

# MATHEMATICS

## Standard V

### Part - II



Government of Kerala  
Department of Education

State Council of Educational Research and Training (SCERT), KERALA  
2016

## The National Anthem

Jana-gana-mana adhinayaka, jaya he  
Bharatha-bhagya-vidhata.  
Punjab-Sindh-Gujarat-Maratha  
Dravida-Utkala-Banga  
Vindhya-Himachala-Yamuna-Ganga  
Uchchala-Jaladhi-taranga  
Tava subha name jage,  
Tava subha asisa mage,  
Gahe tava jaya gatha.  
Jana-gana-mangala-dayaka jaya he  
Bharatha-bhagya-vidhata.  
Jaya he, jaya he, jaya he,  
Jaya jaya jaya, jaya he!

## PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give respect to my parents, teachers and all elders and treat everyone with courtesy.

I pledge my devotion to my country and my people. In their well-being and prosperity alone lies my happiness.

*Prepared by :*

**State Council of Educational Research and Training (SCERT)**

Poojappura, Thiruvananthapuram 695 012, Kerala

*Website :* [www.scertkerala.gov.in](http://www.scertkerala.gov.in)

*E-mail :* [scertkerala@gmail.com](mailto:scertkerala@gmail.com)

Phone : 0471-2341883, Fax : 0471-2341869

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Dear children,

We have learnt much about Numbers and Shapes.

We'll now see larger numbers and fractions.

Work with them and see their peculiarities.

Use them to solve problems.

We'll also see new ideas in Geometry.

And draw new shapes.

Let's think logically, draw precisely.

Find new connections.

And move ahead with confidence.

**Dr. J. Prasad**  
Director  
SCERT

# TEXTBOOK DEVELOPMENT TEAM

## Participants

**Rameshan N. K.**

H.S.A., RGMHS Panoor, Kannur

**Kunhahmmad T. P.**

UPSA, GMUP School, Tiruvallur

**T. P. Prakashan**

HSA, GHSS Vazhakkad,  
Malappuram

**Ravikumar T.S.**

UPSA, GUPS, Anjachavadi,  
Malappuram

**Anita V. S.**

Lecturer, DIET, Thiruvananthapuram

**Susheelan K.**

BRC, Trainer, Thirur, Malappuram

**Vasudevan K. P.**

Master Trainer, IT@School Project, Thrissur

**Veeran Kutty K.**

UPSA, CHMKMAUPS Mundakkulam,  
Malappuram

**Rawayath M. K.**

Teacher, GHS, Bemmannur, Palakkad

**Krishnadas Paleri**

UPSA, GUPS Kodyamme, Kasaragod

## Experts

**Dr. Ramesh Kumar P.**

Asst. Prof. Kerala University

**Dr. Mumtaz N.S.**

Associate Prof. Farook Training College, Kozhikode

## Artist

**Dhaneshan M.V.**, AVS GHSS, Karivelloor, Kannur

**Kunhiraman P.C.**, DIET Ernakulam

**Harikumar K.B.**, Kazhakkuttam, Thiruvananthapuram

**Hari charutha**, Nemam Thiruvananthapuram

## ENGLISH VERSION

**Venugopal C.**

Asst. Professor, Govt. College  
of Teacher Education, Thiruvananthapuram

**Biju G. S**

Asst. Professor,  
College of Engineering, Thiruvananthapuram

**Dr. E. Krishnan**

Prof.(Rtd) University College,  
Thiruvananthapuram

## Academic Co-ordinator

**Sujith Kumar G.**, Research Officer, SCERT



State Council of Educational Research and Training (SCERT)

Vidya Bhavan, Thiruvananthapuram

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Certain icons are used in this textbook  
for convenience



*Computer Work*



*Additional Problems*



*Project*



*Self Assessment*

6

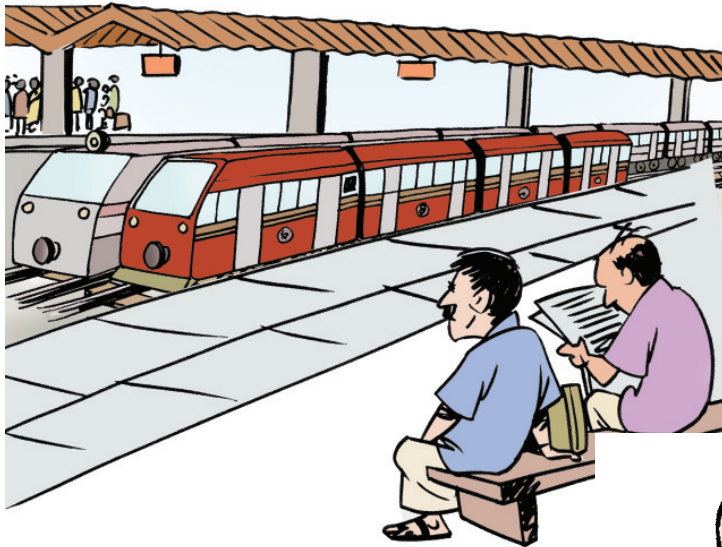
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# Area



## Which is longer?

See the two trains at a station. Which is longer?



Remya has two ribbons. She is trying to find out the longer one. Which is longer?

How do we find out?

Hold them side by side, right?

Here's a picture of two lines in Ravi's notebook:



How do we find out which is longer?

Here we can't get them close together to check.

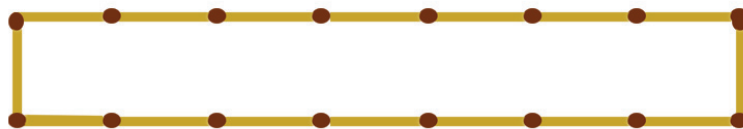
And we can't tell at a glance, like the trains in the first picture.

But we can measure and check, right?

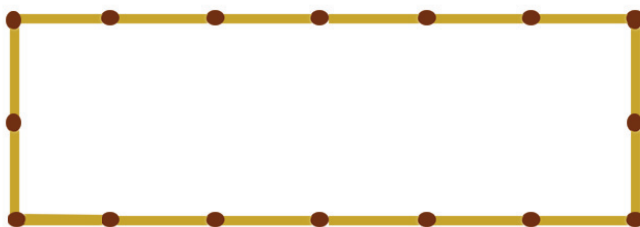


## Matchstick rectangles

Jose and Rahim have 16 matchsticks each. All of the same length. These are the rectangles they made with their matchsticks.



Jose's rectangle



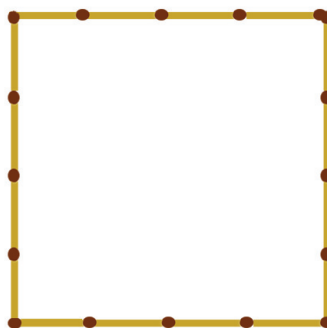
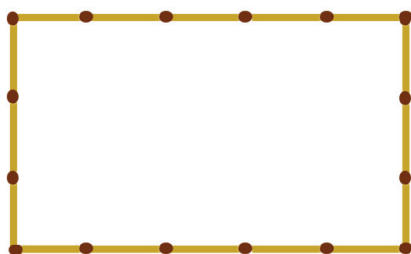
Rahim's rectangle

Which is longer?

Which is wider?

How did you find out?

Can we make other rectangles with 16 matchsticks?



Can we make any other rectangles?

Here we made four rectangles. What can we say about them?

16 matchsticks in each.

The first is 7 matchsticks long and 1 matchstick wide.

The second is 6 matchsticks long and 2 matchsticks wide.

The third is 5 matchsticks long and 3 matchsticks wide.

The fourth is 4 matchsticks long and 4 matchsticks wide.

In all these, is there any relation between the number of matchsticks along the length and width, and the total number of matchsticks?

Find out and write down.

In what all ways can we make rectangles with 20 matchsticks?

Draw these in your notebook.

What about 24 match sticks?

Can we make rectangles with 15 match sticks? Why?



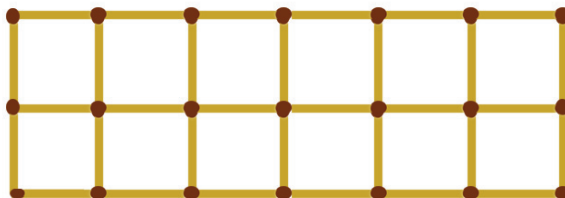
### Squares in rectangle

Remember the rectangles made by Jose and Rahim. Jose split his rectangle into small squares using more matchsticks like this.



How many squares are there?

What about splitting Rahim's rectangle into squares of the same size?



### Eerkkil rectangle

Two *eerkkil* pieces were bent to make rectangles like this:

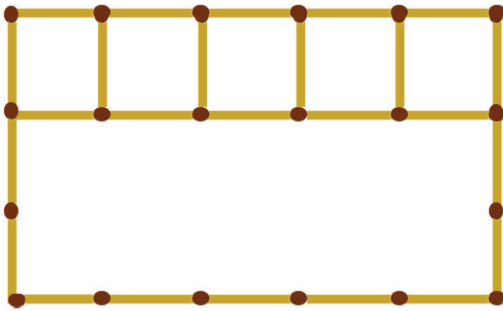


Which rectangle is made with the longer piece? How do we find out?

We need only straighten the pieces and place them side by side.

The rectangle made with the longer piece has greater perimeter, right?

In a rectangle having 5 matchsticks long and 3 matchsticks wide, how many such squares can we make?



How many rows of squares can we make?

How many squares in one row?

What is the total number of squares?

In the other rectangles made with 16 matchsticks, can't we make small squares like this?

Find out the number of squares in each of these.

- How many small squares can we make in a rectangle with 14 matchsticks long and 6 matchsticks wide?

### Paper square

Rani and Veena have rectangular cardboard pieces.

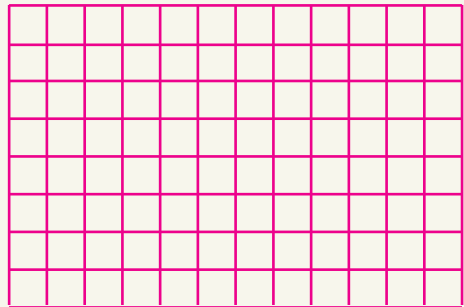
Rani's board is 7 centimetres long and 3 centimetres wide. Veena's board is 6 centimetres long and 4 centimetres wide.

From whose cardboard can we cut out more squares of side 1 centimetre?

Before cutting, let's draw the squares.

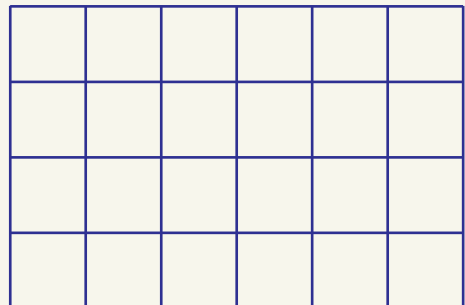
### Different squares

A rectangle is made with 40 sticks of the same length. See how it is split into squares:



How many squares are there?

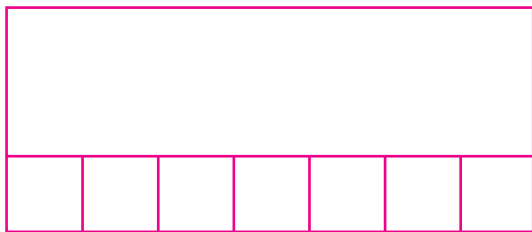
The picture below shows squares made with two sticks to a side.



How many squares now?

Can you fill the rectangle with squares of any other size?

First let's take the cardboard piece of length 7 centimetres and width 3 centimetres?



In one row we can draw 7 squares of side 1 centimetre.

How many rows like this can we make?

Total number of squares = .....

Now, can't you find out how many squares Veena can cut out from her cardboard?



What did you get?

Veena got 3 squares more than Rani.

Why did this happen?

Because Veena's piece has more space within, right?

### Measure of space

Look at the red and blue rectangles.

Which has more space within?

We can see at a glance that the red rectangle has more space inside.

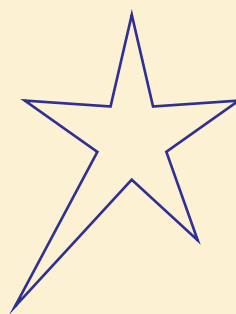
What can you say about the space within two ten rupee notes?

What about if it is a 10 rupees note and a 100 rupees note?

Which has more space within?

