MATHEMATICS

Standard VII

Part - II

Government of Kerala
Department of Education

The National Anthem

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata.
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha.
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata.
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he!

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give respect to my parents, teachers and all elders and treat everyone with courtesy.

I pledge my devotion to my country and my people. In their well-being and prosperity alone lies my happiness.

Dear children,

    We've learned much math.

    Let's now move up higher

    To the world of Arithmetic

    Full of interesting numbers

    And strange relations

    To the new levels of Geometry

    To understand the logic of Math

    And to find the new things

    Let's move ahead with confidence.

Dr. J. Prasad
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Contents

Certain icons are used in this textbook for convenience

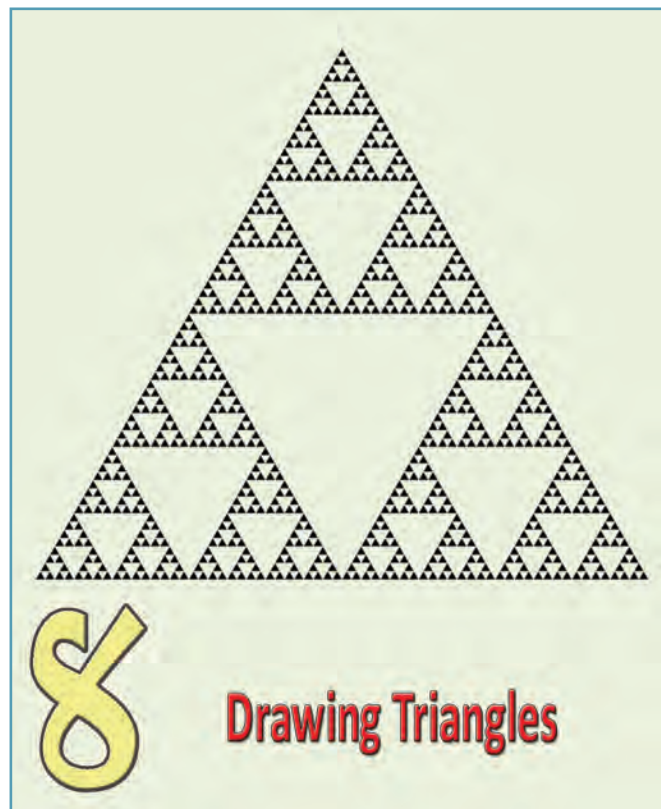 *Computer Work*

 *Additional Problems*

 Project

 Self Assessment

8

Drawing Triangles

Circle and triangle

Euclid, who lived in Greece during the third century BC, is considered the Master of geometry. His book, "The Elements" is the first authoritative text on geometry.

The first result on this book is the construction of an equilateral triangle of specified length for sides. This is Euclid's method:
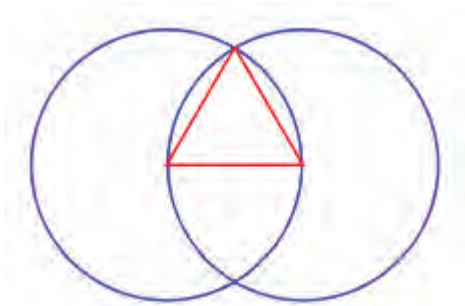
This figure consisting of two intersecting circles is often used as a motif in churches of medieval Europe.
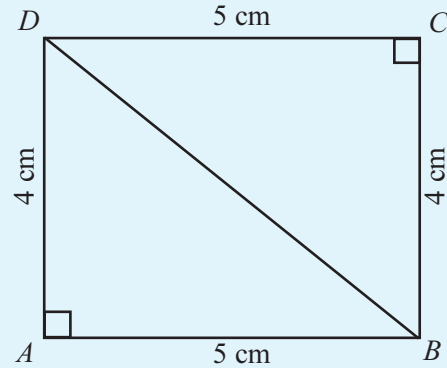

Triangle in a rectangle

Remember how we drew rectangles, using a set square?

Draw rectangle $ABCD$ with $AB = 5$ cm and $BC = 4$ cm.

Let's join a pair of opposite sides.

Now we have two right triangles.

Can you name them? What are the lengths of the perpendicular sides of each?

Now let's draw a right triangle with perpendicular sides of length 6 centimetres and 8 centimetres.

First draw a pair of perpendicular lines and name the point where they meet as A.

Mark B on one of the lines, 6 centimetres from A and mark C on the other line, 8 centimetres from A.

Now we need only join B and C to get the triangle.
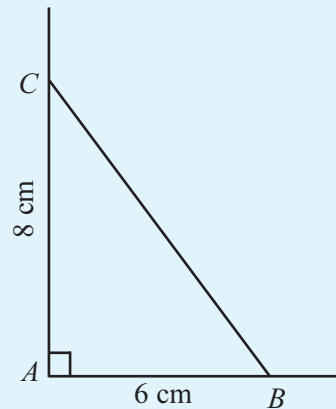
Measure the length of BC and write it next to BC.

Similarly draw a right triangle with lengths of perpendicular sides 5 centimetres and 7 centimetres.
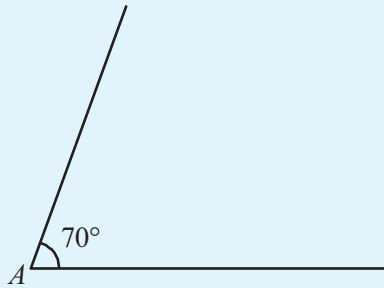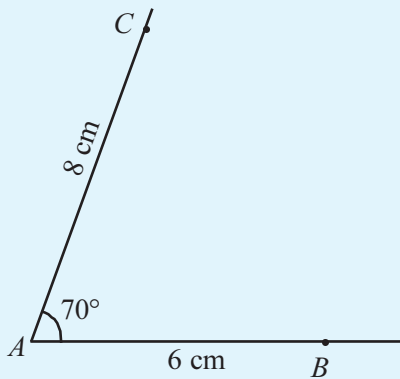
Another triangle

In both the triangles we drew now, the length of two sides were specified; and angle between them was right. If the angle between is not right, how do we draw a triangle?

Let's draw triangle ABC, with $AB = 6$ cm, $AC = 8$ cm and $\angle A = 70°$.

First draw an angle of 70°.

Next mark B on one line, 6 centimetres from A and C on the other line, 8 centimetres from A.

If we join B and C, we have the required triangle.
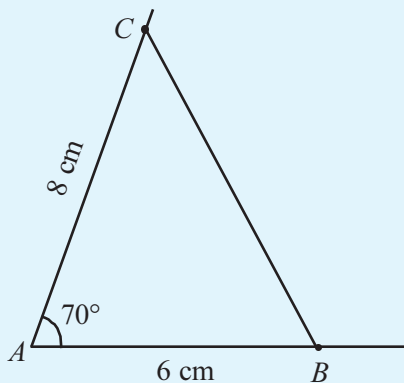
New figures

In the figure used for drawing an equilateral triangle, two triangles can be drawn, one up and the other down.
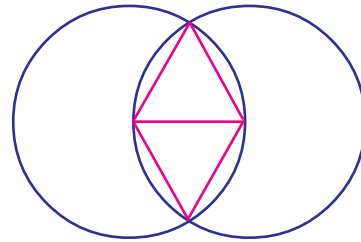
Suppose we erase the middle line:
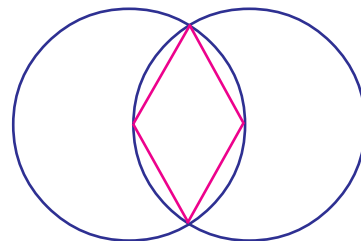
What are the peculiarities of this quadrilateral?

What if we draw three circles, instead of two?

Joining the points of intersections and the centres of two circles, we get a figure like this:

What is its name?

What can you say about the lengths of the sides?

