MODEL EXAMINATION (DEC – 2017)

CLASS: XII   **MATHEMATICS**   **Time: 3 hrs.**

Date……………   **MAX. MARKS : 100**

Name………………………………   **Roll No…….**

(i) *All questions are compulsory.*

(ii) *This question paper contains 29 questions.*

(iii) *Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each*

(iv) *Questions 5-12 in Section B are short-answer type questions carrying 2 marks each.*

(v) *Questions 13-23 in Section C are long-answer-I type questions carrying 4 marks each.*

(vi) *Questions 24-29 in Section D are long-answer-II type questions carrying 6 marks each.*

SECTION A

1. Let $*$ be a binary operation on N given by $a * b = \text{LCM}(a, b)$ for all $a, b \in N$. Find $5 * 7$ (1)

2. Find the direction ratios of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.   (1)

3. Find the value of x if $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$ (1)

4. If $f(x) = |x|$ and $g(x) = |5x - 2|$ then find gof.   (1)

SECTION B

5. Solve $\tan^{-1} \dfrac{1 - x}{1 + x} = \dfrac{1}{2} \tan^{-1} x$, $x > 0$ (2)

6. If $A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$,then find the value of λ for which A^{-1} exists.   (2)

7. Solve for x: $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$, $x \neq \dfrac{\pi}{2}$ (2)

8. If $y = x^4 - 10$ and if x changes from 2 to 1.99, what is the approximate change in y?   (2)

9. Evaluate $\int \dfrac{2x + 1}{(x + 2)(x - 3)} dx$ (2)

10. Find the general solution of differential equation $\dfrac{dy}{dx} = \dfrac{1 - \cos x}{1 + \cos x}$ (2)

11. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude 1 and 2 respectively when $|\vec{a} \times \vec{b}| = \sqrt{3}$ (2)

12. If A and B are two events such that $P\left(\dfrac{A}{B}\right) = p, P(A) = p, P(B) = \dfrac{1}{3}$ and $P(A \cup B) = \dfrac{5}{9}$, then find the value of p. (2)

SECTION C

13. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2 \sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then find $\lim\limits_{t \to 0} \dfrac{f(t)}{t^2}$ (4)

14. Find the value of p and q, for which (4)

$$f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3 \cos^2 x} & , \quad if \ x < \dfrac{\pi}{2} \\ p & , \quad if \ x = \dfrac{\pi}{2} \\ \dfrac{q(1 - \sin x)}{(\pi - 2x)^2} & , \quad if \ x > \dfrac{\pi}{2} \end{cases}$$ is continuous at $x = \dfrac{\pi}{2}$

OR

Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

15. If $y = \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$, then show that $(1 - x^2) \dfrac{d^2y}{dx^2} - 3x \dfrac{dy}{dx} - y = 0$. (4)

16. Determine the interval, where $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing. (4)

OR

Find the equation of tangent to the curve $y = \dfrac{x - 7}{(x - 2)(x - 3)}$ at the point, where it cuts the x-axis.

17. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the following formula: (4)

$C(x) = x^3 - 45x^2 + 600x$, Where x is the number of trees and $C(x)$ is the cost of planting x trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? Use calculus to answer these questions. **Which value is being exhibited by the person?**

18. Evaluate $\displaystyle\int_0^1 \cot^{-1}(1 - x + x^2)\, dx$ (4)

19. Solve the differential equation $\frac{dy}{dx} + y \cot x = 2 \cos x$, $given\ that\ y\left(\frac{\pi}{2}\right) = 0$ (4)

OR

Show that the following differential equation is homogeneous and then solve it.

$$y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$$

20. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that ($\vec{a} - \vec{d}$) is parallel to $(\vec{b} - \vec{c})$, it is being given that $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$. (4)

21. Find the vector and cartesian equation of the line passing through the point (1, 2, −4) and perpendicular to the lines (4)

$$\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7} \text{ and } \frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$$

22. A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white? (4)

23. In a game, a man wins Rs 5 for getting a number greater than 4 and loses Rs 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses? (4)

SECTION D

24. If A = R −{0} and ∗ defined on A such that a∗ b = 2ab then prove that (6)

(i)   '∗' is commutative

(ii)   '∗' is associative

(iii) Write identity element of ∗ on A

(iv)   If inverse exists find the inverse of 'a'

OR

Let $A = R-\{3\}$, $B = R-\{1\}$. Find $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, $\forall x \in A$. Show that f is bijective. Also find (i) x, if $f^{-1}(x) = 4$ (ii) $f^{-1}(7)$

25. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

(6)

$x - y + 2z = 1, 2y - 3z = 1, 3x - 2y + 4z = 2$

OR

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, find A^{-1} using elementary row transformations.

26. Using integration find the area of the region (6)

$\{(x, y): 25x^2 + 9y^2 \le 225 \text{ and } 5x + 3y \ge 15\}$

27. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to $2, 3, -6$. (6)

28. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each executive class ticket and Rs 300 is made on each economy ticket. The airline reserves at least 20 seats for executive class. However, at least four times as many passengers prefer to travel by economy class than by executive class. Determine, how many tickets of each type must be sold in order to maximise the net profit of the airline. Make the above as LPP and solve graphically. (6)

29. Evaluate $\int\limits_{0}^{\pi} e^{2x} . \sin\left(\frac{\pi}{4} + x\right) dx$ (6)

OR

Evaluate $\int \frac{x+2}{\sqrt{x^2 + 5x + 6}} dx$