### Colour them

See the figures below:

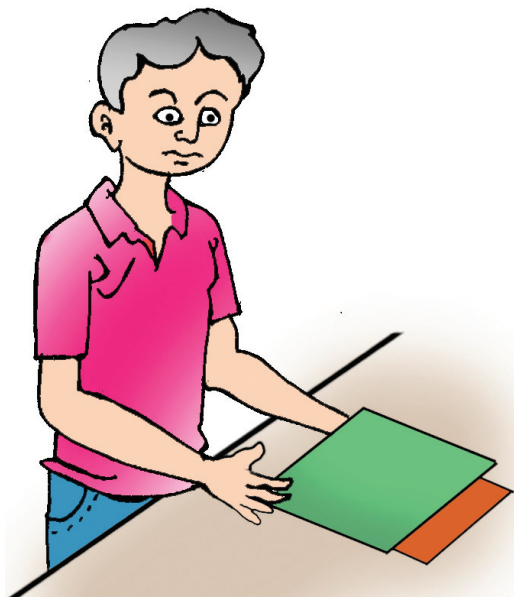


Give same colour to both the pictures.

Which figure needs more colour?



## A number for space



Ravi has two cardboard pieces. He is trying to find out which has more space within.

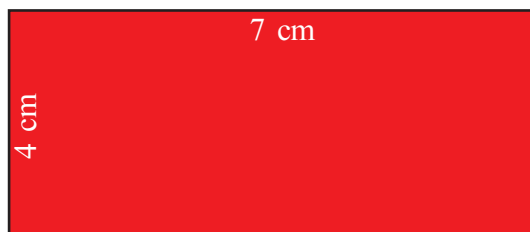
He placed one above the other.

Can he say which one has more space inside? Why?

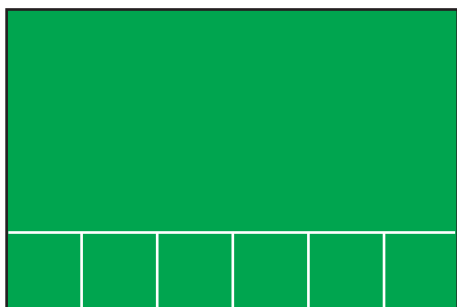
How do we help him?

All we have to do is to see how many squares of the same size can be cut out from each.

For that, let's measure the length and width of each cardboard and write down.



Next let's see how many squares of side 1 centimetre can be drawn within each.



Here there are six squares in a row, and five such rows. So,  $5 \times 6 = 30$  small squares in all.

What about the red rectangle?

$4 \times 7 = 28$  small squares, right?

Now we can say which one has more space inside.

Here we can say that the space inside the red rectangle is equal to the total space within 28 small squares.

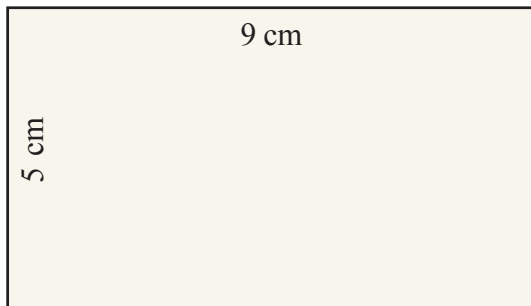
We say that the *area* of a square of side 1 centimetre is 1 square centimetre.

Then we can say that the area of the red rectangle is 28 square centimetres.

What about the area of the green rectangle?

### Area formula

See the rectangle drawn below.



How do we find out its area without drawing small squares?

Imagine a grid of horizontal and vertical lines 1 centimetre apart inside the rectangle.

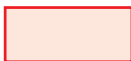
How many squares are there in each row?



How many such rows?



Now can't you say what is the area of the rectangle?



We can write this as;

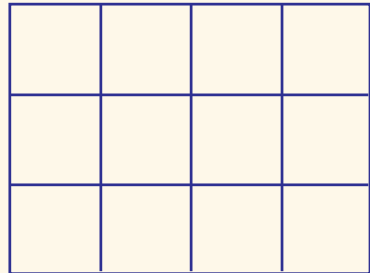
Area of the rectangle = length  $\times$  width

Now can't you easily compute the area of a rectangle of length 20 centimetres and width 10 centimetres?

### Common method

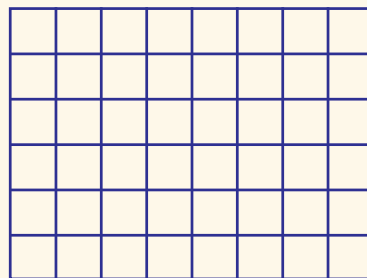
What is the area of a rectangle with length 8 centimetres and breadth 6 centimetres?

Ammu explained it like this:



Area of the rectangle is equal to the area of 12 small squares.

Anu described it like this:



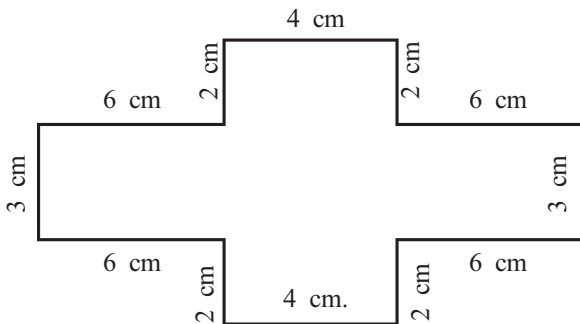
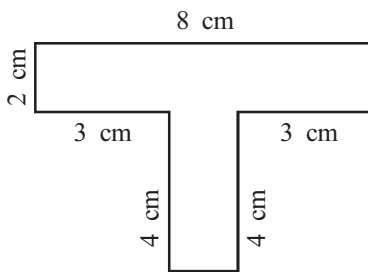
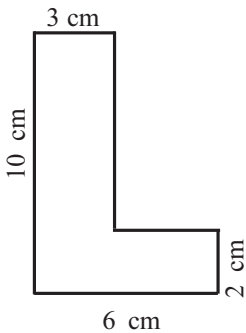
Area of the rectangle is equal to the area of 48 small squares. Both answers are right, aren't they?

But numbers for the area are different.

To avoid such confusion, we generally say areas in terms of squares of side 1 centimetre.

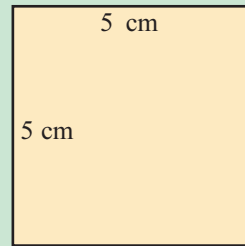
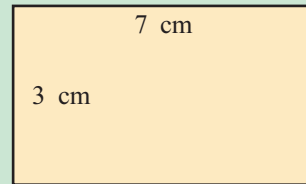
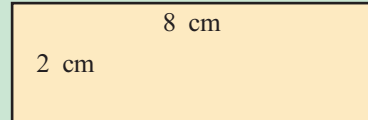
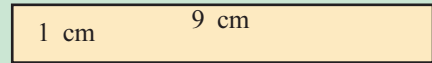


- Find the area of a rectangle of length 15 centimetres and width 8 centimetres.
- All the sides of a rectangle are of 8 centimetres. Find its area.
- The area of a rectangle is 96 square centimetres and its length 12 centimetres. What is its width?
- The area of square is 81 square centimetres. Find the length of a side.
- Find the area of the figures given below:



## Perimeter and Area

Find the perimeter and area of the rectangles, drawn below:



Do the perimeters change? What about the areas?

Which rectangle has the largest area?

Usually we talk about the size of rectangles based on its area.

In practical situations, the use of perimeter or area depends on the context.

For example, to fence a rectangular plot, we consider its perimeter. If we are thinking of farming in the plot, it is the area that we consider.



## Project

If the length of a rectangle is doubled without changing the width, what would be the change in area? What if width is doubled without changing the length? What if both length and width are doubled? If the length and width of a rectangle are multiplied by the same number, what would be the change in area?



- If the length of a rectangle is 6 centimetres and the width is 5 centimetres then what is its area? Without changing its width, if the length is increased to 12 centimetres, what would be the relation between the area of the new rectangle and the area of the original rectangle?
- Length of a rectangle is 10 centimetres and width 8 centimetres. If its length and width are doubled, what would be the area of the new rectangle. How many times the area of the original rectangle is the area of the new rectangle?
- Perimeter of a rectangle is 48 centimetres and its width is 9 centimetres. Calculate its area.
- A rectangle of area 40 square centimetres is to be drawn and its length and width in centimetres should be natural numbers. Find all possibilities.

## Large squares

If the length of one side of a square is 1 metre, we say that its area is 1 square metre.

Can you say how many square centimetres is 1 square metre?

You know how many centimetres is 1 metre.

Thus, the sides of this square are 100 centimetres each.

So, how many square centimetres is its area?

## Large measures

For measuring larger plots of lands we will use larger measurements.

The area of a square with side 10 metres is called an are.

100 ares is called a hectare.

So, how many square meters make 1 hectare?

## Larger area

Can you imagine a square of side 1 kilometre? Its area is called 1 square kilometre. The area of very large regions are expressed in square kilometres. The area of our country is 32,87,263 square kilometres.

The area of our state is 38,863 square kilometres. The area of Palakkad, the largest district in our state is 4480 square kilometres and that of Alappuzha, which is the smallest district in our state is 1414 square kilometres.

Try to find the area of your Panchayat.

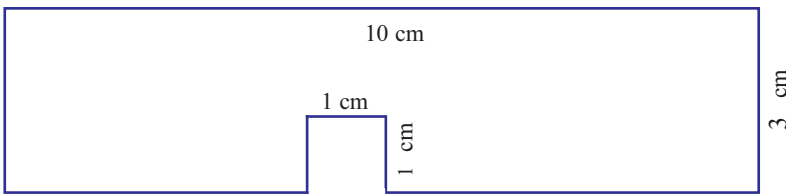


1 square metre =  $100 \times 100 = 10000$  square centimetres.

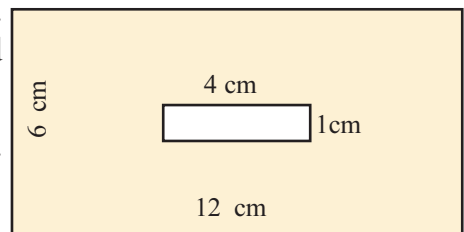
Thus, 1 square metre is the area formed by 10000 small squares each of area 1 square centimetre.



- What is the area of a rectangle of length 5 metres and width 1 metre, in square metre? How much in square centimetres?
- The length and width of a rectangular plot are 40 metres and 25 metres. What is its area in square metres?
- The length and width of a rectangle are 6 metres and 50 centimetres. Find its area in square centimetres. How much is it in square metres?
- From a cardboard piece as shown in the figure, how many squares of side 1 centimetre can be cut out?



- From a rectangular cardboard, 36 squares of side 1 centimetre can be cut off. What are the possible lengths and widths in centimetres, if both are to be natural numbers?
- The sides of a square are 10 centimetres each. If each side is increased by 5 centimetres, what would be the area of the new square?
- The length of a side of a square cardboard is 14 centimetres. From its four corners, squares of side 1 centimetre are cut off. What is the area of the remaining portion? And its perimeter?
- Find the area of the coloured part in the figure.



### Population density

People live in different parts of the world. Some places are very much crowded; and there are places where very few people live.

An estimate of the number of people living in 1 square kilometre within a place is called population density in that place. For example, population density in Kerala is 859 per square kilometre. Among other states, it is highest in Bihar - 1102 per square kilometres; and lowest in Arunachal Pradesh - 17 per square kilometre. In India as a whole, it is 382 per square kilometre.

## Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none"><li>• Finding the area of a rectangle by drawing unit squares in it.</li><li>• Explaining the method of finding the area of a rectangle.</li><li>• Finding the area of a rectangle using the formula.</li><li>• Solving practical problems using ideas related to area.</li><li>• Switching between various units while doing practical problems.</li></ul>			

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# Inside Numbers



### Colour the boxes

Arun and his friends are designing the cover page of the Math Club Magazine.

Neethu suggested drawing coloured squares in rows and columns.

They decided to colour all squares in the first row.