Inside a circle

You know how to divide a circle into six equal parts, using a corner of one of the set squares in the geometry box.
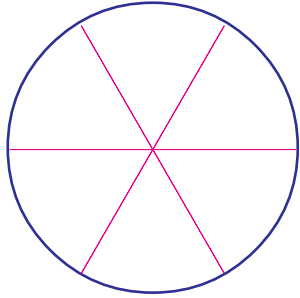
Joining the ends of these lines, we get a figure like this:
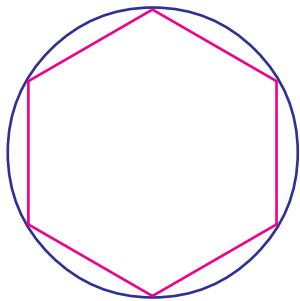
What if we join three alternate points?
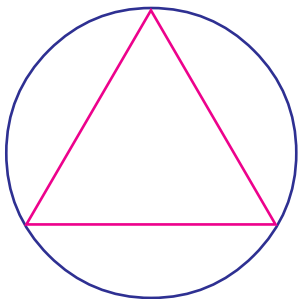
If we join the points left also, we get a star like this:
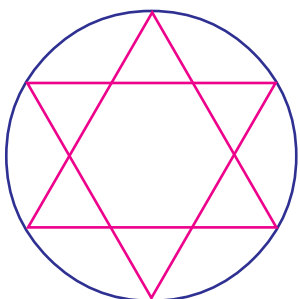
Now draw triangles of these specifications:

- MN = 6 centimetres, $\angle M$ = 70°, ML = 5 centimetres.
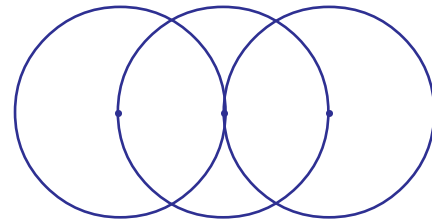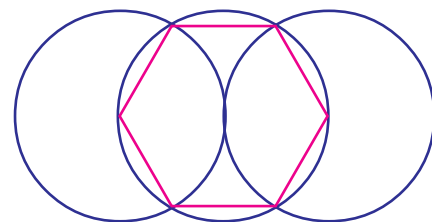
- PQ = 7 centimetres, QR = 7 centimetres, $\angle Q$ = 50°.

- XY = 6.5 centimetres, $\angle Y$ = 110°, YZ = 7.5 centimetres,

- CD = 5 centimetres, DE = 5 centimetres, $\angle D$ = 60°.

Another angle

In all the triangles we have drawn so far, the lengths of two sides and the included angle were specified.

Can we draw a triangle, if some other angle is specified?

For example, we want to draw triangle XYZ with XY = 5 centimetres, YZ = 3 centimetres and $\angle X$ = 30°.

First, we draw a rough sketch and write the specifications:
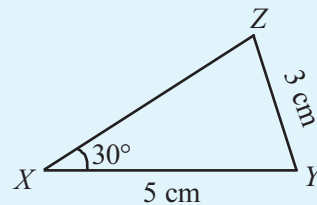
To draw the actual triangle, we start by drawing the line XY of length 5 centimetres:
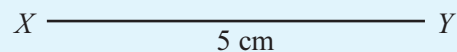
Next, we draw an angle of 30° at X.
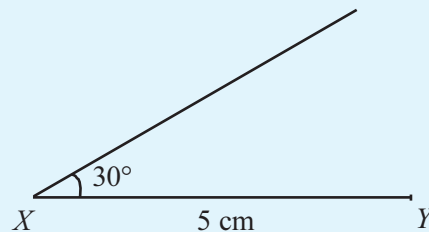
Now we must locate Z. It should be 3 centimetres from Y; and also should be on the upper line.

All points at a distance of 3 centimetres from *Y* are on the circle centred at *Y* of radius 3 centimetres. Let's draw this circle:
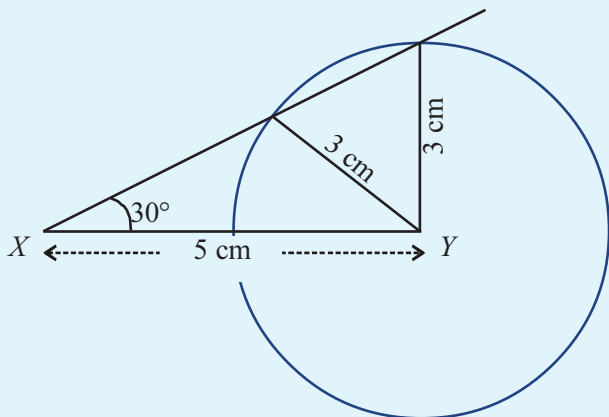
How many points are on both the circle and the upper line?

Taking one of them as *Z*, we get a triangle as specified:
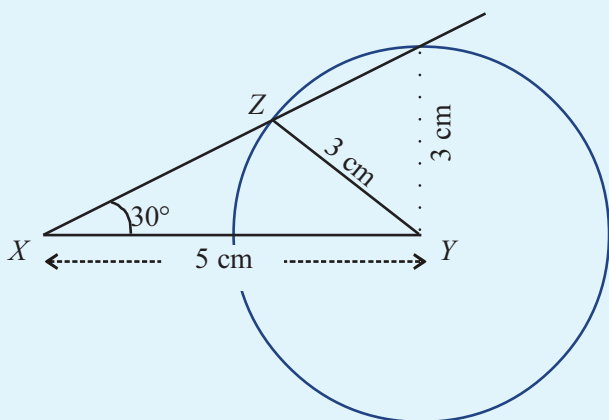
What if we take the other point as *Z*?
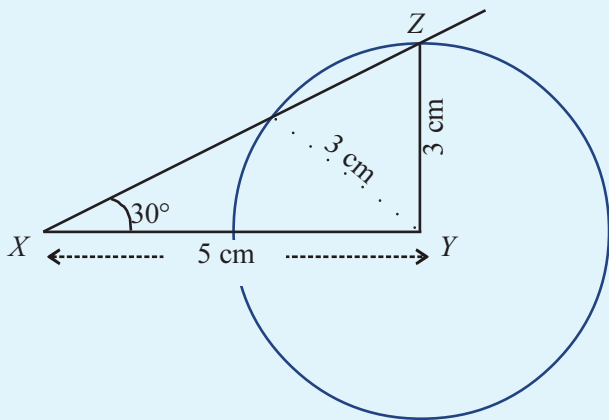
Sides and angles

Draw a triangle of sides 6 centimetres, 3 centimetres and the angle between them 60°.

Measure the other two angles.

Now draw a triangle of sides 8 centimetres and 4 centimetres (with angle between them 60° as before). Do the other angles change?

What is the relation between the sides here? Let's draw such triangles using GeoGebra. For this, first create a slider a with Min = 0 and Max = 10. Using the **Segment with Given Length** tool, draw line AB of length 2a. Use the **Angle with given size** to draw a line AB' slanted at 60° through A. Next choose the **Circle with Centre and Radius tool** and click on A. In the dialog box, give a as the radius. Mark the point where the circle cuts the slanted line as C.
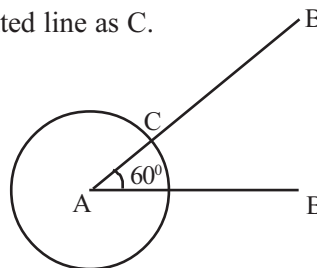
Now we can hide the lines, angles and circle in the figure. Using the **Polygon** tool, draw triangle ABC. Select the **Distance or Length** tool and click on the sides of the triangle to get their lengths; select **Angle** tool and click within the triangle to get the anlges.

Use slider to change the value of a.

How do the sides change?

What about the angles?

Angles and sides

Draw a line of 6 centimetres and draw another line slanted at 30° at one end. With the other end as centre, draw some circles of different radii:

30°

6 cm
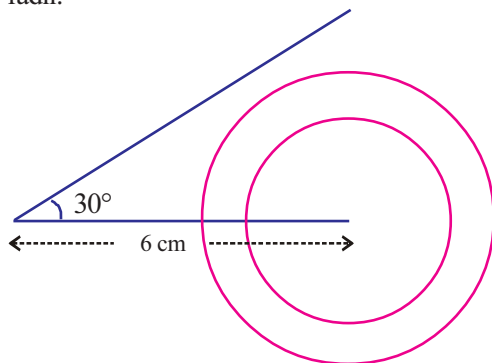
What is the minimum radius for which the circle meets the upper line?

For what all radii do the circle cut the upper line at two points?

We want to draw ΔABC with $AB = 6$ cm and $\angle B = 30°$. What should be the minimum length of AC?

For what all lengths of AC do we get two such triangles?

Let's do this using GeoGebra. Draw line AB of length 6 units and line AB' such that $\angle BAB' = 30°$. Make a slider a and use the **Circle with Centre and Radius** tool to draw a circle centred at B, of radius a. Change the value of a using the slider. For what all values of a does the circles meet AB'?

