PRE BOARD EXAMINATION , JAN/FEB - 2018

CLASS : XII | SUBJECT – MATHEMATICS | M.Marks :100

DATE : | SET - A | TIME : 3 HRS

General Instructions:

- All questions are compulsory.
- The question paper consists of **29** questions divided into **4** sections – A , B ,C and D. Section A comprises of **4** questions of **1** mark each, section B comprises of **8** questions of **2** marks each, section C comprises of **11** questions of **4** marks each, and section D comprises of **6** questions of **6** marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- There is no overall choice. However, internal choice has been provided in **3** questions of **4** marks each and **3** questions of **6** marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculator is not permitted.

SECTION – A

1. Write the value of the determinant $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$.

2. Find the value of the integral $\int (\log(logx) + \frac{1}{logx})dx$.

3. Write the intercepts cut off by the plane $2x + y - z = 5$ on x-axis.

4. For what value of k is the function

$$f(x) = \begin{cases} \frac{sin5x}{3x} + cosx \ ; & x \neq 0 \\ k \ ; \ x = 0 \end{cases}$$ continuous at x=0.

SECTION -B

5. Differentiate : $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ with respect to x.

6. Find the value of x and y if: $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

7. If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$, find the value of k.

8. If $C = 0.003x^3 + 0.02x^2 + 6x + 250$ gives the amount of carbon pollution in air in an area on the entry of x number of vehicles, then find the marginal carbon pollution in the air when 3 vehicles have entered in the area and write which value does the question indicate.

9. Find the angle between the lines
$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\,(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\bar{r} = 7\hat{i} - 6\hat{k} + \mu\,(\hat{i} + 2\hat{j} + 2\hat{k})$.

10. A die is tossed thrice. Find the probability of getting an odd number at least once.

11. A company produces two types of goods A and B that require gold and silver. Each unit of type A require 3g of silver and 1g of gold while that of type B require 1g of silver and 2g of gold.The company can procure a maximum of 9g silver and 8g of gold. If each unit of type A bring a profit of ₹40 and that of type B ₹50, formulate LPP to maximize profit.

12. Write the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$.

SECTION – C

13. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

OR

If x = sint and y = sinpt, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

14. Find $\int \frac{e^x}{(2+e^x)(4+e^{2x})}dx$

OR

Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$

15. Solve the differential equation $(\cot^{-1}y + x)dy = (1+y^2)dx$

16. Find a unit vector perpendicular to both the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$.

17. Solve for x: $\tan^{-1}3x + \tan^{-1}2x = \frac{\pi}{4}$

18. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$ then find the values of a and b.

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ verify that $A^2 - 4A - 5I = 0$

19. A box has 20 pens out of which 2 are defective. Calculate the probability that out of 5 pens drawn one by one with replacement at most 2 are defective.

20. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$

21. Suppose a girl throws a die. If she gets a 5 or 6 she tosses a coin 3 times and notes the number of heads. If she gets 1,2,3 or 4 she tosses a coin once

and notes whether a head or tail is obtained. If she obtains exactly one head what is the probability that she throws 1,2,3 or 4 with the die.

22. Determine graphically the minimum value of the objective function $Z = -50x + 20y$ subject to $2x - y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$; $x, y \geq 0$

23. Show that the four points A, B, C & D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-(\hat{j} + \hat{k})$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

SECTION – D

24. Using properties of determinants prove that

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

25. If $f; g : R \rightarrow R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$ for all $x \in R$. Then find fog and gof. Hence find fog(-3), fog(5) and gof(-2).

OR

Let $A = R \times R$, let * be a binary operation on A defined by $(a,b) * (c,d) = (ad+bc, bd)$ for all $(a,b), (c,d), \in R \times R$.

i. Show that * is commu~~t~~tative on A
ii. Show that * is associative on A
iii. Find the identity element of * in A

26. Show that a cylinder of given volume which is open at the top has minimum total surface area when its height is equal to the radius of its base.

27. Using integration find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and x- axis.

OR

Find the area of the smaller region bounded by the ellipse $4x^2 + 9y^2 = 36$ and the line $2x + 3y = 6$

28. Solve the differential equation $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$ given that $y = 1$ when $x = 1$

29. Find the equation of the plane which contains the line of intersection of the plane $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k}) - 4 = 0$ and $\vec{r}.(2\hat{i}+\hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+ 3\hat{j} - 6\hat{k}) + 8 = 0$.

OR

Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

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