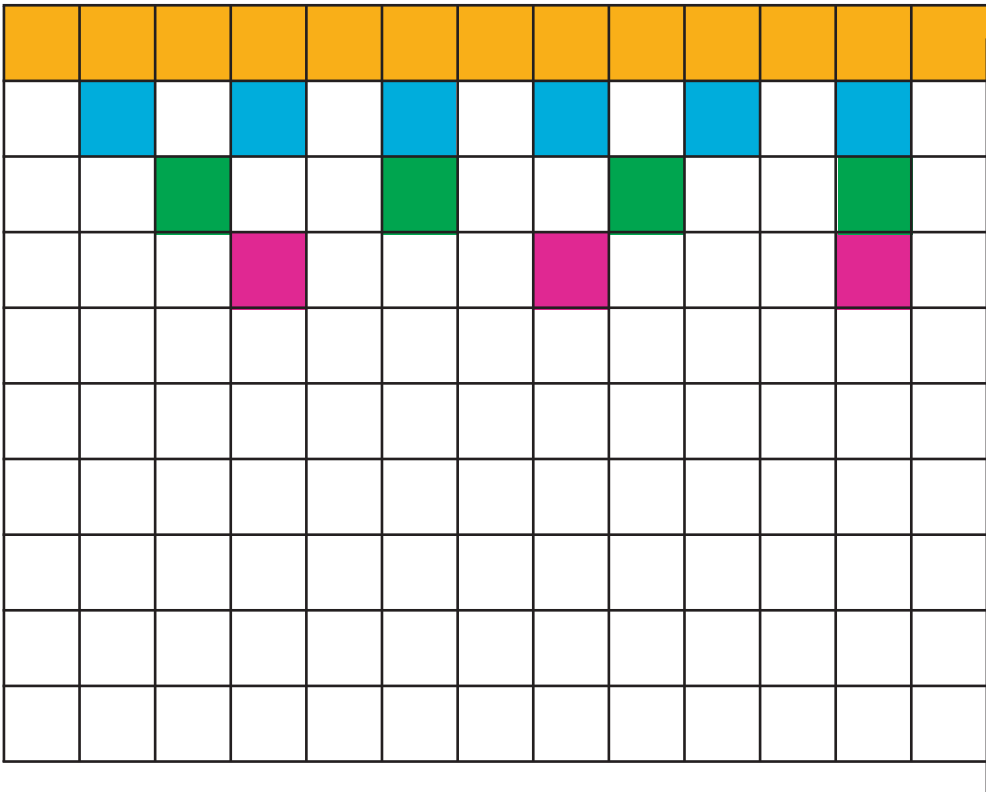
Remya suggested colouring only alternate squares in the second row.

Salma said, "let's colour only every third square in the third row."

What about fourth and fifth rows?

See how the squares are coloured in the first four rows.

Can you find the squares left to colour and complete the design?



Complete the table:

Which are the squares coloured in the second row?	2, 4, ...
Which are the squares coloured in the third row?	3, 6, ...
Which are the squares coloured in the third column?	
Which are the squares coloured in the sixth column?	
Which are the squares coloured in the eighth column?	
In which columns are only two squares coloured?	

If we draw some more rows and columns and colour them in the same way, which are the squares coloured in the second row?

2, 4, 6, 8, ...

What is the peculiarity of this number pattern?

These are the numbers got, starting from 2 and repeatedly adding 2.

In other words, these are the numbers got by multiplying 1, 2, 3,... by 2.

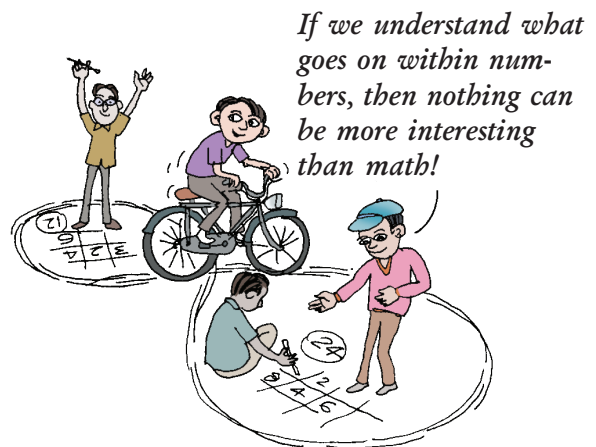
That is, 2, 4, 6, 8, .... are all multiples of 2.

See the third row.

The squares coloured are 3, 6, 9, 12, ....

These are numbers obtained by multiplying 1, 2, 3, .... by 3.

That is, these are all multiples of 3.



Let's write some multiples of 1 to 10 in this table.

Number	Multiples									
1	1	2	3	4	5	6	-	-	-	-
2	2	4	-	-	-					
3					15					
4										
5										
6										
7			21							
8										
9								72		
10				40						

From the table, find answers to these questions:

Which of the numbers are multiples of 1?

10 is a multiple of which all numbers?

Which of the numbers are multiples of both 2 and 3?

Is 56 a multiple of 7?

Which of the numbers are common among the multiples of 5 and 10?

What is the least multiple of 8?

Is every number, the least multiple of itself?



- Write four multiples each of the numbers 12, 20, 36 and 45.

- Ancy and Anna are placing hurdles in the track for the school sports meet. Hurdles are placed 11 metres apart from the starting point. Write in order, the distance to each hurdle from the starting line.



- Razia is climbing the stairs in a building. The height of each step is 25 centimetres. Find out the height to each step from the ground in centimetres.

### Let's measure

Najeem and Manoj are measuring milk. Najeem has a measuring jar of 2 litres and Manoj has a measuring jar of 5 litres.

What are the quantities of milk Najeem can measure out?

Najeem can measure out 2 litres, 4 litres, 6 litres, 8 litres, 10 litres, and so on, right?

Similarly Manoj can measure out 5 litres, 10 litres, 15 litres, 20 litres, and so on.

Can both of them measure out 2 litres of milk?

What about 5 litres?

What are the quantities both of them can measure out?

The quantities Najeem can measure out are

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, and so on.

Similarly Manoj can measure out 5, 10, 15, 20, 25, and so on.

From these, we see that both of them can measure out 10 litres. The other quantities both can measure out are 20 litres, 30 litres, 40 litres, ...



### Division by 4

Can we divide 535 exactly by 4?

No, because 535 is an odd number.

What about 536?

Without actual division, how can we decide?

$$536 = 500 + 36$$

100 is a multiple of 4 and so 500 is also a multiple of 4.

36 is also a multiple of 4.

So, 536 is a multiple of 4.

Since 100, 1000, 10000, ... are all multiples of 4, to verify whether a number is a multiple of 4, we need only check whether the number formed by the last two digits is a multiple of 4 or not.

## Common multiple

What if the measuring jars of Najeem and Manoj are of 3 litres and 4 litres?

Using the 3 litre jar, we can measure out 3 litres, 6 litres, 9 litres, 12 litres and so on.

3, 6, 9, ... are all multiples of 3. Similarly using the 4 litre jar, the quantities which we can measure out are multiples of 4.

Hence the quantities that can be measured out using both these jars are the multiples of both 3 and 4.

12, 24, 36, ... are called the *common multiples* of 3 and 4.

Among them 12 is least.

So, 12 is called the *least common multiple* of 3 and 4.

Similarly, how do we find out the least common multiple of 6 and 8? Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, ...

From these, common multiples of 6 and 8 are 24, 48, 72, ...

Since 24 is the least among this, it is the least common multiple.



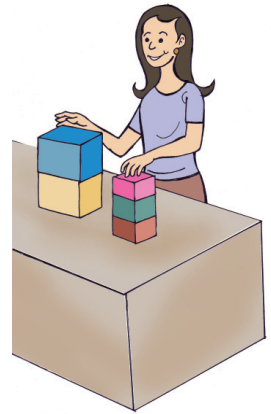
- Write the common multiples of the number pairs given below and find the least common multiple.
  - 2, 5
  - 4, 6
  - 3, 7
  - 5, 10
  - 8, 6
  - 9, 12
  - 12, 14
  - 9, 18
- For the school anniversary, the gateway is decorated with blinking lights. The green lights glow every 4 seconds and the blue lights glow every 6 seconds. They glow together at 8 o' clock. At what time do they glow together next?



- Meenu and Asha are playing on a tiled floor. From one end, Meenu put a button on every second tile and Asha put a ring on every third tile. Which is the first tile with both a button and ring? What about the other tiles with both?



- Anju is making two stacks, one with cubes of sides 4 centimetres and the other with cubes of sides 9 centimetres. To make the heights of the stacks equal, what should be the least possible height of each?



### Multiple of multiples

Write all natural numbers starting from 1 and circle the numbers which are multiples of 2:

1, (2) 3, (4) 5, (6) 7, 8, 9, 10, 11, 12, 13, 14, 15, ...

Next, draw squares around the multiples of 4:

1, (2) 3, (4) 5, (6) 7, (8) 9, 10, 11, 12, 13, 14, 15, ...

Here all the numbers in squares are in circles also.

That is, multiples of 4 are all multiples of 2 also.

On the other hand, is every multiple of 2, a multiple of 4 also?

Now let's check whether all the multiples of 3 are multiples of 2 also.

As before, draw circles around multiples of 2 and squares around multiples of 3.

1 (2) (3) (4) 5 (6) 7 (8) (9) (10) 11 (12) 13, ...

From this, we can see that not every multiple of 3 is a multiple of 2.

Also, not every multiple of 2 is a multiples of 3.

Now verify the following:

- Is every multiple of 3, a multiple of 6 also?
- Is every multiple of 6, a multiple of 3 also?
- Is every multiple of 4, a multiple of 8 also?

- Is every multiple of 8, a multiple of 4 also?
- Is every multiple of 4, a multiple of 6 also?
- Is every multiple of 6, a multiple of 4 also?

### Factors

6 is a multiple of 2. Another way of saying this is that 2 is a *factor* of 6.

Similarly,

6 is a multiple of 3.

3 is a factor of 6.

This 2 and 3 are factors of 6.

Suppose we want to check whether 45 is a multiple of 3.

3 should be multiplied by what to get 45?

We need only divide 45 by 3.

$$45 \div 3 = 15$$

This shows that,  $15 \times 3 = 45$

Therefore, 45 is a multiple of 3.

From this, we can see that 3 is a factor of 45.

- From the pairs of numbers given below, find those in which the second number is a factor of the first number.
  - 12, 6      ▪ 50, 5      ▪ 45, 7
  - 35, 9      ▪ 62, 8      ▪ 42, 6

### Let's draw a rectangle

12 is a multiple of 2 and 6.

So 2 and 6 are factors of 12.

### Multiples of 8

Since 100 is exactly divisible by 4, to check whether a number is a multiple of 4, we need only check whether the number formed by the last two digits of this number is a multiple of 4.

So how do we check whether a number is a multiples of 8?

For example, is 6424 a multiple of 8?

100 is not a multiple of 8.

But 1000, 10000, 100000 ..., are all multiples of 8.

So 6000 is a multiple of 8 .

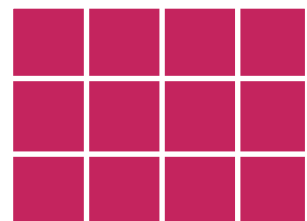
$$6424 = 6000 + 424$$

Now we need only check whether 424 is a multiple of 8 or not.

$$424 = 53 \times 8$$

Thus 6424 is a multiple of 8.

Haven't you found a trick to check whether a number is a multiple of 8?



Does 12 have any other factors? How do we find out?

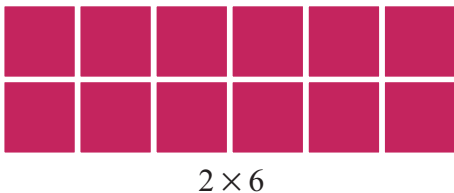
Cut out 12 squares of the same size.

In what all ways can we arrange these 12 squares to construct different rectangles?

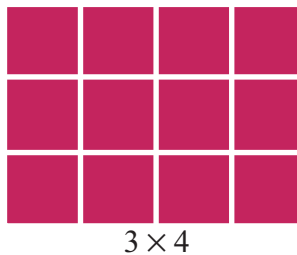
Look at the pictures of all these arranged in a single row:



What if we arrange them in two rows?



We can also arrange them in 3 rows.



Any other way to make rectangle?

From these rectangles, we see that 1, 2, 3, 4, 6, 12 are all factors of 12.

Similarly find all the factors of 24 and write them down:

$$24 = 1 \times 24$$

$$24 = 2 \times 12$$

$$\underline{\quad} = \underline{\quad} \times \underline{\quad}$$

$$\underline{\quad} = \underline{\quad} \times \underline{\quad}$$

.....

Factors of 24 are 1,     ,     ,     ,     ,

    ,     

### Multiple of 9 and digital root

How do we check whether a number is a multiple of 9? We need only check whether the sum of the digits is a multiple of 9. Or check whether the digital root of the number is 9 or not.

Why does this work?

For example, take the number 342.

$$342 = 3 \times 100 + 4 \times 10 + 2$$

We can write this as

$$(3 \times 99 + 3) + (4 \times 9 + 4) + 2.$$

$3 \times 99$  and  $4 \times 9$  are multiples of 9.

The remaining is  $3 + 4 + 2$ .

If this sum is a multiple of 9, then 342 is a multiple of 9.

Here  $3 + 4 + 2 = 9$ . So, 342 is a multiple of 9.

What about a four digit number?

For example,

$$8631 = 8 \times 1000 + 6 \times 100 + 3 \times 10 + 1$$

$$= (8 \times 999 + 8) + (6 \times 99 + 6) + (3 \times 9 + 3) + 1$$

$8 + 6 + 3 + 1 = 18$ , a multiple of 9.

So, 8631 is a multiple of 9.

Check this for other four digit numbers.