Suppose in this problem, we change the length of YZ to 4 centimetres?

Do we again get two triangles?

If YZ is to be 2.5 centimetres, how many triangles do we get?

What if we take it as 2 centimetres?

X 30°

5 cm

2 cm

Y
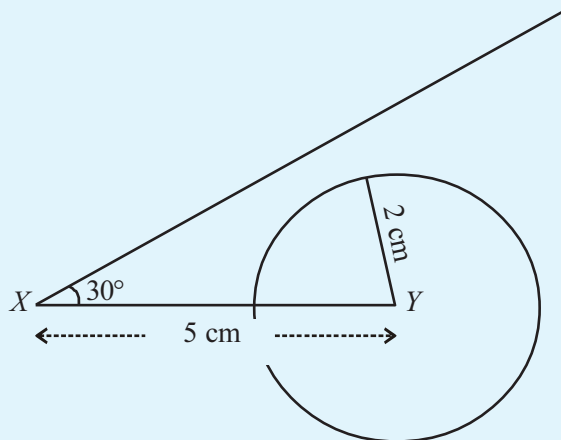
Can we draw a triangle?

Suppose we take the length of YZ as 6 centimetres?

Now try to draw triangles of these specifications.

- $AB = 5$ centimetres, $BC = 6$ centimetres, $\angle A = 40°$
- $PQ = 8$ centimetres, $PR = 7$ centimetres, $\angle Q = 50°$
- $XY = 4$ centimetres, $YZ = 6$ centimetres, $\angle X = 70°$

Two angles

Can we draw a triangle if one side and two angles are specified?

Let's see how we can draw triangle STU with $ST = 5$ centimetres, $\angle S = 60°$, $\angle T = 70°$.

We first draw a rough sketch:

U

60°   70°
S   5 cm   T
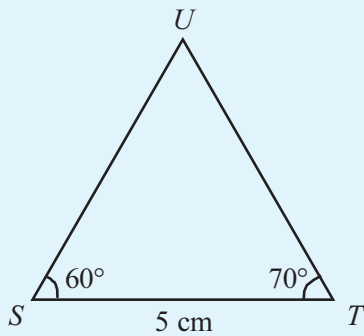
Let's start by drawing line *ST* of length 5 centimetres.

S   5 cm   T
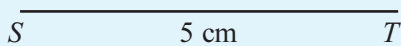
Now we want to find the position of *U*.

Draw a line through *S* of inclination 60° and a line through *T* of inclination 70°.

The point where they meet is *U*.

U

60°   70°
S   5 cm   T
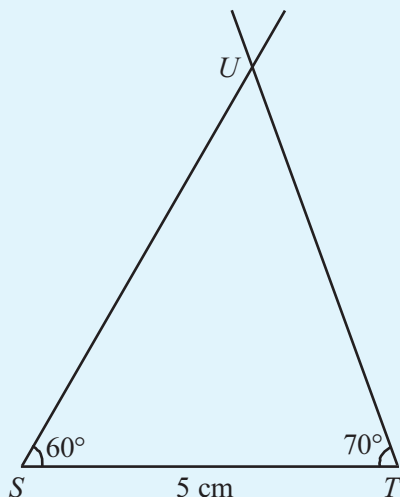
Draw triangles of the following specifications.

- *YZ* = 7 centimetres, ∠*Y* = 45°, ∠*Z* = 65°
- *MN* = 6.5 centimetres, ∠*M* = 60°, ∠*N* = 55°
- Draw △*ABC* with *AB* = 7 centimetres, ∠*A* = 60° and ∠*B* = 60°. How much is ∠*C* ? Measure the length of *BC* and *CA*.
- Draw △*PQR* with *PQ* = 4.5 centimetres, ∠*P* = 70° and ∠*Q* = 70°. How much is ∠*R*? Measure the length of *PR* and *RQ*.

Parallel triangles

At the two ends of a line, draw angles of 70° and 80° to make a triangle:

70°   80°

What is its third angle?

Draw lines parallel to the sides of the triangle and make another triangle.

70°   80°
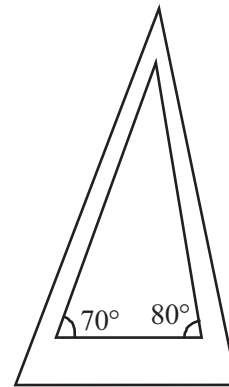
Measure the angles of this triangle. Draw other triangle like this and check. Do the angles change?

Let's do this using GeoGebra. Make slider a with Min = 0 and Max = 2. Draw a triangle using the **Polygon** tool. Mark a point D within it. Select the tool **Dilate Object from Point by Factor** and click within the triangle and on D. In the dialog box give the value of Factor as a and give OK.

Use the slider to change a. Using the **Angle** tool, the angles of both triangles can be displayed.

Change the position of D to one of the vertices and see what you get.

Unchanging relations

Can we draw a triangle with AB = 6 and AC =2 BC? Let's draw such triangle with GeoGebra. Draw line AB of the length 6. Make a slider a with suitable Min and Max values. Draw the circle of radius a centred at B and the circle of radius 2a centred at A. Mark the points C, D where these circles cut each other. Draw the lines AC and BC. Now we can hide the circles. Change the value of a using the slider. (We can also right click on the slider and select **Animate**). Right click on C and in the dropdown menu, check the **Trace on** option. What is the path of the point C? Draw the lines AD, BD. Draw the path of D also. This path can be seen more clearly by making the change smaller. (Right click on the slider and choose Properties → Slider → Increment)

Draw triangles with the relation AC = 3BC or 2AC = 3AC, instead of AC = 2BC. In all these, what are the paths of C and D?

What if AC = BC?

Suppose in the last problem above, $\angle R$ is to be 70º, instead of $\angle Q$?

In the problem so far, one side and the two angles on this side were specified.

Here we are given $\angle P$ and $\angle R$.

We need $\angle P$ and $\angle Q$.

How do we calculate $\angle Q$?

$$\angle Q = 180° - (70° + 70°) = 40°$$

Now can't we draw the triangle?

Three sides

We can draw triangles of three specified sides also.

Let's try $\triangle ABC$ with $AB = 5$ cm, $BC = 3$ cm and $AC = 4$ cm.

We draw rough sketch for reference.
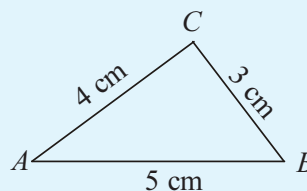
First we draw AB of length 5 centimetres.
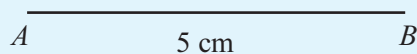
Now we must locate C.

It is 4 centimetres away from A and 3 centimetres away from B.

All points which are 4 centimetres away from A are on the circle centred at A, of radius 4 centimetres.

Likewise, if we draw the circle centred at B of radius 3 centimetres, we get all points 3 centimetres away from B.
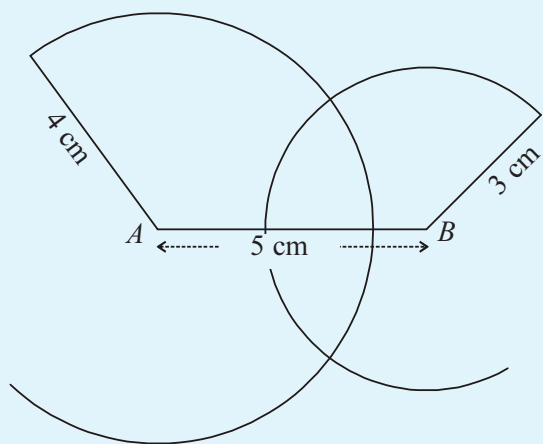
Both the points where these circles cut are at a distance of 4 centimetres from A and 3 centimetres from B.

