PRE BOARD EXAMINATION , JAN/FEB - 2018

CLASS : XII | SUBJECT – MATHEMATICS | M.Marks :100

DATE | SET - B | TIME : 3 HRS

General Instructions:

- All questions are compulsory.
- The question paper consists of 29 questions divided into 4 sections – A , B ,C and D. Section A comprises of 4 questions of 1 mark each, section B comprises of 8 questions of 2 marks each, section C comprises of 11 questions of 4 marks each, and section D comprises of 6 questions of 6 marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- There is no overall choice. However, internal choice has been provided in 3 questions of 4 marks each and 3 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculator is not permitted.

SECTION – A

1. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8$, write the value of x.

2. If $\int \left(\frac{x-1}{x^2}\right) e^x dx = f(x)e^x + c$ then write the value of f(x).

3. For what value of k is the function

$$f(x) = \begin{cases} \frac{\sin 5x}{3x} + \cos x \ ; & x \neq 0 \\ k \ ; & x = 0 \end{cases}$$ continuous at x=0.

4. Find the sum of the intercepts cut off by the plane 2x + y - z=5 on the co-ordinate axes.

SECTION -B

5. Differentiate : sin^2x with respect to e^{cosx}.
6. The radius r of a right circular cylinder is increasing uniformly at the rate of 0.3 cm/sec and its height h is decreasing at the rate of 0.4cm/sec. When r = 3.5cm and h = 7cm find the rate of change of the curved surface area of the cylinder (use $\pi = \frac{22}{7}$).
7. Find $\int \frac{sec^2x}{cosec^2x}$ dx
8. Let A is a square matrix such that $A^2 = A$, then find the value of $(I + A)^3 - 7A$.

9. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$

10. Two cards are drawn at random and without replacement from a pack of 52 cards. Find the probability that both the cards are black.

11. A company produces two types of goods A and B that require gold and silver. Each unit of type A require 3g of silver and 1g of gold while that of type B requires 1g of silver and 2g of gold.The company can procure a maximum of 9g silver and 8g of gold. If each unit of type A bring a profit of ₹40 and that of type B ₹50, formulate LPP to maximize profit. What are the good qualities should be possessed by the company owner to get maximum profit?

12. Evaluate $\sin\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)$.

SECTION – C

13. If $x = acos\theta + b\, sin\theta$ and $y = asin\theta - bcos\theta$, show that

$$y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$$

OR

If $y = x^{sinx} + (sinx)^{cosx}$ find $\frac{dy}{dx}$.

14. Find $\int e^{2x} \sin(3x+1)dx$.

OR

Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x} dx$

15. Solve the differential equation : $y^2dx + (x^2 - xy + y^2)dy = 0$

16. Find a unit vector perpendicular to both the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

17. Prove that : $\sin^{-1}(\frac{4}{5}) + \sin^{-1}(\frac{5}{13}) + \sin^{-1}(\frac{16}{65}) = \frac{\pi}{2}$

18. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$ then find the values of a and b.

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ verify that $A^2 - 4A - 5I = 0$

19. Three cards are drawn at random without replacement from a well shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence find the mean of the distribution.

20. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$

21. Given three identical boxes I, II and III each contains two coins. In box I, both coins are gold coins ,in box II , both are silver coins and in the box III there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what the probability that the other coin in the box is also of gold.

22. Determine graphically the minimum value of the objective function $Z = -50x + 20y$, subject to $2x - y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$; $x, y \geq 0$

23. Show that the four points A, B, C & D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-(\hat{j} + \hat{k})$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

SECTION – D

24. Let $A = R - \{3\}$ $B = R - \{1\}$. Let f: A→B be defined by $f(x) = \frac{x-2}{x-3}$ for all $x \in A$, show that f is bijective. Also find

i. x, if $f^{-1}(x) = 4$

ii. $f^{-1}(7)$

OR

Let A = R X R, let * be a binary operation on A defined by (a,b) * (c,d) = (ad+bc, bd) for all (a,b), (c,d), ∈ R X R.

i. Show that * is commuttative on A

ii. Show that * is associative on A

iii. Find the identity element of * in A

25. Find the vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Hence find whether the plane thus obtained contains the line $\frac{x+2}{5} = \frac{y-3}{4} = \frac{z}{5}$ or not.

OR

Find the vector and cartesian equation of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

26. Using matrix method, solve the following system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

$x, y, z \neq 0$

27. Solve the differential equation : $\frac{dy}{dx}$ - $3y\cot x = \sin 2x$; given y=2 when $x = \frac{\pi}{2}$.

28. Show that a cylinder of given volume which is open at the top has minimum total surface area when its height is equal to the radius of its base.

29. Find the area of the region $\{(x, y): y^2 \leq 4x,\ 4x^2 + 4y^2 \leq 9\}$

OR

Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

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