In a similar manner, can you explain the method to check whether a number is a multiple of 3?



- Find all the factors of the numbers given below:  
10    18    25    16    36    13
- Rahim has 28 pens. He wants to pack them with the same number of pens in all packets. In what all ways can he do it?
- There are 30 students in a class. For physical training, they are asked to stand in rows, the same number of students in all rows. In what all ways can this be done?
- Jincy has 42 cubes of same size. She wants to put them in stacks, all stacks of the same height. In what all ways can she do this?
- What is the smallest factor of 48? What is its largest factor? What are the other factors? What is the total number of factors?
- Check whether each statement below are true or false:

1 is a factor of every number.

The largest factor of a number is the number itself.

All the numbers except 1 have more than 2 factors.

The number of factors of all numbers except 1 is an even number.

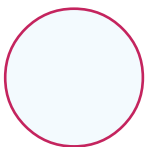
1 is the only number having one factor.

### Without division

From the numbers below, pick out those for which 10 is a factor, those which 5 is a factor and those for which 2 is a factor. Write them in the circle.

50    18    45    40    28    14    25    70

12    20    25    6    9    8    10    5



Numbers with 10  
as a factor



Numbers with 5  
as a factor



Numbers with 2 as  
a factor

What is the peculiarity of numbers in each circle?

Is this true for other numbers for which 10, 5, 2 is a factor? Check it out!

Number	Digit in the one's place
10 as a factor	
5 as a factor	
2 as a factor	

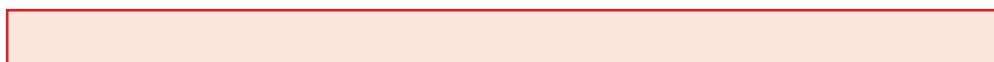
How do we check whether 3 is a factor of a number?

The numbers with 3 as a factor are 3, 6, 9, 12 and so on.

Find their digital roots.

Find the digital roots of other numbers for which 3 is a factor.

Similarly, what is the digital root of the numbers for which 9 is a factor?



For each number in the table below put  mark for factor and  if not a factor.

Number	Factor				
	2	3	5	9	10
12	✓	✓	x	x	x
35					
30					
55					
60					
215					
240					
316					

### Composite and prime

13 is exactly divisible by 1 and 13. Is there any other number which divides 13 exactly?

What are the other numbers which are divisible only by 1 and the number itself?

1, 2, 3, 5, 7, 11, ... are such numbers.

Numbers (other than 1) which are not divisible by numbers other than 1 and the number itself are called *prime numbers*.

In another way, we can say that the only factors of such numbers are 1 and the number itself.

But the numbers 4, 6, 8, 9, 10, ... are divisible by numbers other than 1 and the number itself. Such numbers are called *composite numbers*.

1 is neither composite nor prime.

Separate the following numbers into composite numbers and prime numbers.

9, 17, 26, 23, 45, 31,

36, 29, 48, 64, 41, 51

Find all prime numbers less than 100.

### Prime factors

How can we express 10 as a product of different numbers?

$$1 \times 10$$

$$2 \times 5$$

What about 30?

$$1 \times 30$$

$$2 \times 15$$

$$3 \times 10$$

$$6 \times 5$$

$$2 \times 3 \times 5$$

Here 10 and 30 are expressed as a product of different numbers.

### Sieve of numbers

- For finding the prime numbers, a method developed by Erathosthenees, who lived before Christ, is still used.
- Write all the numbers from 1 to 50.
- Since all the numbers are multiple of 1, it is not considered.
- Keep the next number and strike off all the multiples of this number. Continue this process.
- The remaining numbers are prime numbers.
- For finding the prime numbers less than 100, multiples of up to which number should we strike off?



### Perfect Numbers

Find the sum of all factors of 6 except 6. It is 6 itself.

$$6 = 1 + 2 + 3$$

Numbers having this property are called perfect numbers.

There is one more perfect number less than 50.

Find that perfect number.

The next perfect number is 496.

Until the year 2013, only 48 perfect numbers were found.

### Highly composite numbers

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6

The numbers 2, 4 and 6 have a peculiarity.

The number of factors of 2 is more than the number of factors of 1. Also, 4 has more factors than any smaller number. Such numbers are called highly composite numbers.

We can include 1 also to the collection of highly composite numbers.

Which is the next highly composite number?

We can write these numbers as products of prime numbers only.

That is,

$$10 = 2 \times 5$$

$$30 = 2 \times 3 \times 5$$

Here 2 and 5 are the prime factors of 10.

Similarly 2, 3, 5 are the prime factors of 30.

How can we write 24 as a product of prime numbers?

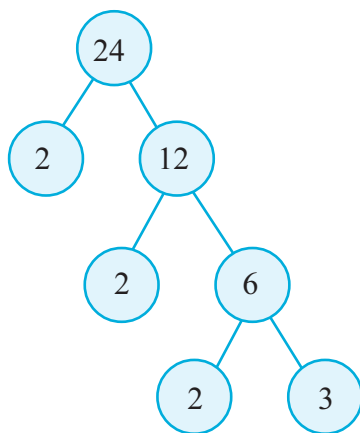
$$24 = 2 \times 12$$

$$12 = 2 \times 6$$

Since,  $6 = 2 \times 3$ , we can write  $24 = 2 \times 2 \times 2 \times 3$

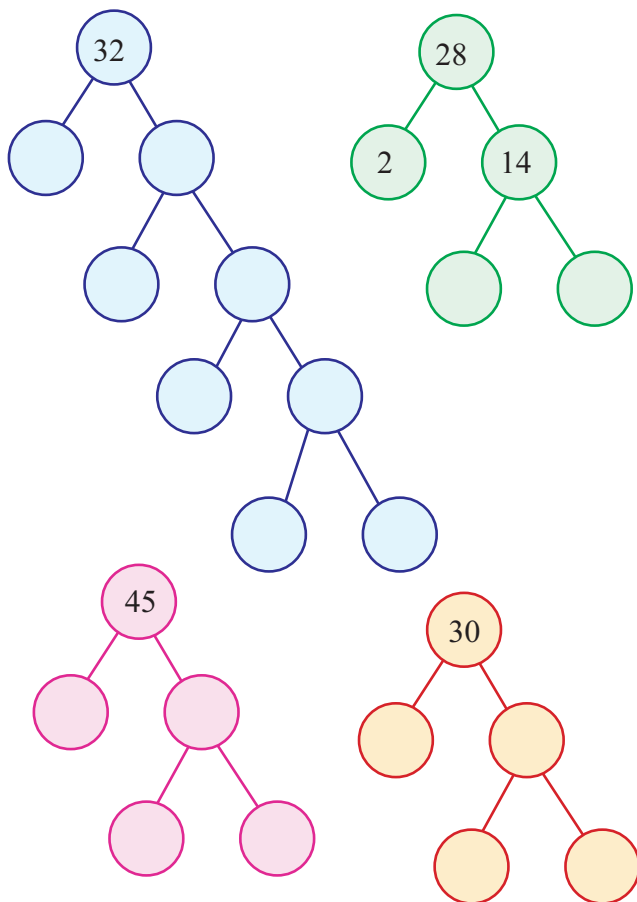
Thus,

$$24 = 2 \times 2 \times 2 \times 3$$



Write the numbers up to 20 as a product of prime numbers and write the prime factors of each number.

- Write the numbers below as a product of prime numbers.



### Basic factors

How can we express 252 as a product of prime numbers?

Here the digit in the ones place is 2. So, 2 is a factor. To find the other factors of 252 divide 252 by 2.

$$252 = 2 \times 126$$

In 126, the digit in the ones place is 6.

So, 2 is a factor.

$$126 = 2 \times 63$$

The digital root of 63 is 9 and 3 is a factor of 9.

### A method to find prime numbers

Write the numbers from 1 to 50 in 6 columns as shown below.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50				

- Keeping 2, strike off the other multiples of 2.
- Keeping 3, strike off the other multiples of 3.
- Similarly keeping 5 and 7, strike off the other multiples of 5 and 7.
- The remaining are the prime numbers.



So, 3 is a factor of 63.

$$63 = 3 \times 21$$

We can write  $21 = 7 \times 3$

We can shorten this as:

$$\begin{array}{r|l} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

That is,  $252 = 2 \times 2 \times 3 \times 3 \times 7$

Write the following numbers as the product of prime numbers.

- 145      • 210      • 100
- 168      • 225      • 288

### Highest common factor

What are the factors of 16?

They are, 1, 2, 4, 8, 16.

What about 12?

1, 2, 3, 4, 6, 12

The factors common to 12 and 16 are 1, 2 and 4.

1, 2 and 4 are called the common factors of 12 and 16.

The largest among them is 4 and it is called the *highest common factor* of 12 and 16.

We can find it in another way.

We know to write 16 and 12 as products of prime numbers.

$$16 = 2 \times 2 \times 2 \times 2$$

$$12 = 2 \times 2 \times 3$$

### Multiple of 11

Is 462, a multiple of 11.

For this, we can divide 462 by 11.

Without division, what is the way?

$$\begin{aligned} 462 &= 4 \times 100 + 6 \times 10 + 2 \\ &= 4 \times (99 + 1) + 6(11 - 1) + 2 \\ &= 4 \times 99 + 4 + 6 \times 11 - 6 + 2 \\ &= (4 \times 99 + 6 \times 11) + 4 - 6 + 2 \end{aligned}$$

99 and 11 are multiples of 11. Therefore, 462 is a multiple of 11 only if  $4 - 6 + 2$  is a multiple of 11. But  $4 - 6 + 2 = 0$ . Therefore 462 is a multiple of 11.

Another example:

$$\begin{aligned} 2596 &= 2 \times 1000 + 5 \times 100 + 9 \times 10 + 6 \\ &= 2(1001 - 1) + 5(99 + 1) + 9(11 - 1) + 6 \\ &= 2 \times 1001 - 2 + 5 \times 99 + 5 + 9 \times 11 - 9 + 6 \\ &= (2 \times 1001 + 5 \times 99 + 9 \times 11) - 2 + 5 - 9 + 6 \end{aligned}$$

1001, 99, 11 are all multiples of 11. Therefore 2596 is a multiple of 11. Thus if the difference between the sum of the digits in the ones, hundreds, ten thousands, ... and the sum of the digits in the tens, thousands, lakh, ... is a multiple of 11, then the number is a multiple of 11.

What are the common numbers among the prime factors of 12 and 16?

$$16 = 2 \times 2 \times 2 \times 2$$

$$12 = 2 \times 2 \times 3$$

2 and 2, right?

So the highest common factor of 12 and 16 is  $2 \times 2 = 4$ .

How do we find the highest common factor of 24 and 18?

You know how to write 24 as a product of prime numbers.

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$$24 = 2 \times 2 \times 2 \times 3$$

Now write 18 as a product of prime numbers.

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$18 = 2 \times 3 \times 3$$

From these, find the common factors of 24 and 18. They are

$$24 = 2 \times 2 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$2 \times 3 = 6$$

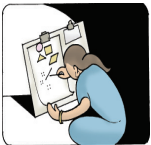
Thus the highest common factor is 6.



- Find the common factors and highest common factor of the number pairs given below:
  - 28, 20      ● 15, 25      ● 28, 36
  - 36, 45      ● 32, 40      ● 18, 24
- Jose is measuring out coconut oil to Siyad and Meera. Siyad need 12 litres of oil and Meera, 16 litres. Jose has jars to measure out various quantities. What is the largest jar he can use for both Siyad and Meera?

**Seminar:** Prepare and present a seminar paper on ‘History of prime numbers.’

- Write five multiples of each of the numbers below.  
32      23      55      60
- Find the least common multiple of the pairs below.
  - 12, 15      ■ 20, 30      ■ 7, 8      ■ 8, 16
- Find the factors of the numbers below:  
25      37      48      100
- Write the following numbers as the product of prime numbers.  
25      60      58      125  
160      204      190      92
- Find the common factors and the highest common factor of the number pairs below:
  - 36, 48      ■ 44, 64      ■ 24, 56



By taking different number pairs, find the relation between the product of two numbers, their least common multiple and the highest common factor.

## Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none"> <li>• Recognizing and explaining the idea of common multiples of numbers.</li> <li>• Recognizing and explaining the idea of common factors of numbers.</li> <li>• Solving practical problems using the idea of common multiples and common factors.</li> <li>• Classifying numbers as prime and composite based on their factors.</li> <li>• Explaining the method of expressing a number as the product of its prime factors.</li> <li>• Checking whether a number is a multiple of 2, 3, 4, 5, 6, 8, 9, 10 without actual division.</li> <li>• Explaining the relationship of two numbers with their least common multiple and highest common factor.</li> </ul>			

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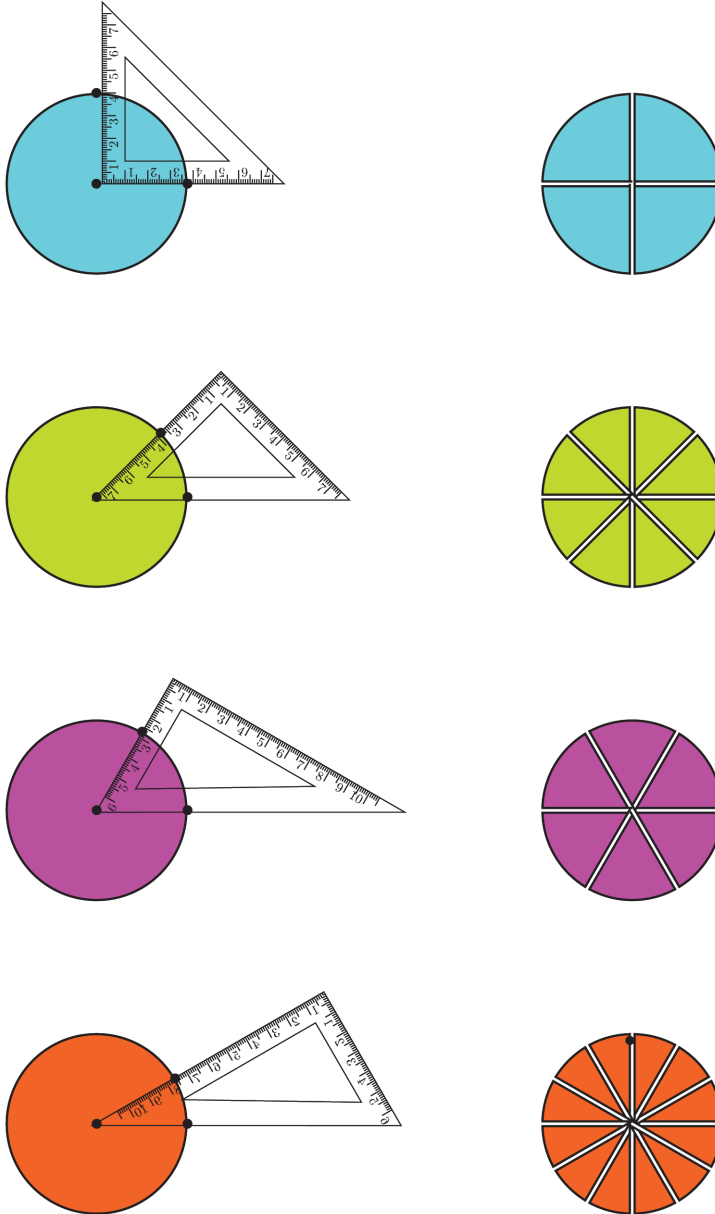
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## Joining Parts



## Parts of a circle

You have seen that a circle can be divided into various equal parts by using a setsquare.



Leela teacher came to the class with four boxes containing  $\frac{1}{4}$ 's,  $\frac{1}{8}$ 's,  $\frac{1}{6}$ 's and  $\frac{1}{12}$ 's of different circles of the same size. She divided the class into four groups and gave one box to each group. The task to each group is to make a half circle with their pieces.



$$\frac{1}{2} = \frac{2}{4}$$



$$\frac{1}{2} = \frac{4}{8}$$



$$\frac{1}{2} = \frac{3}{6}$$



$$\frac{1}{2} = \frac{6}{12}$$

Can't you also make circular pieces like this?

In what all ways can you make  $\frac{1}{4}$  of a circle?

What about  $\frac{1}{3}$ ?

In what all ways can we make  $\frac{2}{3}$ ?

### Numerator and Denominator

Each one of two equal parts is called a half and is written  $\frac{1}{2}$ .

2 parts from 4 equal parts, joined together is also half. That is, 2 of 4 and 1 of 2 are both half.

That is,  $\frac{1}{2} = \frac{2}{4}$

What about 3 of 6 equal parts?

Thus,

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$$

We can continue this as much as we want.

For example, from 100 equal parts, how many parts should we take to make half?

We can write this as,

$$\frac{1}{2} = \dots\dots\dots$$

We can say it in a different way. In the fraction  $\frac{50}{100}$ , the number 100 denotes the total number of parts and the number 50 denotes the number of parts taken.

The number 100 in  $\frac{50}{100}$  is called the *denominator* and 50 is called the *numerator*.

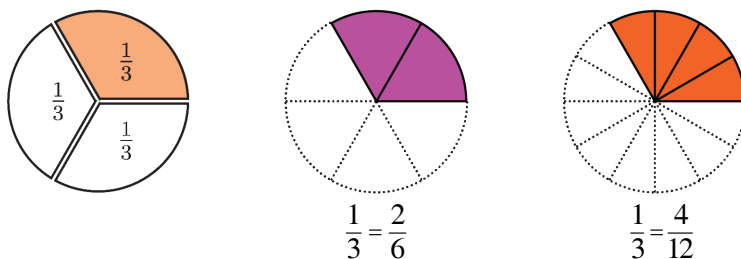
Thus in the different forms of half like,

$$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \dots,$$

when the denominator changes as 2, 4, 6, 8, ... the numerator should change to 1, 2, 3, 4, ...

Now let's take one-third.

Look at this picture:



From it, we see that  $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$

Now look at this picture:



A ribbon is divided into 9 equal parts. How many parts should we take to get  $\frac{1}{3}$ ?



Thus we see that

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$

We can continue this also.

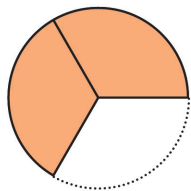
For example, from a ribbon cut into 15 equal parts, how many parts should we take to make  $\frac{1}{3}$ ?



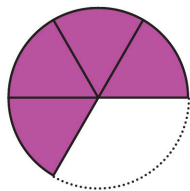
How do we write this?

$$\frac{1}{3} = \dots\dots\dots$$

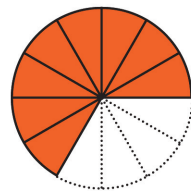
Using the pieces of circles used earlier, in what all ways can we make  $\frac{2}{3}$ ?



$$\frac{2}{3}$$



$$\frac{2}{3} = \frac{4}{6}$$



$$\frac{2}{3} = \frac{8}{12}$$

If a ribbon is divided into 9 equal parts, how many parts should we take to get its  $\frac{2}{3}$ ?



$$\frac{1}{3} = \frac{3}{9}$$



$$\frac{2}{3} = \frac{6}{9}$$

What do we see from all these?

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$

We can continue like this. In the different forms of  $\frac{2}{3}$ , what are the denominators?

These are all multiples of what number?

And the numerators?

Can we take any multiple of 3 as the denominator?

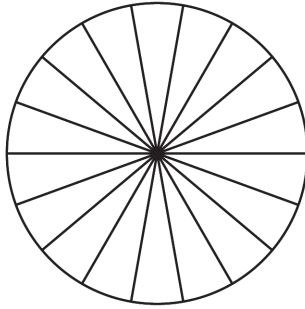
If the denominator is to be 24, what should be the numerator?

Can we take any multiple of 2 as the numerator?

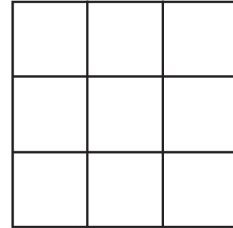
If the numerator is to be 24, what should be the denominator?



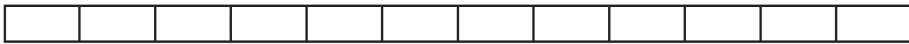
- Colour the specified part in each figure. From it find another form of the fraction.



$$\frac{1}{6}$$

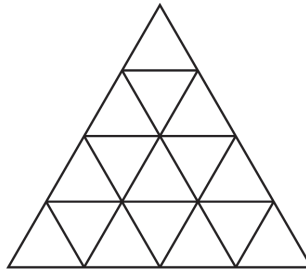


$$\frac{2}{3}$$



$$\frac{3}{4}$$

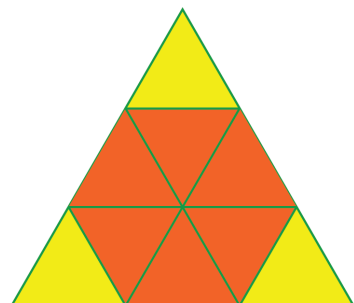
- Colour  $\frac{1}{4}$  of this triangle.



Try to do it in different ways with your friends. What form of  $\frac{1}{4}$  do we get from this?

- In the figure, what part of the triangle is coloured red?

Write this fraction with denominator 3.



- Draw a circle and cut it into 12 equal parts. Join these parts to make  $\frac{1}{4}, \frac{3}{4}, \frac{1}{6}, \frac{5}{6}$  of this circle. Write these fractions with denominator 12.
- Draw a circle and cut it into 8 equal parts. Is it possible to make  $\frac{1}{3}$  of the circle using these parts? What about  $\frac{2}{3}$ ? What about  $\frac{3}{4}$ ?

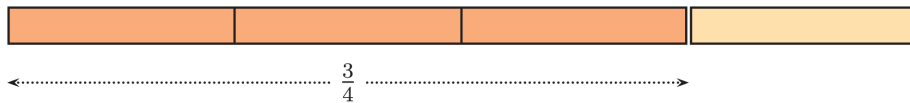
### One fraction, several forms

We have seen that by changing the numerator and denominator, we can find different forms of a fraction. How do we find such forms of  $\frac{3}{4}$ ?

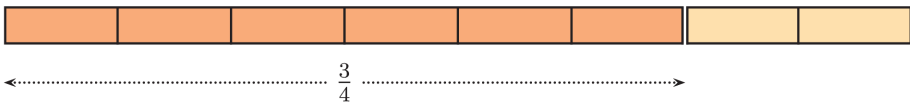
Divide a long ribbon into 4 equal parts.



3 such pieces joined together, makes its  $\frac{3}{4}$ .



What if we halve each of these four pieces?

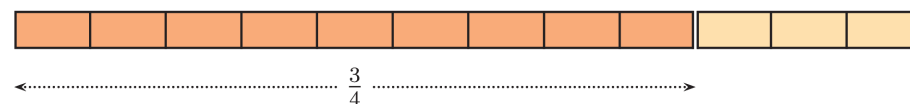


Now the ribbon is divided into 8 equal parts; and 6 of them make  $\frac{3}{4}$ .

That is,

$$\frac{3}{4} = \frac{6}{8}$$

Instead of dividing each of the four parts into two equal parts, suppose we divide each into three equal parts:



Don't we get another form of  $\frac{3}{4}$ ?

$$\frac{3}{4} = \frac{9}{12}$$

What if we divide each of the first four parts into four equal parts?

Let's think, without any picture:

Total number of pieces is  $4 \times 4 = 16$

Number of parts in  $\frac{3}{4}$  is  $3 \times 4 = 12$

Thus,

$$\frac{3}{4} = \frac{12}{16}$$

That is, to get a number equal to  $\frac{3}{4}$ , we note how many times 4 is the total number of parts and then take that many times 3.

In other words, to get different forms of  $\frac{3}{4}$ , we can take any multiple of 4 as denominator and the numerator should be the same multiple of 3.

For example, from the multiples

$$4 \times 25 = 100 \text{ and } 3 \times 25 = 75$$

we get

$$\frac{3}{4} = \frac{75}{100}$$

Isn't it true for all fractions? For example, take  $\frac{2}{5}$ .

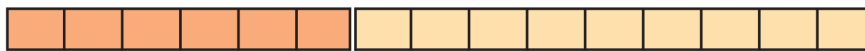


$$\leftarrow \dots \frac{2}{5} \dots \rightarrow$$



$$\leftarrow \dots \frac{2}{5} \dots \rightarrow$$

$$\frac{2}{5} = \frac{4}{10}$$



$$\leftarrow \dots \frac{2}{5} \dots \rightarrow$$

$$\frac{2}{5} = \frac{6}{15}$$

What is the general principle we see here?

If we multiply the numerator and denominator of a fraction by the same number, we get another form of this fraction.

Let's look at another thing. Consider the fraction  $\frac{18}{24}$ . Its numerator and denominator are even numbers; that is 2 is a factor of both.

$$24 = 12 \times 2 \quad 18 = 9 \times 2$$

So from what we saw above,

$$\frac{18}{24} = \frac{9}{12}$$

Do 9 and 12 have any common factor?

We see that,  $12 = 4 \times 3$ ,  $9 = 3 \times 3$

So,

$$\frac{9}{12} = \frac{3}{4}$$

Thus,

$$\frac{18}{24} = \frac{3}{4}$$

What do we see here? If the numerator and denominator of a fraction have a common factor, and if we divide the numerator and denominator by this number, we will get another form of this fraction.

In the above example, we first wrote  $\frac{18}{24}$  as

$\frac{9}{12}$ ; then reduced the numerator and

denominator again to form  $\frac{3}{4}$ .

So  $\frac{3}{4}$  is called the form of  $\frac{18}{24}$  in *lowest terms*.

We can't reduce the numerator and denominator any more (why?). So we say that the fraction  $\frac{18}{24}$  is reduced to its lowest terms as  $\frac{3}{4}$ . In general, to reduce a fraction to lowest terms, we divide out all common factors of the numerator and denominator.

Now try these problems;

• Fill in the blanks:

•  $\frac{3}{5} = \frac{\dots}{30}$

•  $\frac{5}{6} = \frac{20}{\dots}$

•  $\frac{45}{75} = \frac{3}{\dots}$

•  $\frac{42}{48} = \frac{\dots}{8}$

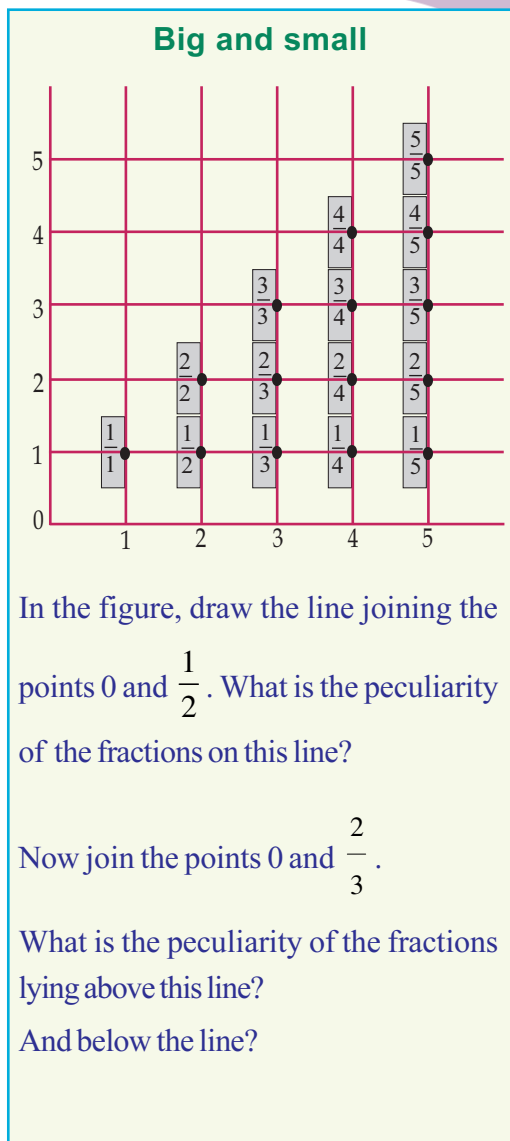
• Write the following fractions with denominator 10, 100 or 1000.

•  $\frac{1}{2}$

•  $\frac{2}{5}$

•  $\frac{3}{4}$

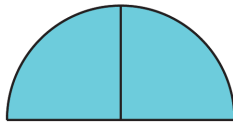
•  $\frac{5}{8}$



- Can we write the fractions  $\frac{1}{3}$  and  $\frac{2}{3}$  with denominator 10, 100 or 1000?
- Write each of the following pairs of fractions with the same denominator:
  - $\frac{1}{2}, \frac{1}{4}$
  - $\frac{1}{2}, \frac{1}{3}$
  - $\frac{1}{3}, \frac{1}{4}$
  - $\frac{1}{3}, \frac{2}{5}$

### Let's join

Two of four equal parts of a circle, joined together makes half a circle:

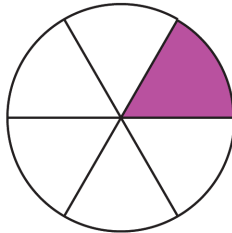


That is, quarter circle is joined with quarter circle makes half circle; that is, quarter and quarter make half. We can write this as

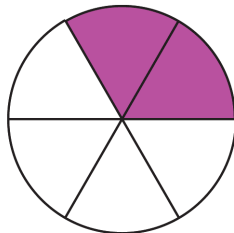
$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

What about joining together two of the pieces got on dividing a circle in six equal parts?

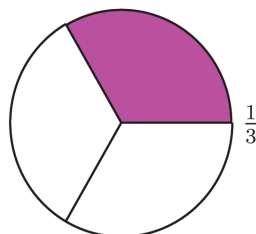
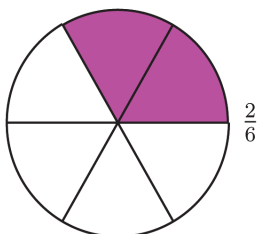
Draw a circle and mark 6 equal parts. Colour one part.



Colour one more part.



Now  $\frac{2}{6}$  circle is coloured and  $\frac{2}{6}$  is another form of  $\frac{1}{3}$ , right?



This also, we can write as a sum:

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Suppose we join together two of eight equal pieces of a circle.

What fraction of the circle do we get?

Can you do this in your head?

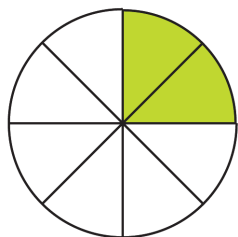
2 parts of 8 equal parts means  $\frac{2}{8}$ ; also,

$$\frac{2}{8} = \frac{1 \times 2}{4 \times 2} = \frac{1}{4}$$

Thus,

$$\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

We can also see this by drawing a circle and colouring:



### Half and half of half

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 1$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = 1$$

.....  
 .....  
 .....

Write some more lines like this.

What fraction of a circle do we get on joining together its  $\frac{1}{8}$  and  $\frac{3}{8}$ ?

Of 8 equal parts,  $1 + 3 = 4$  parts are taken.

That is,  $\frac{4}{8}$ . Can't we reduce the numerator and denominator?

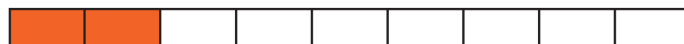
$$\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

Draw a picture of this sum using parts of a circle.

Take a long ribbon and divide it into 9 equal parts:



Colour two of the parts:



Next colour 4 more parts:



Now  $2 + 4 = 6$  parts are coloured.

This can be put in this way; first we coloured  $\frac{2}{9}$  of the ribbon, and then  $\frac{4}{9}$  of it;  $\frac{6}{9}$  of the ribbon in all.

We can write this as the sum of fractions:

$$\frac{2}{9} + \frac{4}{9} = \frac{6}{9}$$

We can reduce  $\frac{6}{9}$  to lowest terms:

$$\frac{6}{9} = \frac{2 \times 3}{3 \times 3} = \frac{2}{3}$$



That is,

$$\frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$$

Look at this picture:



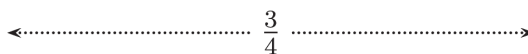
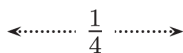
How many parts are coloured red?

How many green?

In all, how many parts are coloured?

What sum of fractions do we get here?

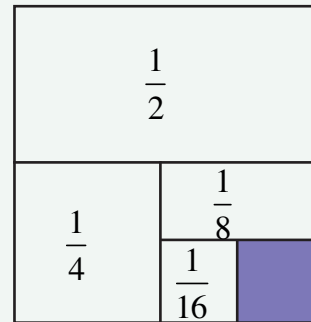
$$\frac{1}{8} + \frac{5}{8} = \frac{6}{8} = \frac{3}{4}$$



### Sum in geometry

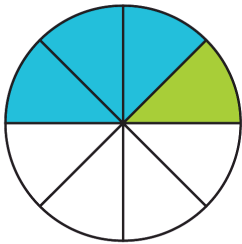
In the figure, a square is divided into different parts. What part of the square is unshaded?


Without adding the fractions, can you find the answer?





In each picture below, write the different coloured parts and total coloured parts as fractions. Also write the sum of the fractions got from the picture in their lowest terms.



 colour  parts

 colour  parts

Total  parts

$$\boxed{\phantom{0}} + \boxed{\phantom{0}} = \boxed{\phantom{0}} = \boxed{\phantom{0}}$$

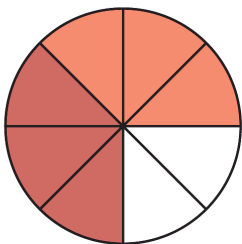



 colour  parts

 colour  parts

Total  parts

$$\boxed{\phantom{0}} + \boxed{\phantom{0}} = \boxed{\phantom{0}} = \boxed{\phantom{0}}$$




 colour  parts

 colour  parts

Total  parts

$$\boxed{\phantom{0}} + \boxed{\phantom{0}} = \boxed{\phantom{0}} = \boxed{\phantom{0}}$$



 colour  parts

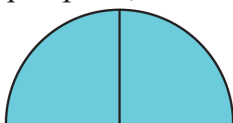
 colour  parts

Total  parts

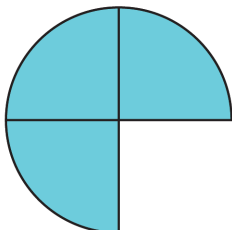
$$\boxed{\phantom{0}} + \boxed{\phantom{0}} = \boxed{\phantom{0}} = \boxed{\phantom{0}}$$

## Addition of fractions

If a circle is divided into four equal parts, and two of the parts are joined together, we get half a circle.



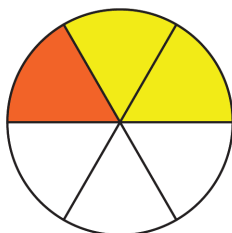
What if one more part is joined with this?



We get three-fourths of a circle. Thus, half and quarter makes three-fourths.

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

Now look at this figure:



A circle is divided into 6 equal parts. Two parts are coloured yellow and one part coloured red. Altogether  $1 + 2 = 3$  parts are coloured. We can write this as the sum of fractions.

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$$

In this, we can reduce the fractions to lowest terms as.

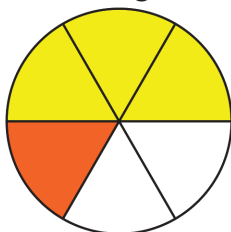
$$\frac{2}{6} = \frac{1}{3} \text{ and } \frac{3}{6} = \frac{1}{2}$$

Then we get,

$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$



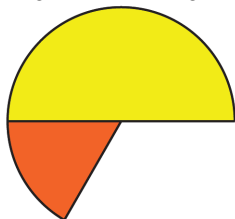
What sum of fractions do we get from this figure?



$$\frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

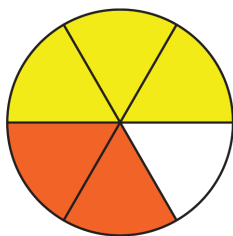
If we reduce fractions as  $\frac{3}{6} = \frac{1}{2}$  and  $\frac{4}{6} = \frac{2}{3}$ , then we can write this as

$$\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$$



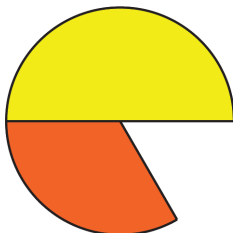
What if the picture is like this?

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$



If  $\frac{3}{6}$  and  $\frac{2}{6}$  are reduced to lowest terms, then it becomes

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$



Now let's think about what fraction of a circle we get on joining together  $\frac{1}{4}$  and  $\frac{3}{8}$  of it, without drawing a picture.

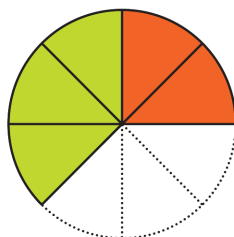
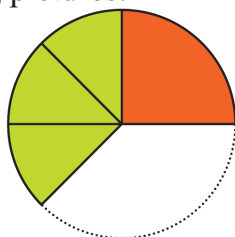
If the pieces are all of the same size, we can easily add up. How about seeing the  $\frac{1}{4}$  of the circle as the two  $\frac{1}{8}$ 's joined together?

$\frac{3}{8}$  is 3 such pieces joined together.

So, altogether  $2 + 3 = 5$  parts out of 8 equal parts; that is,  $\frac{5}{8}$ .

