and "B coming on time" are independent. Find the probability of only one of them coming to school on time.

SECTION C

27. Consider f : $R_+ \rightarrow [-9,\infty)$ given by f(x) = $5x^2 + 6x - 9$. Prove that f is invertible. Hence find f^{-1}.

28. If y=$\log(x)^{\cos x} + \frac{x^2+1}{x^2-1}$,find $\frac{dy}{dx}$

OR

Differentiate $\tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$ w.r.t $\sin^{-1}(\frac{2x}{1+x^2})$

29. Find the particular solution of the following differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ given that y=1 when x=0.

30. Evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$$

31. In a school there are 1000 students out of which 430 are girls. It is known that out of 430,10%of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl.

OR

Three cards are drawn at random (without replacement) from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution.

32. There are two types of fertilizers A and B. A consists of 12% nitrogen and 5% phosphoric acid whereas B consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions farmer finds that he needs at least 12kg of nitrogen and 12kg of phosphoric acid for his crops. If A costs Rs.10 per kg and B costs Rs.8 per kg then graphically determine how much of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost?

SECTION D

33. Using matrices solve the following system of equations 2x+3y+3z=5, x-2y+z=-4 and 3x-y-2z=3.

OR

If a, b, c are all non- zero and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$ then prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1 = 0$

34. Using integration find the area of the region $\{(x, y): x^2 + y^2 \leq 2ax,\ y^2 \geq ax,\ x, y \geq 0\}$

35. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 tan^2 \alpha$.

OR

A window is of the form of a semi-circle with a rectangle on its diameter. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

36. Find the vector and Cartesian equations of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$, $\vec{r} \cdot (2\hat{i} + 5\hat{j} - 3\hat{k}) = 9$ such that the intercepts made by the plane on X-axis and Z-axis are equal.

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