Either of them can be used to draw the required triangle:
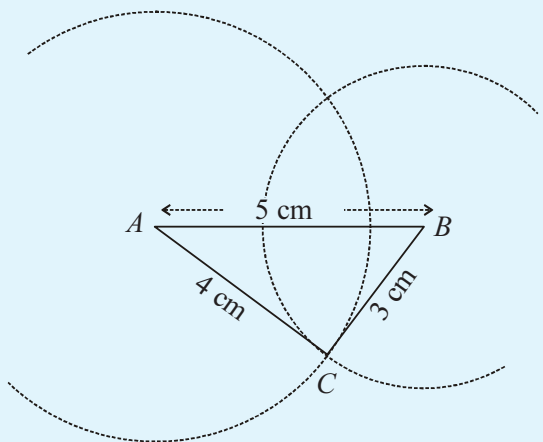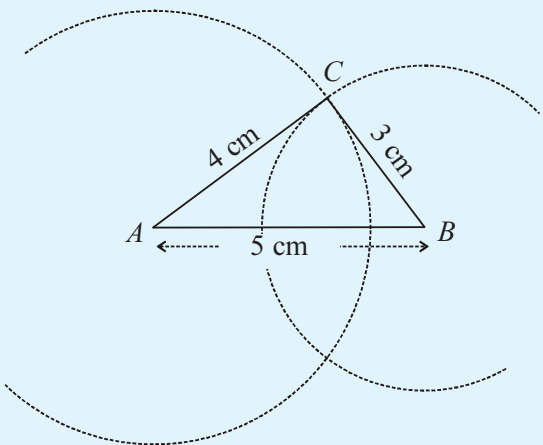
Look at this picture:
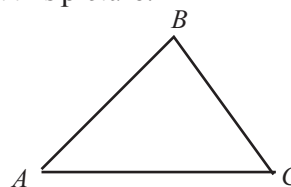
We can go from A to C, straight along the line AC; or we can first go to B along AB and then to C along BC. Which path is shorter?

Do you see any relation connecting the sides of a triangle from this?

Sticky math

Take two thin sticks of the same length. Break one of them into two pieces.

Can we make a triangle with these three sticks?

Now break off a small bit from the long stick.

Now can you make a triangle?

Unchanging perimeter

Can you draw a triangle of perimeter 15 centimetres? Let's see how we do it in GeoGebra. We first make two sliders to control the length of sides. Make sliders a and b with Min = 0 and Max = 7.5. Use the **Segment with Given Length** tool to draw the line AB of length a. Then what should be the sum of the lengths of the other two sides?

The perimeter should be 15. So,

$$AC + BC = 15 - AB = 15 - a$$

If the length of one of these sides is b, then what must be the length of the other? We use this to draw the other two sides. With A as centre, draw a circle of radius b, and with B as centre draw a circle of radius 15 - a - b. Mark the points C, D where the circles cut each other. Draw triangle ABC, using the **Polygon** tool. Choose the **Distance or Length** tool and click within the triangle to get its perimeter. Change the value of a and b using the sliders. Don't we get different triangles of same perimeter?

Let's see how we can get a nice picture with this set up. Draw the lines AD and BD also. Give the **Trace On** option for the lines AC, BC, AD, BD and the points C, D. Fix some convenient value for a and animate the slider for b. Look at the picture got. What are the path of C and D?

Now can't you draw triangles of the following specifications?

- PQ = 5 centimetres, QR = 5 centimetres, PR = 4 centimetres.

- XY = 7.5 centimetres, YZ = 6.5 centimetres, XZ = 5.5 centimetres.

- DE = 7 centimetres, EF = 7 centimetres, DF = 7 centimetres.

- Draw ΔABC with AB = 6 centimetres, AC = 5 centimetres, $\angle A = 85°$.

- Draw ΔPQR with PQ = 5 centimetres, $\angle Q = 60°$ PR = 7 centimetres. Measure the third side and write the length in the figure.

- Draw ΔMNT with MN = 8 centimetres, $\angle M = 60°$ $\angle N = 50°$.

- Draw ΔXYZ with XY = 6 centimetres, YZ = 7 centimetres, XZ = 7 centimetres.

Can you draw a triangle of sides 5 centimetres, 4 centimetres and 10 centimetres?

How about sides 5 centimetres, 4 centimetres and 9 centimetres?

Sides 5 centimetres, 4 centimetres and 8.5 centimetres?

If two sides are to be 5 centimetres and 4 centimetres, the third side should be less than how much centimetres?

What is the relation between the lengths which can be used to draw a triangle?

Why is it that we cannot draw triangles of some specified lengths of sides?

Now check which of the lengths below can be the sides of a triangle.

- 8 centimetres, 6 centimetres, 13 centimetres.

- 2 centimetres, 5 centimetres, 8 centimetres.

- 5 centimetres, 4 centimetres, 9 centimetres.

- 4 centimetres, 6 centimetres, 7 centimetres.

Unchanging angles

Can you draw triangle ABC with AB = 5 and $\angle$C = 60°? Let's do it with GeoGebra.

Draw line AB of length 5. Make an Angle slider a. Choose the **Angles with Given Size** tool and click first on B and then on A. In the pop-up window, give a as the size of the angle. Now we get a point B' with $\angle$BAB' = a. With the same tool, click first on A and then on B; in the pop-up window, give the size of the angle as 120 - a and select the clockwise option also. We now get another point A'. Join AB' and BA' using the **Ray Through Two Points** tool. Mark the point C where these lines intersect. Draw triangle ABC, using the **Polygon** tool. Now we can hide all lines and points other than those of the triangle. We can see the angles of the triangle by choosing the **Angle** tool and clicking within the triangle. change the value of a and see what happens. Choose **Trace on** for lines AC, BC and the point C and animate the slider. What is the path of C?

Do this with other angles instead of 60° at C. We can also use a slider to change the angle at C.

Looking back

Achievements	On my own	With teacher's help	Must improve
• Drawing a triangle with two sides and an angle specified.			
• Drawing a triangle with one side and two angles specified.			
• Drawing a triangle with three sides specified.			
• Justifying logically why triangles of certain specifications cannot be drawn.			
• Drawing geometric figures precisely and accurately			
• Using GeoGebra for drawing geometric figures.			

9

Ratio

Same shape

In both the rectangles below, the length is 1 centimetre more than the breadth.

1 cm
2 cm

4 cm
5 cm
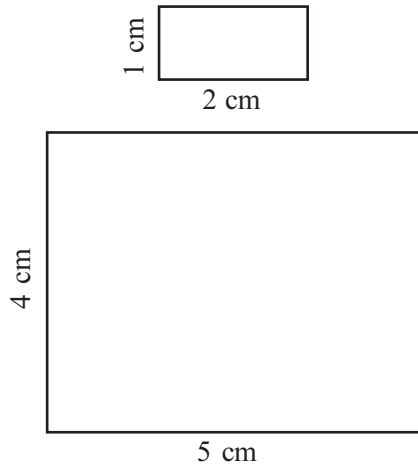
But not only the length of these are different, they look different also. In the larger rectangle, the sides look almost the same. Draw a rectangle of length 50 centimetres and breadth 49 centimetres in a larger sheet of paper. It looks almost a square, doesn't it?

In the first rectangle above, length is double the breadth. Now look at this rectangle.

2.5 cm
5 cm
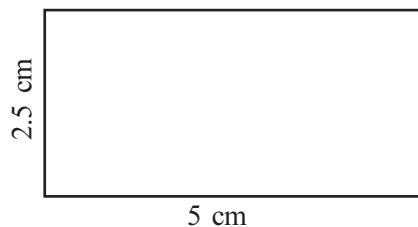
Again the length is double the breadth. Even though it is larger than the first, they have the same shape, right?

Length and width

Look at these rectangles:

2 cm
4 cm

2.5 cm
5 cm

3 cm
6 cm
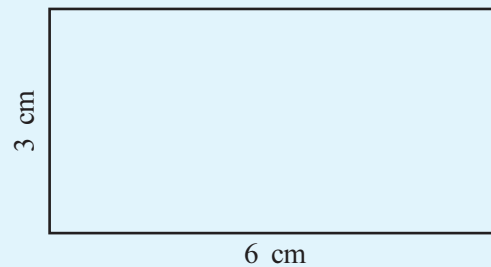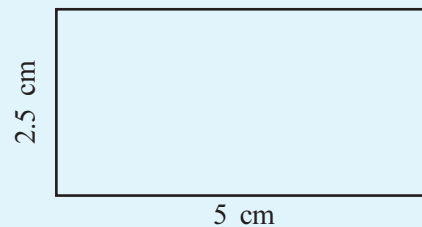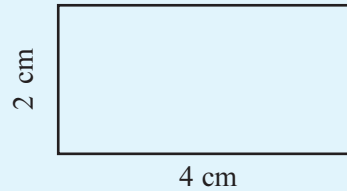
Is there any common relation between the lengths and widths of all these?

In all these, the length is twice the width, right? (We can also say width is half the length)

In the language of mathematics, we state this fact like this:

In all these rectangles, the width and length are in the *ratio* one to two.

In writing, we shorten the phrase "one to two" as 1 : 2; that is

In all these rectangles, the width and length are in the ratio 1 : 2.

In the rectangle of width 1 centimetre and length 2 centimetres, the length is twice the width. In a rectangle of width 1 metre and length 2 metres also, we have the same relation. So, in both these rectangles, width and length are in the ratio one to two (1 : 2).

We can state this in reverse: in all these rectangles, length and width are in the ratio two to one (2 : 1).

What is the width to length ratio of the rectangle below?

2 cm | 6 cm (rectangle)
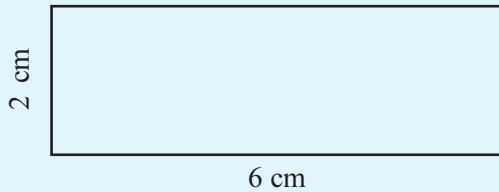
And what about this rectangle?

1.5 cm | 4.5 cm (rectangle)
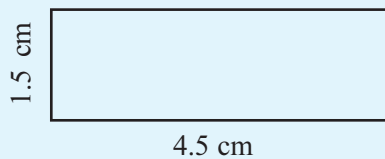
In both, the length is 3 times the width. So what is the ratio of the width to the length?

What if the width is 2 centimetres and length is 1 metre?

How many times the width is the length?

1 metre means 100 centimetres. So in such a rectangle, the ratio of the width to the length is 1 : 50.

Now look at these rectangles:

4 cm | 6 cm (rectangle) 1 cm | 1.5 cm (rectangle)
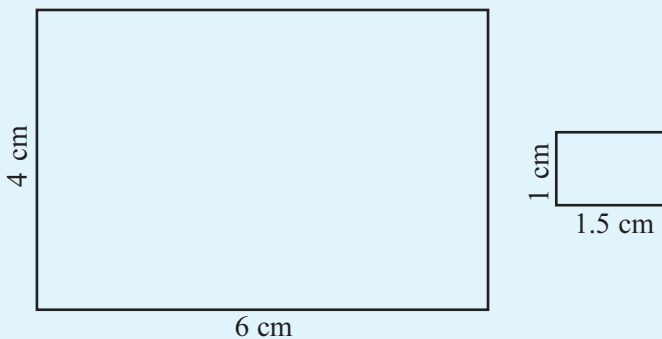
Changing scale

Look at this photo:
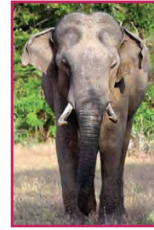
The shorter side is 2 centimetres and the longer side, 3 centimetres; that is, the longer side is

$1\dfrac{1}{2}$ times shorter side.

Suppose we make the shorter side 3 centimetres and longer side 4.5 centimetres.

Still, the longer side is $1\dfrac{1}{2}$ times the shorter side.

Now suppose we   change the shorter side to 3 centimetres and increase the longer side also by 1 centimetre, making it 4 centimeters.
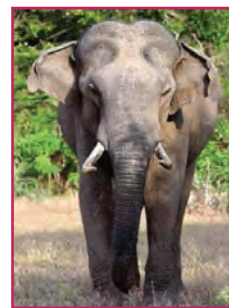
Does the picture look right?

TV Math

The sizes of TV sets are usually given as 14 inch, 17 inch, 20 inch and so on. What does it mean?

The TV screen is a rectangle; and these are lengths of the diagonals of the screen.

Does it determine the size of the screen? Rectangles of different width and lengths can have the same diagonal:
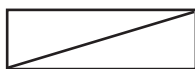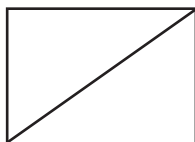
In the modern TV sets, the ratio of length to width is 16 : 9. In the earlier days, it was 4 : 3.

See their difference in two TV screens of the same diagonal length.

4 : 3

16 : 9

In both, the length is one and a half times the width.

How do we say this as a ratio?

We can say one to one and a half; but usually we avoid fractions when we state ratios.

Suppose we take the width as 2 centimetres.

What is $1\frac{1}{2}$ times 2?

2 cm

3 cm
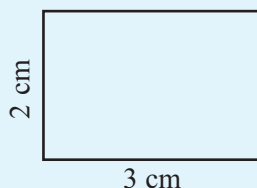
So, we say that in rectangles of this type, the ratio of width to length is two to three and write 2 : 3.

Can't we say here that the ratio is 4 : 6 ?

Nothing wrong in it; but usually ratios are stated using the least possible counting numbers.

So how do we state using ratios, the fact that the length of a rectangle is two and a half times the width?

If the width is 1 centimetre, then the length is $2\frac{1}{2}$ centimetres.

What if the width is 2 centimetres?

Length would be 5 centimetres.

So, we say that width and length are in the ratio 2 to 5.

What if the length is one and a quarter times the width?

If the width is 1 centimetre, the length is $1\frac{1}{4}$ centimetre.

If the width is 2 centimetres then the length is $2\frac{1}{2}$ centimetres.

Still we haven't got rid of fractions.

Now if width is 4 centimetres, what would be the length?

So, in all such rectangles, the width and length are in the ratio 4 : 5.

Do you notice another thing in all these?

If we stretch or shrink both width and length by the same factor, the ratio is not changed. For example, look at this table.

Width	Length
3 cm	9 cm
6 cm	18 cm
1 m	3 m
$\frac{1}{2}$ m	$1\frac{1}{2}$ m
$1\frac{1}{2}$ m	$4\frac{1}{2}$ m

In all these, the length is 3 times the width; or the reverse, the length is $\frac{1}{3}$ of the width.

In terms of ratio, we say width and length are in the ratio 1 : 3 or length and width are in the ratio 3 : 1.

- For all rectangles of dimensions given below, state the ratio of width to length, using the least possible counting numbers:
  - width 8 centimetres, length 10 centimetres
  - width 8 metres, length 12 metres
  - width 20 centimetres, length 1 metre
  - width 40 centimetres, length 1 metre
  - width 1.5 centimetres, length 2 centimetres

Flags

When we draw our National Flag, not only should the colours be right, the ratio of width to length should also be correct. This ratio is 2 :3.

That is, in drawing our flag, if the length is taken as 3 centimetres, the width should be 2 centimetres.

This ratio is different for flags of other countries. For example, in the flag of Australia, this ratio is 1:2.

And in the flag of Germany, it is 3 : 5.

Without fractions

When quantities like length are measured using a definite unit, we may not always get counting numbers; and it is this fact which led to the idea of fractions.

In comparing the sizes of two quantities, one question is whether both can be given as counting numbers, using a suitably small unit of measurement. It is this question that leads to the idea of ratio.

For example, suppose the length of two objects are found as $\frac{2}{5}$ and $\frac{3}{5}$, when measured using a string. If $\frac{1}{5}$ of the string is taken as the unit, we can say the length of the first is 2 and that of the second is 3. This is the meaning of saying the ratio of the lengths is 2 : 3.

Suppose the length of two objects are $\frac{1}{3}$ and $\frac{1}{5}$ of the string.

To get both lengths as counting numbers, what fraction of the string can be taken as a unit of measurement?

- In the table below two of the width, length and their ratio of some rectangles are given. Calculate the third and fill up the table.

Width (cm)	Length (cm)	Ratio
6	8	
3		3 : 4
1		3 : 4
	1	3 : 4
6	15	
2		2 : 5
1		2 : 5
	1	2 : 5

- What does it mean to say that the width to length ratio of a rectangle is 1 : 1 ? What sort of a rectangle is it?

Other quantities

There are two ropes, the shorter $\frac{1}{3}$ metre long and the longer, $\frac{1}{2}$ metre long. What is the ratio of their lengths?

We can do this in different ways.

We can check how much times $\frac{1}{3}$ is $\frac{1}{2}$.

$$\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$$

So, the length of the longer rope is $\frac{3}{2}$ times that of the shorter.

That is, $1\frac{1}{2}$ times.

If the length of the shorter rope is taken as 1, the length of the longer is $1\frac{1}{2}$; if 2, then 3.

So the length of the shorter and longer rope are in the ratio 2 : 3.

