PRE-BOARD EXAMINATION (2019-20)

GRADE: XII [CBSE] TOTAL MARKS: 80

MATHEMATICS

DATE: 27/01/2020 TIME: 3Hours

General Instructions:-

(i) All the questions are compulsory.

(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4marks each. Section D comprises of 4 questions of 6 marks each.

(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION – A

Q1 - Q10 are multiple choice type questions. Select the correct option.

1. Let A be 3x3 matrix such that $|A|$= - 2, then $|-2A^{-1}|$ is equal to
(a) 4 (b) -4 (c) 8 (d) -2

2. The matrix P = $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a
(a) square matrix (b) diagonal matrix (c) scalar matrix (d) all the above

3. If $\vec{a}\ .\ \vec{b}$ = $\vec{a}\ .\ \vec{c}$ and $\vec{a}$ x $\vec{b}$ = $\vec{a}$ x $\vec{c}$, $\vec{a}$ ≠0, then
(a) $\vec{b}-\vec{c}$ is parallel to $\vec{a}$ (b) $\vec{b}-\vec{c}$ is perpendicular to $\vec{a}$ (c) $\vec{b}-\vec{c}=0$ (d) none of these

4. Let A and B be two events such that P(A) = 0.6, P(B) = 0.2 and P(A/B) = 0.5, then P(A'/B') equals
(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

5. Maximum value of the function Z = 11x +7y, subject to the constraints : x ≤ 3, y ≤ 2, x≥ 0 And y≥ 0 is

(a) 33 (b) 14 (c) 54 (d) 47

6. Which of the following corresponds to the principal value branch of $\tan^{-1}x$?

(a) $(\frac{-\pi}{2}, \frac{\pi}{2})$ (b) $[\frac{-\pi}{2}, \frac{\pi}{2}]$ (c) $(\frac{-\pi}{2}, \frac{\pi}{2})$ - {0} (d) $(0, \pi)$

7. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum is 3

(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

8. If $\int_0^1 \frac{e^t}{1+t}$ dt = a, then $\int_0^1 \frac{e^t}{(1+t)^2}$ dt is equal to

(a) a -1 +$\frac{e}{2}$ (b) $a + 1 - \frac{e}{2}$ (c) $a - 1 - \frac{e}{2}$ (d) a +1 +$\frac{e}{2}$

9. The reflection of the point (α, β, γ) in the xy-plane is

(a) $(\alpha, \beta, 0)$ (b) $(0, 0, \gamma)$ (c) $(-\alpha, -\beta, -\gamma)$ (d) $(\alpha, \beta, -\gamma)$

10. A plane makes intercepts 4 and 3 respectively on x-axis and z-axis. If it is parallel to y-axis, then its equation is

(a) 3x +4z =12 (b) 3z + 4x =12 (c) 3y + 4z =12 (d) 3z +4y =12.

(Q11 - Q15) Fill in the blank

11. Let R be the equivalence relation in the set Z of integers given by R = { (a,b): 2 divides a –b}. The equivalence class [0] is -----------------.

12. The value of b for which the function f(x) = $\begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^2 + 3bx, & 1 < x < 2 \end{cases}$ is continuous at every point of its domain is ------------------------.

13. If A and B are symmetric matrices of same order, then AB is symmetric if and only if ------------.

14. The values of k for which the function f(x) = kx^3 + 5 decreasing is -------------.

OR

The interval in which the function f(x) = x - $\frac{1}{x}$ increasing is ------------.

15. $\vec{a}.(\vec{b} \times \vec{a})$ = ------------

OR

If $(\hat{\imath} + 3\hat{\jmath} + 9\hat{k}) \times (3\hat{\imath} - p\hat{\jmath} + q\hat{k}) = \vec{0}$, then the values of p and q are -----------.

(Q16 - Q20) Answer the following questions

16. If x ∈ N and $\begin{vmatrix} x & 3 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$, find the value of x.

17. Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \cos^2 x\,dx$.

18. Evaluate $\int \frac{\sin^6 x}{\cos^8 x}$ dx **OR** $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}$ dx

19. Evaluate $\int_0^{\frac{\pi}{2}} \log \tan x\, dx$

20. Find the order and degree of differential equation: $\frac{d^4y}{dx^4} + \sin(\frac{d^3y}{dx^3}) = 0$.

SECTION – B

21. Prove that $\sin^{-1}\frac{3}{5} = \frac{1}{2}\tan^{-1}\frac{24}{7}$

OR

Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$.

22. Solve the differential equation $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$.

23. Show that the function f defined by $f(x) = \tan^{-1}(\sin x + \cos x)$ is strictly increasing in the interval $\left(0, \frac{\pi}{4}\right)$.

24The two adjacent sides of a parallelogram are $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the unit vectors along the diagonals of the parallelogram.

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

25.If a plane meets the coordinate axes in the points A, B, C and the centroid of the triangle ABC Is (α, β, γ), find the equation of the plane.

26. A die is thrown 3 times, if the first throw is a four, then find the probability of getting 15 as a sum.

SECTION – C

27. Let f: W → W be defined as $f(n) = \begin{cases} n-1, \text{if } n \text{ is odd} \\ n+1, \text{if } n \text{ is even} \end{cases}$. Show that f is invertible. Also, find the inverse of f.

28. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then prove that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

It is given that for the function $f(x) = x^3 + bx^2 + ax + 5$ on [1,3], Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$ .Find the values of a and b.

29. Find the particular solution of the following differential equation ,
Cos y dx + $(1 + 2e^{-x})$ sin y dy =0; $y(0) = \frac{\pi}{4}$.

30. Evaluate $\int_0^2 (x^2+3)\,dx$ as limit of sums.

31. A problem in Mathematics is given to four students A, B, C and D. Their chances of solving the problem, respectively are $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{2}{3}$. What is the probability that (i) the problem will be solved ? (ii) atmost one of them solve the problem?

OR

Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1black and 1red balls ; Bag III contains 4 white, 3black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III.

32. If a 20 year old girl drives her car at 25 km/hr, she has to spend Rs 4 per km on petrol. If she drives her car at 40 km/hr, the petrol cost increases to Rs 5 per km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Using LPP find the maximum distance she can travel.

SECTION – D

33. Using the properties of determinants, prove the following:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$

OR

If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

x + 2y +z = 4; -x + y + z = 0; x – 3y + z = 2.

34. Using integration, find the area of the region in the first quadrant bounded by the curve y = $x\,|x|$, the circle $x^2 + y^2 = 2$ and y axis.

35. Find the shortest distance between the line x- y +1= 0 and the curve $y^2 = x$.

OR

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

36. Find the equation of the plane through the line $\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ and parallel to the line $\frac{x+1}{2} = \frac{1-y}{4} = \frac{z+2}{1}$ .Hence, find the shortest distance between the lines.

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