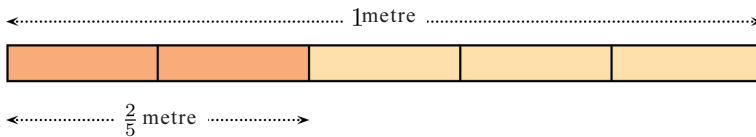
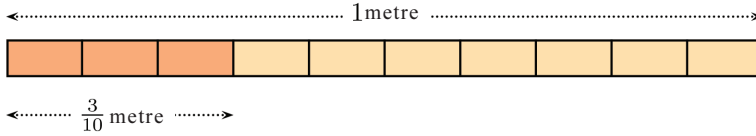
$$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$$

Let us see this using pictures:

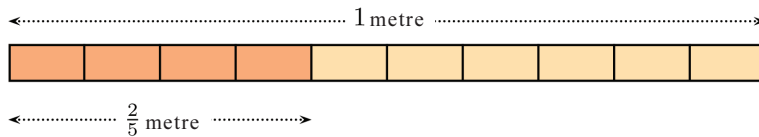
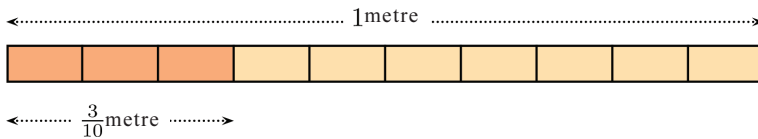


Another problem: a ribbon of length  $\frac{3}{10}$  metre and a ribbon of length  $\frac{2}{5}$  metre are joined end to end. What is the total length?

$\frac{3}{10}$  metre is 3 parts of 10 equal parts of a metre and  $\frac{2}{5}$  metre is 2 parts of 5 equal parts of a metre; but these parts are not alike:



We can also think of  $\frac{2}{5}$  metre as 4 parts of 10 equal parts of a metre.



So, altogether  $4 + 3 = 7$  equal parts. That is,  $\frac{7}{10}$  metre.

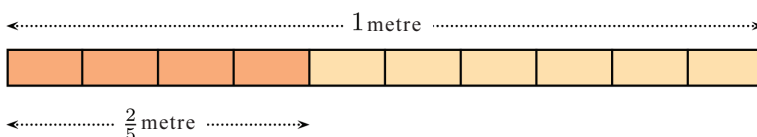
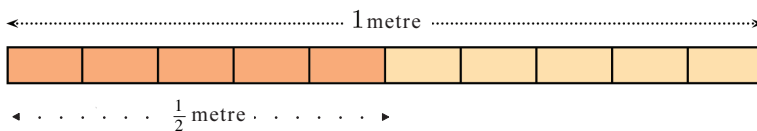
$$\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$

What if  $\frac{1}{2}$  metre and  $\frac{2}{5}$  metre are joined together?

We can take  $\frac{2}{5}$  metre as 4 parts of 10 equal parts of a metre. What about  $\frac{1}{2}$  metre?

5 parts out of 10 equal parts of a metre make  $\frac{1}{2}$  metre. So  $4 + 5 = 9$  parts in all.

That is,  $\frac{9}{10}$  of a metre.



$$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$$

What common method do we see in all these?

To find the sum of two fractions, we must put them into forms with the same denominator.

So, how do we compute  $\frac{1}{3} + \frac{2}{5}$ ?

First put them in forms with the same denominators.

In the different forms of  $\frac{1}{3}$ , the denominators are all multiples of 3.

In the different forms of  $\frac{2}{5}$ , the denominators are all multiples of 5.

So, the same denominator we want should be a multiple of 3 and 5.

For that, we need only take  $3 \times 5 = 15$ .

$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15}$$

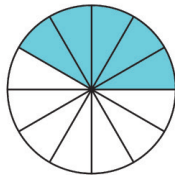
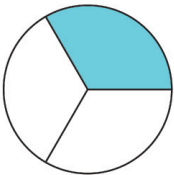
$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

Now we can find the sum:

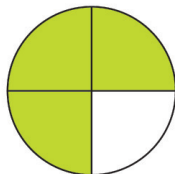


$$\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$$

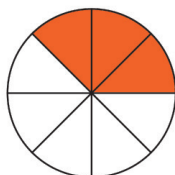
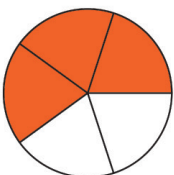
- In each pair of circles below, if the coloured parts are joined together, what fraction of the circle do we get?



$$\square + \square = \square + \square = \square = \square$$



$$\square + \square = \square + \square = \square = \square$$



$$\square + \square = \square + \square = \square = \square$$

- Compute the sums below:

$$\bullet \frac{1}{4} + \frac{1}{8}$$

$$\bullet \frac{2}{3} + \frac{1}{6}$$

$$\bullet \frac{2}{3} + \frac{1}{4}$$

$$\bullet \frac{1}{8} + \frac{5}{6}$$

### Some more sums

A jar contains three quarters of a litre of milk. Half a litre milk is poured into it. How much milk does it contain now?

Suppose the half litre is poured separately, a quarter of a litre at a time; when the first quarter of a litre is poured in, the jar contains one litre (three quarters and a quarter); when another quarter of a litre is poured in, it contains one and a quarter litres.

How about writing this as sum of fractions?

$$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{1}{4} + \frac{1}{4} = 1 + \frac{1}{4} = 1\frac{1}{4}$$

What, if we do this by making the denominators same as before?

$$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}$$

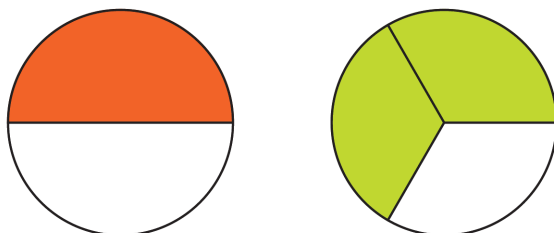
$\frac{5}{4}$  is another form of,  $1\frac{1}{4}$ , isn't it?

Suppose three-quarters of a litre is added to three-quarters of a litre itself?

Three-quarters and a quarter make one; remaining to add is half; one and a half in all:

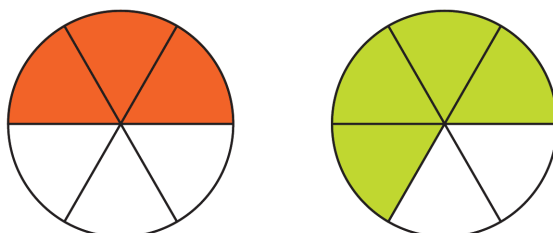
$$\frac{3}{4} + \frac{3}{4} = \frac{3}{4} + \frac{1}{4} + \frac{1}{2} = 1 + \frac{1}{2} = 1\frac{1}{2}$$

Draw two circles of the same size, colour half of the first circle and two thirds of the second circle:

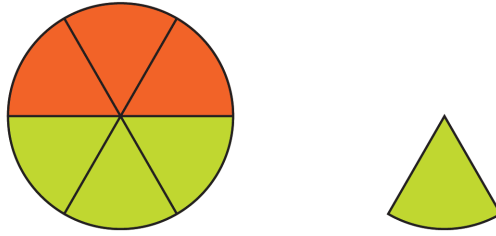


If the coloured parts are cut out and joined together, it makes more than a full circle, isn't it?

Suppose we cut them like this.



We can join the pieces to make a full circle and a piece left over, like this.



Let's write the math:

$$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Another Problem: Anoop and his father went to buy cloth for shirts; one and a half metres for Anoop and two and a quarter metres for his father. What is the total length of cloth they should buy?

We can calculate like this: one and two makes three; half and quarter makes three-fourths; three and three fourths in all.

That is,

$$1\frac{1}{2} + 2\frac{1}{4} = \left(1 + \frac{1}{2}\right) + \left(2 + \frac{1}{4}\right) = (1 + 2) + \left(\frac{1}{2} + \frac{1}{4}\right) = 3 + \frac{3}{4} = 3\frac{3}{4}$$

There is another way:

$$1\frac{1}{2} = \frac{3}{2} \quad 2\frac{1}{4} = \frac{9}{4}$$

So,

$$1\frac{1}{2} + 2\frac{1}{4} = \frac{3}{2} + \frac{9}{4} = \frac{6}{4} + \frac{9}{4} = \frac{15}{4} = \frac{(3 \times 4) + 3}{4} = 3 + \frac{3}{4} = 3\frac{3}{4}$$



- A jar contains one and a half litres of milk. Another jar contains two and a quarter litres of milk. How much milk in all?
- If two ribbons, each of length one and a half metres are joined end to end, then what is the total length?
- Sarala bought one and a half kilograms of beans and  $\frac{3}{4}$  kilograms of yam. What is the total weight?
- Calculate the sums below:

•  $\frac{5}{6} + \frac{1}{3}$     •  $\frac{7}{8} + \frac{1}{4}$     •  $\frac{3}{4} + \frac{1}{3}$     •  $\frac{5}{6} + \frac{1}{4}$     •  $2\frac{1}{3} + 3\frac{1}{2}$

## How can we subtract?

From a rod, three-fourths of a metre long, a piece quarter of a metre long is cut off. What is the length of the remaining part?

Three-fourths of a metre means half a metre and quarter of a metre together. Removing quarter of a metre from it leaves half a metre. We can write it like this:

$$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

We can do this subtraction in another way, as in the case of addition.

$$\frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

If half a metre is cut off from three-fourths of a metre, what remains is quarter of a metre.

$$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

As in the case of addition, we can do this by making the denominators same.

$$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{3-2}{4} = \frac{1}{4}$$

What if one-third of a metre is cut off from half a metre?

We must find out  $\frac{1}{2} - \frac{1}{3}$ . Let's make the denominators same.

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{3-2}{6} = \frac{1}{6}$$

That is, what remains is  $\frac{1}{6}$  metre.

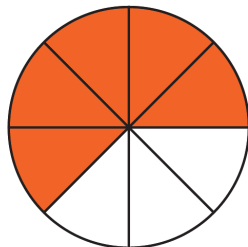
From one litre of milk, quarter of a litre is used. How much is left?

One litre is quarter and three-fourths. So what is left is three-fourths of a litre. How do we write this?

$$1 - \frac{1}{4} = \left(\frac{1}{4} + \frac{3}{4}\right) - \frac{1}{4} = \frac{3}{4}$$

We can also do it like this:  $1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

Look at this picture:



Write one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 in each box to make the sum correct.

In how many ways can we do this?

Without repeating the numbers, in how many ways can we write this?

$$\begin{array}{r} \square \\ \hline \square \end{array} + \begin{array}{r} \square \\ \hline \square \end{array} = 1$$



How many parts are coloured?

How many parts remain to be coloured?

We can write this as:  $1 - \frac{5}{8} = \frac{3}{8}$

We can compute it like this:  $1 - \frac{5}{8} = \frac{8}{8} - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

### Another Problem

From a yam weighing two and a half kilograms, a piece weighing one and a quarter kilograms is cut out. What is the weight of the remaining piece?

We can do it in our heads like this:

One kilogram taken away from two kilograms leaves one kilogram; and a quarter of a kilogram taken away from half a kilogram leaves quarter of a kilogram; so what remains finally is one and a quarter kilograms.

We can write this as

$$2\frac{1}{2} - 1\frac{1}{4} = \left(2 + \frac{1}{2}\right) - \left(1 + \frac{1}{4}\right) = (2 - 1) + \left(\frac{1}{2} - \frac{1}{4}\right) = 1\frac{1}{4}$$

There is another way to doing this:

$$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$$

and

$$1\frac{1}{4} = 1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

so that

$$2\frac{1}{2} - 1\frac{1}{4} = \frac{5}{2} - \frac{5}{4} = \frac{10}{4} - \frac{5}{4} = \frac{5}{4} = 1\frac{1}{4}$$

In the cloth problem done earlier, one and a half metres was bought for Anoop and two and a quarter for his father. How much more than Anoop's is the cloth bought for his father?

Here we can't subtract half a metre from quarter of a metre. So let's think it in different way.

To one and a half metre, if we add half a metre, we get two metres; adding quarter of a metre again gives two and a quarter metres. We added half and quarter, which gives three quarters. And this is how much more father bought: That is,

$$2\frac{1}{4} - 1\frac{1}{2} = \frac{3}{4}$$

The ideas above can be written like this.

$$2\frac{1}{4} - 1\frac{1}{2} = 2 + \frac{1}{4} - 1\frac{1}{2} = \left(1\frac{1}{2} + \frac{1}{2}\right) + \frac{1}{4} - 1\frac{1}{2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

We can compute it like this also:

$$2\frac{1}{4} - 1\frac{1}{2} = \frac{9}{4} - \frac{3}{2} = \frac{9}{4} - \frac{6}{4} = \frac{3}{4}$$



- From a string one and three-fourths of a metre long, half a metre is cut off. What is the length of the remaining piece?
- From a pumpkin weighing three and a half kilograms, a piece weighing one and three-fourths kilograms was cut off. What is the weight of the remaining piece?
- Anu drew a circle and coloured  $\frac{5}{12}$  of it. What fraction of the circle remains to be coloured?
- In a 10 litre bucket, there is  $3\frac{3}{4}$  litres of water. How much more water is needed to fill the bucket?
- A panchayat laid  $14\frac{3}{4}$  kilometres of new roads last year.  $16\frac{1}{4}$  kilometres of road is laid this year. How much more is laid this year?
- Vinod bought 20 metres of string. From this, he first cut out  $5\frac{3}{4}$  metres and later  $6\frac{1}{2}$  metres. How much length is left?
- One-third of a tank is filled with water. When another 100 litres of water was poured in, it became half full. How many litres can the tank hold in all?
- There are two taps opening into a tank. If only the first tap is opened, the tank fills in 10 minutes. If only the second tap is opened, it takes 15 minutes to fill the tank.
  - If only the first tap is opened, what fraction of the tank would be filled in one minute?
  - If only the second tap is opened, what fraction of the tank would be filled in one minute?
  - If both taps are opened, what fraction of the tank would be filled in one minute?
  - If both taps are opened, how much time would it take to fill the tank?
- The milk society got  $75\frac{1}{4}$  litres in the morning and  $55\frac{1}{4}$  in the evening. Of this,  $15\frac{1}{4}$  litres was sold. How much milk is left?

## Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none"> <li>• Formulating a method to find different forms of a fractions and explaining it.</li> <li>• Reducing a fraction to lowest terms.</li> <li>• Explaining the sum of fractions using pictures and in practical situations</li> <li>• Explaining the method of adding fraction of different denominators by making the denominators equal and solving practical problem.</li> <li>• Explaining the method of subtracting fractions by making denominators equal and solving practical problems.</li> </ul>			

# 9

## Let's Read Pictures



## Quiz competition

Kalyani looked curiously at the quiz master recording points in the Maths Club Quiz.

If a team correctly answers a question given to them, they get a star (★); if they pass the question and another team answers it correctly, that team gets a triangle (▲)

"This is a good idea" - thought Kalyani.

Look at the score board at the end of the quiz competition.

Team	Points	Total points
A	★ ★ ★ ★ ▲ ▲ ▲ ▲	
B	★ ★ ★ ▲ ▲	
C	★ ★ ★ ★ ★ ▲ ▲ ▲	
D	★ ★ ▲ ▲ ▲	

Which team won? Kalyani could not understand.

The quiz master said, "10 points for ★ and 5 points for ▲."

"Sir, I'll tell the total score", Kalyani jumped out.

How did Kalyani find the total score of each team?

Team A got four ★'s and three ▲'s.

$$\text{Score for four } \star \text{'s} = 4 \times 10 = 40$$

$$\text{Score for four } \blacktriangle \text{'s} = 4 \times 5 = 20$$

$$\text{Total score for team A} = 40 + 20 = 60$$

Similarly find the scores of the other teams.

Which team won?

### Tally Mark

In ancient times, lines were used to denote numbers.

| - 1

|| - 2

||| - 3

|||| - 4

||||| - 5

To identify and count larger numbers easily, lines are split into groups of 5. For example, we can write 23 as





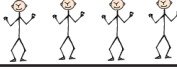



|||| |


Have you noticed quiz masters marking scores like this?


These markings are called tally marks.

## How many students?

The table below shows the number of students in different divisions of class 5.

5A		
5B		
5C		
5D		

 - 5 Boys

 - 5 Girls

In which division is the number of girls more?

How many girls are there in that division?







How many boys are there in 5 C? How many girls?

What is the total number of students in each division?

How many more girls than boys are there in all?

## School Library






The table below shows the number of books in the panchayat UP School.

Novel	
Short story	
Poem	
Drama	
Autobiography	
Others	

 - 100 books

## Picture representation

The picture below shows world population in different years.

1850	
1930	
1980	
2000	
2011	

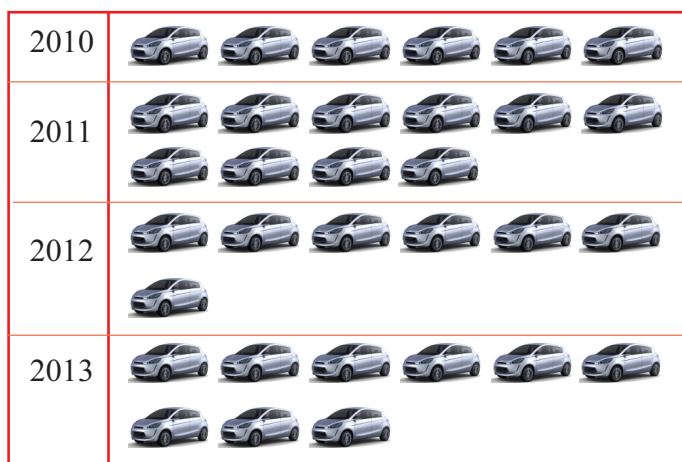
Here the picture of a man is used to indicate 100 crore people. This makes comparison easy.

What type of books is the most?

How many books does the library have in all?

### How many cars?

The picture below shows the number of cars manufactured by a company from 2010 to 2013.



 -10000 cars

In which year did they make the most number of cars? How many?

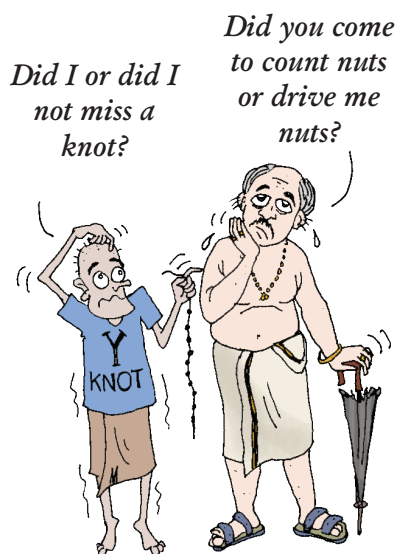
How many more cars did they make in 2011 than in 2013?

We have seen some instances where pictures are used to represent numerical information. Such a picture is called *pictograph* or *pictogram*. It is a convenient method to exhibit information involving large numbers. It also makes comparison easy.

### String math

In the past, in many countries, numbers were recorded as knots in a string. For example, in many parts of our place, when coconuts are counted, a knot is made in a string for every hundred nuts.

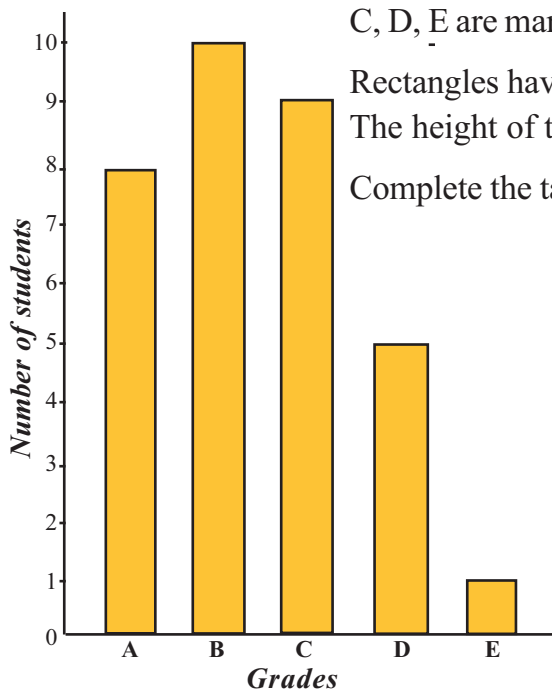
The Inca people, who lived in South America during the 13<sup>th</sup> century, had an extensive system of keeping numerical records using bunch of strings with various knots. Such a bunch is called a *quipu*.



## Another picture

The picture below shows the number of children in a class who got various grades in mathematics in the half yearly examination. The numbers 1, 2, 3, 4, ... marked on the

vertical line shows the number of students. The grades A, B, C, D, E are marked on the horizontal line.



Rectangles having the same width are drawn on each grade. The height of the rectangle shows the number of students.

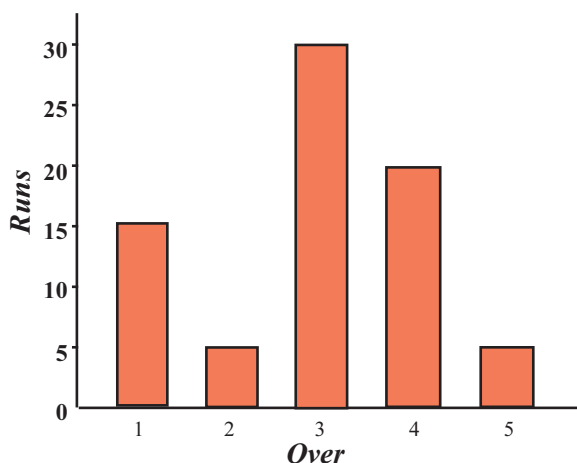
Complete the table using this picture:

Grade	Number of students
A	.....
B	.....
C	9
D	.....
E	.....

This method of showing numerical information using rectangles is called *bar graph* or *bar diagram*.



- The picture below shows the runs scored by the Indian cricket team in the first 5 overs of a match.



In which over did the team score the most runs?

What is the total runs scored in the first three overs?

What is the total runs scored in the first five overs?



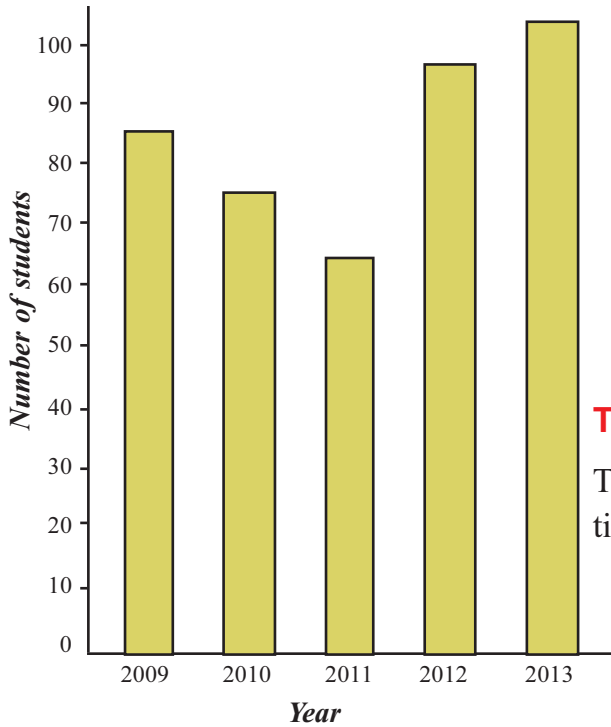
## Bar diagrams with a computer

Information we get or collect can be shown in pictures in various ways. They help us to analyse the information quickly. It is easy to draw a bar diagram using a spreadsheet program.

For this, open Application → Office → OpenOffice.org → Spreadsheets → Libre Office Calc. Type in the numbers in the table. Click on any cell and select Insert → Chart → Bar graph to draw the bar diagram.

## Enrolment

- The enrolment of children in class 1 of a school in five years is given in the bar graph below.



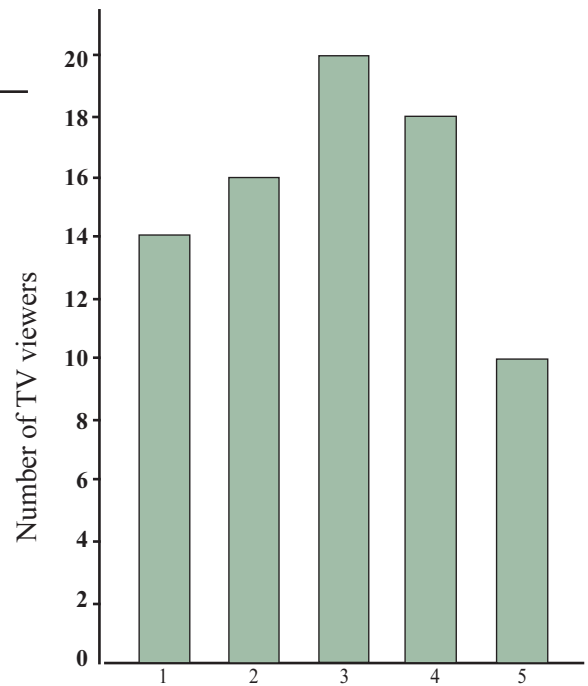
In which year did the most number of children join?

In which the enrolment is more, 2012 or 2013? How many more?

In which year did the enrolment is least? How many less than the previous year?

## Time for TV

The bar diagram below shows information collected on TV viewing.



- How many people watch TV for exactly one hour?
- How many spend 3 hours for watching TV?
- How many spend more than 2 hours for watching TV?
- Make more questions based on this.

## Looking back



Achievements	On my own	With teacher's	Must
<ul style="list-style-type: none"> <li>Collecting and classifying data from a pictogram.</li> <li>Collecting and classifying data from a bar graph.</li> <li>Interpreting and classifying data from graphs.</li> </ul>			