We can also think in another manner. As in the case of the width and length of rectangles, we can imagine the length to be stretched by the same factor; and the ratio won't change.

Suppose we double the length of each piece of rope.

Then the length of the shorter is $\frac{2}{3}$ metres and that of the longer 1 metre. This doesn't remove fractions.

By what factor should we stretch to get rid of fractions?

How about 6 ?

6 times $\frac{1}{3}$ is 2.

6 times $\frac{1}{2}$ is 3.

The shorter is now 2 metres and the longer is 3 metres. So the ratio is 2 : 3.

There is yet another way; we can write

$$\frac{1}{3} = \frac{2}{6} \qquad \frac{1}{2} = \frac{3}{6}$$

That is, we can think of the shorter rope as made up of 2 pieces of $\frac{1}{6}$ metres each and the longer one as 3 of the same $\frac{1}{6}$ metres. In this way also, we can calculate the ratio as 2 to 3.

Now look at this problem : to fill a can we need only half the water in a bottle.

Circle relations

Look at the pieces of circles below:

The smaller piece is $\frac{1}{4}$ of the circle and the larger piece is $\frac{1}{2}$ of the circle.

So, the larger piece is twice the size of the smaller. That is, the ratio of the sizes of small and large pieces is 1 : 2.

Now look at these pieces:
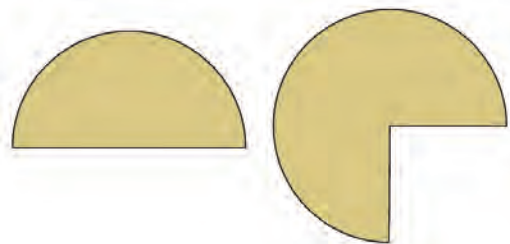
What is the ratio of their sizes?

Let's measure each using $\frac{1}{4}$ of circles.

The smaller figure has two such pieces.

What about the larger?
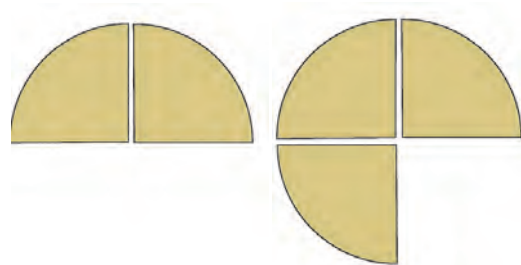
So what is the ratio of the sizes of these two?

Moving Ratio

Have you taken apart toy cars or old clocks? There are many toothed wheels in such mechanisms. See this picture:
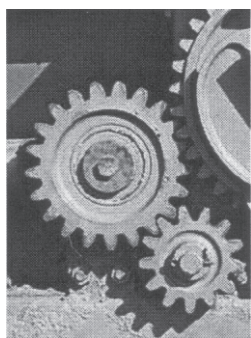
It is a part of a machine. In the two whole wheels we see, the smaller has 13 teeth and the larger has 21. So during the time that the smaller circle makes 21 revolutions, the larger one would have made only 13 revolutions.

The speed of rotation of machines is controlled by arranging such wheels with different number of teeth.

They are all toothed wheels; but they have lost much of their bite!

To fill a larger can, we need three quarters of the bottle. What is the ratio of the volume of the smaller can to the larger can ?

Here, we can write

$$\frac{1}{2} = \frac{2}{4}$$

So, if we pour $\frac{1}{4}$ of the bottle 2 times, we can fill the smaller can; to fill the larger can, $\frac{1}{4}$ of the bottle should be poured 3 times. So the volumes of the smaller and larger cans are in the ratio 2 : 3.

Another problem: Raju has 200 rupees with him and Rahim has 300. What is the ratio of the money Raju and Rahim have?

If we imagine both to have the amounts in hundred rupee notes, then Raju has 2 notes and Rahim, 3 notes. So the ratio is 2 : 3.

Let's change the problem slightly and suppose Raju has 250 rupees and Rahim, 350 rupees.

If we think of the amounts in terms of 50 rupee notes, then Raju has 5 and Rahim has 7.

The ratio is 5 : 7.

What if the amounts are 225 and 325 rupees?

Imagine each amount as packets of 25 rupees. Raju has $225 \div 25 = 9$ packets and Rahim has $325 \div 25 = 13$ packets. The ratio is 9 : 13.

Let's look at one more problem: in a class, there are 25 girls and 20 boys. What is the ratio of the number of girls to the number of boys?

If we split the girls and boys separately into groups of 5, there will be 5 groups of girls and 4 groups of boys. So the ratio is 5 to 4.

In this way, calculate the required ratios and write them using the least possible counting numbers, in these problems:

- Of two pencils, the shorter is of length 6 centimetres and the longer, 9 centimetres. What is the ratio of the lengths of the shorter and the longer pencils?

- In a school, there are 120 boys and 140 girls. What is the ratio of the number of boys to the number of girls?

- 96 women and 144 men attended a meeting. Calculate the ratio of the number of women to the number of men.

- When the sides of a rectangle were measured using a string, the width was $\frac{1}{4}$ of the string and the length $\frac{1}{3}$ of the string. What is the ratio of the width to the length?

- To fill a larger bottle, $3\frac{1}{2}$ glasses of water are needed and to fill a smaller bottle, $2\frac{1}{4}$ of glasses are needed. What is the ratio of the volumes of the large and small bottles?

Ratio of mixtures

To make *idlis*, Ammu's mother grinds two cups of rice and one cup of urad dal. When she expected some guests the next day, she took four cups of rice. How many cups of urad should she take?

To have the same consistency and taste, the amount of urad must be half the amount of rice.

So for four cups of rice, there should be two cups of urad.

We can say that the quantities, rice and urad must be in the ratio 2 :1

Now another problem on mixing: to paint the walls of Abu's home, first 25 litres of green and 20 litres of white were mixed.

Sand and Cement

In construction of a building, sand and cement are used in definite ratios, but the ratios are different depending on the purpose. When one bowl of cement and five bowls of sand are mixed, the ratio of cement to sand is 1 : 5

When one sack of cement is mixed with five sacks of sand, the ratio is the same. But to set a brick wall, this much cement may not be needed. In this case, the ratio may be 1 : 10 or 1 : 12.

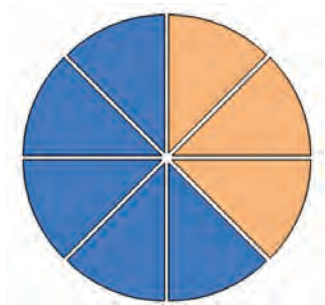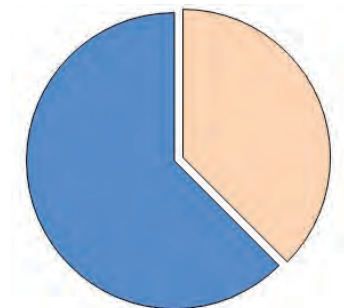## Ratio of Parts

We can use ratio to compare parts of a whole also. For example, in the picture below, the lighter part is $\frac{3}{8}$ of the circle, and the darker part is $\frac{5}{8}$ of the circle.

These two parts together make up the whole circle. The ratio of the sizes of these parts is 3 : 5.

So here, the ratio 3 : 5 indicate the two fractions $\frac{3}{8}$ and $\frac{5}{8}$.

Generally in such instances, a ratio of the numbers indicates fractions of equal denominators adding up to 1.

When this was not enough, 15 litres of green was taken. How much white should be added to this?

To get the same final colour, the ratio of green and white should not change.

In what ratio was green and white mixed first?

That is, for 5 litres of green, we should take 4 litres of white.

To maintain the same ratio, for 15 litres of green, how many litres of white should we take?

How many times 5 is 15?

So 3 times 4 litres of white should be mixed.

That is 12 litres.

To get the same shade of green, how many litres of green should be mixed with 16 litres of white?

Now try these problems:

- For 6 cups of rice, 2 cups of urad should be taken to make *dosas*. How many cups of urad should taken with 9 cups of rice?
- To set the walls of Nizar's house, cement and sand were used in the ratio 1:5. He bought 45 sacks of cement. How many sacks of sand should he buy?
- To paint a house, 24 litres of paint was mixed with 3 litres of turpentine. How many litres of turpentine should be mixed with 32 litres of paint?
- In the first ward of a panchayat, the male to female ratio is 10 : 11. There are 3311 women. How many men are there? What is the total population in that panchayat?
- In a school, the number of female and male teachers are in the ratio 5 : 1. There are 6 male teachers. How many female teachers are there?
- Ali and Ajayan set up a shop together. Ali invested 5000 rupees and Ajayan, 3000 rupees. They divided the profit for a month in the ratio of their investments and Ali got 2000 rupees. How much did Ajayan get? What is the total profit?

Division problem

We have seen that to make *idlis*, rice and urad are to be taken in the ratio 2:1. In 9 cups of such a mixture of rice and urad, how many cups of rice are taken?

2 cups of rice and 1 cup of urad together make 3 cups of mixture.

Here we have 9 cups of mixture.

How many times 3 is 9 ?

To maintain the same ratio, both rice and urad must be taken 3 fold.

So 6 cups of rice and 3 cups of urad.

Another problem: In a co-operative society there are 600 members are male and 400 are female. An executive committee of 30 members is to be formed with the same male to female ratio as in the society. How many male and female members are to be there in the committee?

In the society, the male to female ratio is 3 : 2.

3 men and 2 women make 5 in all.

Here we need a total of 30.

How many times 5 to 30?

So there should be $3 \times 6 = 18$ men and $2 \times 6 = 12$ women in the committee.

One more problem: a rectangular piece of land is to be marked on the school ground for a vegetable garden. Hari and Mary started making a rectangle with a 24 metre long rope. Vimala Teacher said it would be nice, if the sides are in the ratio 3 : 5. What should be the length and width of the rectangle?

The length of the rope is 24 metres and this is the perimeter of the rectangle.

If we take the length and width as 3 meters and 5 metres, what would be the perimeter?

Meaning of ratio

If we know only the ratio of two quantities, we can't say exactly how much they are; but they can be compared in several ways.

For examples, suppose the volumes of two pots are said to be in the ratio 2 : 3. We can interpret this in the following ways:

- To fill the smaller pot, we need $\frac{2}{3}$ of the larger pot.

- To fill the bigger pot, we need $\frac{3}{2} = 1\frac{1}{2}$ times the smaller pot.

- Whether we take $\frac{1}{2}$ of the water in the smaller pot, or $\frac{1}{3}$ of the water in the larger pot, we get the same amount of water.

- If we fill both pots and pour them into a large pot, $\frac{2}{5}$ of the total amount of water is from the smaller pot and $\frac{3}{5}$ from the larger.

If the length of two pieces of rope are said to be in the ratio 3 : 5, what all interpretations like this can you make?

Three measures

Look at this triangle:
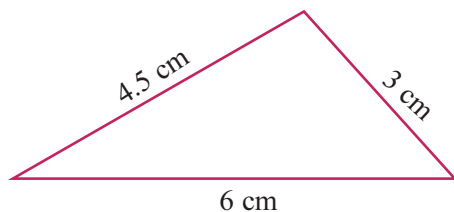
In it, the longest side is double the shortest. The side of medium length is one and a half times the shortest.

Saying this with ratios, the lengths of the shortest and the longest sides are in the ratio 1 : 2 and those of the shortest and the medium are in the ratio 2 : 3.

What is the ratio of the lengths of the medium side and the longest side?

We can say this in another way: if we use a string of length 1.5 centimetres, the shortest side is 2, the medium side is 3 and the longest side is 4.

The can be shortened by saying the sides are in the ratio 2 : 3 : 4.

Why don't you find the ratio?

I don't get the rational!

How much of 16 is 24?

$$\frac{24}{16} = \frac{3}{2} = 1\frac{1}{2}$$

So width should be $1\frac{1}{2}$ times 3 metres; that is

$$3 \times 1\frac{1}{2} = 4\frac{1}{2} \text{ metres.}$$

And length should be $1\frac{1}{2}$ times 5 metres, that is,

$$5 \times 1\frac{1}{2} = 7\frac{1}{2} \text{ metres.}$$

Now try these problems:

- Suhra and Sita started a business together. Suhra invested 40000 rupees and Sita, 30000 rupees. They made a profit of 7000 rupees which they divided in the ratio of their investments. How much did each get?

- John and Ramesh took up a job on contract. John worked 7 days and Ramesh, 6 days. They got 6500 rupees as wages which they divided in the ratio of their investments. How much did each get?

- John and Ramesh took up a job on contract. John worked 7 days and Ramesh, 6 days. They got 6500 rupees as wages which they divided in the ratio of the numbers of days each worked. How much did each get?

- Angles of a linear pair are in the ratio 4:5. What is the measure of each angle?

- Draw a line AB of length 9 centimetres. A point P is to be marked on it, such that the lengths of AP and PB are in the ratio 1 : 2. How far from A should P be marked? Compute this and mark the point.

- Draw a line 15 centimetres long. A point is to be marked on it, dividing the length in the ratio 2 : 3. Compute the distances and mark the point.

- Sita and Soby divided some money in the ratio 3 : 2 and Sita got 480 rupees. What is the total amount they divided?

- In a right triangle, the two smaller angles are in the ratio 1 : 4. Compute these angles.

- Draw a rectangle of perimeter 30 centimetres and lengths of sides in the ratio 1 : 2. Draw two more rectangles of the same perimeter, with lengths of sides in the ratio 2 : 3 and 3 : 7. Compute the areas of all three rectangles.

Triangle Math

How many triangles are there with ratio of sides 2 : 3 : 4?

The lengths of sides can be 2 cm. 3 cm, 4cm.
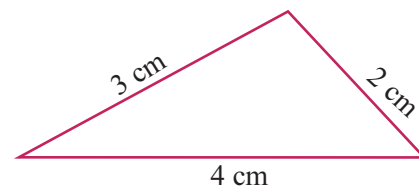
Or they can be 1cm, 1.5 cm, 2cm.
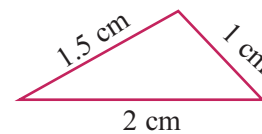
We can take metres instead of centimetres.

Thus we have several such triangles.

In all these, what fraction of the perimeter is the shortest side?

And the side of medium length?

The longest side?

The perimeter of a triangle is 80 centimetres and its sides are in the ratio 5 : 7 : 8. Can you compute the actual lengths of sides?

What if the perimeter is 1 metre?

Looking back

Achievements	On my own	With teacher's help	Must improve
• Stating the ratio of two quantities using the smallest possible counting numbers.			
• Interpreting the ratio of two quantities in different ways.			
• Given the ratio of two quantities and one of the quantities, describing the method to find the other.			
• Dividing a quantity in a specified ratio.			
• Solving practical problems involving ratios.			

10

Money Math
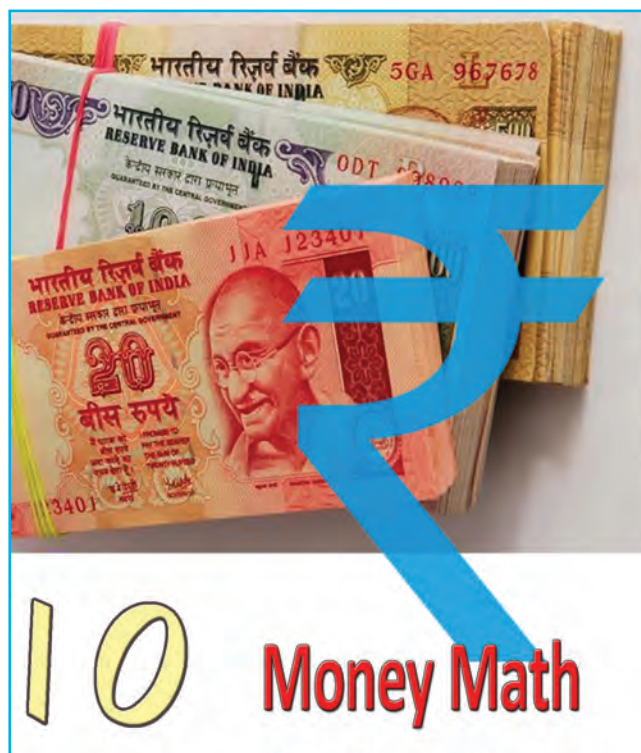
Math and commerce

Various commercial activities started very early in human history. At first it was just exchange of goods, such as two goats for one cow or five mangoes for a jackfruit.

At a later stage, different forms of currency was used to denote the value of goods, instead of the actual goods themselves. Much computation with numbers was necessary to keep such momentary transactions exact. Thus such computations also became a part of mathematics education.

Vegetable prices

The table gives the prices of some vegetables in Thiruvananthapuram and Nagercoil.

Price per kg		
Item	**Thiruvananthapuram**	**Nagercoil**
Beetroot	Rs. 35	Rs. 24
Cabbage	Rs. 45	Rs. 30
Carrot	Rs. 60	Rs. 50
Green chilli	Rs. 76	Rs. 60

Why are the prices different?

• Transportation cost

•

•

Majid is a vegetable merchant. He bought yam at 20 rupees a kilogram. He sold them on the spot at 25 rupees a kilogram. How much profit did he make in this sale?

• How many kilograms of yam did he buy?

• How much money did he get on selling?

• How much money did he spent on buying?

• How much is the profit?

The next day also, Majid bought 200 kilograms of yam at 20 rupees a kilogram and paid 200 rupees to take them to another market. There he sold them at 25 rupees a kilogram. How much is his profit?

Here, the vehicle fare also should be added to the amount paid for the yam to find the total amount he spent.

At Oruma Cooperative Society, 100 kilogram of wheat grains was bought at 25 rupees a kilogram. 500 rupees was spent in washing, drying, grinding and packaging. 100 packets were made, to be sold at 35 rupees a packet. 20 packets of powder was spoiled and the remaining sold. Did they make a profit or suffer a loss in this sale? How much?

- Thomas bought 10 cents of land at 75000 rupees a cent. He spent 50000 rupees to build a wall around and 60000 rupees to dig a well. He sold the plot at 90000 rupees a cent. Was it a gain or loss for him? How much?

- A trader bought 20 quintals of rubber sheets at 19850 rupees a quintal. He spent 3000 rupees to take the load to the shop. Since prices went down, he had to sell them at 18250 rupees a quintal. How much money did he lose?

Selling fruit

This is the price list at Saji's Fruit shop:

Item	Price per kg
Orange	Rs. 60
Grape	Rs. 52
Apple	Rs. 110
Mango	Rs. 65

Quintal, ton, tonne

In the past, different units were used in different parts of the world to measure length, weight and so on. Now these have been unified, with most countries using the metric system.

A unit of weight called *quintal* was in use in many countries during the olden days, denoting hundred times the basic unit of weight. Once the metric system was adopted, a quintal was fixed as 100 kilograms.

In the past, a *ton* in England meant 2240 pounds (1016 kilograms in the current system). In the metric system, 1000 kilograms is called a *tonne*. To distinguish between the two, a tonne is also known as a *metric ton*.

According to the common prefixes used in the metric system, a tonne, is a mega gram (1000000 grams).

Any way I'll need a new unit may be two!

Chains of Commerce

In today's world, there are numerous links in the chain from the producer of goods to the end-user. That means the goods reach the end-user after a series of transactions. Simply put, those who buy and store goods on a large scale from various producers and sell them to other shops and traders, are called *wholesale* dealers. At the other end, those who sell directly to the end users are called *retail* sellers. There may be numerous other transactions in between. Depending on the expenses, the prices may also increase at every stage.

Selling grapes is good business

So, you've nothing to gripe about!

He buys oranges and mangoes at 50 rupees a kilogram, grapes at 40 rupees a kilogram and apples at 100 rupees a kilogram. Which fruit gets him the most profit?

Oranges bought for 50 rupees are sold for 60 rupees and mango bought for 50 rupees are sold for 65 rupees. Of these, mangoes are more profitable, since they return more for the same amount spent.

When apples bought for 100 rupees are sold for 110 rupees he gets a profit of 10 rupees.

When oranges bought for 50 rupees are sold for 60 rupees, how much profit does he get?

How do we decide which is more profitable?

He gets the same profit of 10 rupees, on selling oranges bought for 50 rupees and apples bought for 100 rupees. Of these, oranges are more profitable, since he spends less on them.

Grapes bought for 40 rupees are sold for 52 rupees

Oranges bought for 50 rupees are sold for 60 rupees.

Which of these is more profitable?

Let's suppose he buys each for 100 rupees.

2 kilograms of oranges can be bought for 100 rupees and they are sold for $60 \times 2 = 120$ rupees; the profit is 20 rupees.

How many kilograms of grapes can he buy for 100 rupees?

He can buy 2 kilograms for 80 rupees and with the remaining 20 rupees another $\frac{1}{2}$ kilogram; altogether $2\frac{1}{2}$ kilograms.

For how much does he sells these?

$$52 \times 2\frac{1}{2} = 104 + 26 = Rs.\ 130$$

So the profit is 30 rupees.

Here we find that grapes fetch more profit, by assuming the amount spent on each to be 100 rupees.

Such computations can be made easier by reducing to percentages.

Let's do these in percentages.

In selling oranges, the profit is $\frac{10}{50} = \frac{1}{5}$ of the amount spent.

$\frac{1}{5}$ means $\frac{1}{5} \times 100 = 20$ percent.

In the case of grapes, the profit is $\frac{12}{40} = \frac{3}{10}$ of the amount spent.

In terms of percentage, $\frac{3}{10} \times 100 = 30\%$.

Similarly,

Profit for apples is $\frac{10}{100} \times 100 = 10\%$

Profit for mangoes is $\frac{15}{50} \times 100 = 30\%$

Thus the most profitable deals are in grapes and mangoes, each of these giving 30% profit.

Let's look at another problem:

- A man bought coconuts for 650 rupees and sold these for 598 rupees. What is the percent of loss?

  Loss is 52 rupees.

  It is $\frac{52}{650} = \frac{2}{25}$ of the amount spent.

  As a percent, $\frac{52}{650} \times 100 = 8\%$

Maximum Retail Prices

Many things such as water and other drinks, grains and other food stuff, soaps and tooth pastes are all sold in bottles and packets. In India, there is a law which requires the producers of all such packaged goods to print the maximum price which the retailers can charge for them. It is called maximum retail price (MRP). It includes all taxes. Often retailers sell at a price less than this. But if a customer is charged more than this, he can make a complaint to the authorities.

Uncle ! Does MRP mean Maximum Reduced Print?

A different deal

A man buys pencils at 12 for 10 rupees and sells there at 10 for 12 rupees. Does these result in a profit or loss? What percent?

- An almirah bought for 5000 rupees was sold for 5600 rupees. What is the profit percent?

- A TV bought for 12000 rupees was sold for 10200 rupees. What is the loss percent?

- Akhil is a fish vendor. One day he bought 12 kilograms of fish at 140 rupees a kilogram. He spent 120 rupees to take this to the shop. 4 kilograms of fish were spoilt and he sold the rest at 180 rupees a kilogram. Did he gain or lose money in this ? What percent?

- In Omega stores, Ceiling fans sold at 1728 rupees a piece fetches 128 rupees as profit, while pedestral fans sold at 2616 rupees a piece fetches 216 rupees as profit. Which sale is more profitable?

- A retailer buys 150 kilograms of pepper at 400 rupees a kilogram and sells it at a profit of 60 rupees a kilogram.

    - What is the total amount spent in buying?

    - What is the total amount got?

    - What is the total profit?

    - What is the profit percent?

Some other sums

A trader bought an electric iron for 1200 rupees. He wants 12% profit on selling it. At what price should he sell it?

Here, the iron is bought for 1200 rupees.

And the profit wanted is 12% of this.

That is, $1200 \times \dfrac{12}{100} = 144$ rupees.

Now to find the sale price, we need only add the profit to 1200 rupees.

We can also directly compute 112% of 1200 rupees.

$$1200 \times \frac{112}{100} = 1344 \text{ rupees}$$

If a sale results in 10% loss, what percent of the investment is the sale price?

Calculate the sale price of each in the table below:

Investment	Profit/Loss
1500	15% profit
2400	20% loss
8000	8% loss
1650	13% profit

A trader sold a bicycle for 4500 rupees, at 10% loss. What amount did he originally spent on it?

Since the loss is 10%, the sale price is 90% of the original investment. Thus

$$\text{Original investment} \times \frac{90}{100} = 4500$$

From this, we can compute the original investment as

$$4500 \times \frac{10}{9} = 5000 \text{ rupees}.$$

- Compute the investment:

Sales price	Profit/loss
4440	11% profit
8280	8% loss
6160	12% loss
1695	13% profit

- 10 kilogram of tomatoes were bought for 270 rupees. