SECOND PRE-BOARD EXAMINATION (2017-18)
CLASS: XII

Subject: MATHEMATICS **Date: 17.01 .2018**

Time allowed: 3 Hours. Maximum Marks: 100

General instructions:

(1) **All** questions are **compulsory**.
(2) This question paper contains **29** questions.
(3) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(4) Question **5 - 12** in **Section B** are short-answer type questions carrying **2** mark each.
(5) Question **13 - 23** in **Section C** are long-answer-I type questions carrying **4** mark each.
(6) Question **24 - 29** in **Section D** are long-answer-II type questions carrying **6** mark each.

Section – A
Questions 1 to 4 carry 1 mark each.

1.If $|A^{-1}|=\frac{1}{3}$ and $A_{3\times 3}$ matrix then find $|2A|$.

2.If $A=\{1,2,3\}$ add minimum number of ordered pairs to the relation $R=\{(1,2),(23)\}$ so that it is symmetric and transitive.

3.Find the value of $tan^{-1}\left(tan\frac{13\pi}{6}\right)+Cos^{-1}\left(Cos\frac{7\pi}{6}\right)$.

4.If $a\hat{\imath}+3\hat{\jmath}-b\hat{k}$ is parallel to vector obtained by joining the points (1,2,3) and (2,-4,1) then find a,b.

Section – B
Questions 5 to 12 carry 2 marks each.

5.If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then find k if $A^2 = kA - I$, where I=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6.If $f(x) = \begin{cases} ax + b & if\ x < 3 \\ cx + 3 & if\ x \geq 3 \end{cases}$ is differentiable at x = 3. Find a,b,c.

7.If $x = \sqrt{a^{Sin^{-1}t}}, y = \sqrt{a^{Cos^{-1}t}}$, show that $\frac{dy}{dx} = \frac{-y}{x}$.

8.Evaluate $\int \frac{Cos2x}{(Cosx+Sinx)^2}$ dx

9.Evaluate $\int_0^2 e^x\, dx$ as the limit of a sum.

10.Find differential equation of family of circles touching the coordinate axes in the first quadrant.

11.Find the volume of the parallelepiped whose co initials side vectors are given by $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}, 2\hat{\imath} - \hat{\jmath} + \hat{k}\ and\ 3\hat{\imath} + 2\hat{k} - \hat{\jmath}$.

12.A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Section – C

Questions 13 to 23 carry 4 marks each.

13.Consider $f: R_+ \to [-5, \infty[\ given\ by\ f(x) = 9x^2 + 6x - 5$. Show that f is invertible hence find $f^{-1}(x)$.

14.Solve the equation $2tan^{-1}(Cosx) = tan^{-1}(2Cosecx)$.

15.Prove that $\begin{vmatrix} a + bx & c + dx & p + qx \\ ax + b & cx + d & px + q \\ u & v & w \end{vmatrix} = (1 - x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

OR

Prove that $\begin{vmatrix} 3a & b - a & c - a \\ a - b & 3b & c - b \\ a - c & b - c & 3c \end{vmatrix} = 3(a + b + c)(ab + bc + ca)$

16.Differentiate the function $y = Sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.

17.Find the slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point (2, -1).

18.Evaluate $\int_0^{\pi} \frac{x}{a^2Cos^2x+b^2Sin^2x} dx$

OR

Evaluate $\int \frac{e^{tan^{-1}x}}{(1+x^2)^2}$ dx

19.Express the vector $\vec{a} = 5\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{\imath} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

20.Evaluate $\int \left\{\log(logx) + \frac{1}{(logx)^2}\right\} dx$

21.Find the angle between the lines$\frac{x-1}{2} = \frac{y}{3}; z = 6\ and\ x = 2; \frac{y}{1} = \frac{z}{2}$. Find the shortest distance between them if they are skew lines.

22.The probability of a shooter hitting a target is ¾. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

OR

Find the probility distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distributioin.

23.Assume that the chances of a patient having a heart attack is 40%. It is also assumed that the meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options and patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Section – D

Questions 24 to 29 carry 6 marks each.

24.If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\ and\ B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}\ then\ find\ (AB)^{-1}$.

25.A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m. , find the dimensions of the rectangle that will produce the largest area of the window.

26.Using integration, compute the area bounded by the line

$x + 2y = 2, y - x = 1\ and\ 2x + y = 7$

OR

Find the ratio of the areas in to which curve $y^2 = 6x$ divides the region bounded by $x^2 + y^2 = 16$

27. Show that the differential equation $2ye^{x/y}dx + \left(y - 2xe^{x/y}\right)dy = 0$ is homogeneous and find its particular solution when x = 0 when y = 1.

OR

Find the particular solution of the differential equation

$\frac{dy}{dx} + yCotx = 2x + x^2Cotx\ given\ that\ y = 0, when\ x = \frac{\pi}{2}$.

28.Find the equation of the plane containing the lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y-2}{1} = \frac{z+2}{-2}$. Also find the distance of this plane from the point (1,1,1).

OR

Prove that the image of the point (3,-2,1) in the plane 3x – y + 4z = 2 lies on the plane, x + y +z +4=0.

29.(a) Maximise and minimise Z = 5x +10y for $x + 2y \leq 120, x + y \geq 60$ $x - 2y \geq 0, x \geq 0, y \geq 0$. (4 Marks)

(b)A man has Rs. 1500 for purchase of rice and wheat. A bag of rice and a bag of wheat costs Rs. 180 and Rs. 120, respectively. He has storage capacity of 10 bages only. He earns a profit of Rs. 11 and Rs. 9 per bag of rice and wheat, respectively. Formulate an LPP to maximize the profit. (2 Marks)

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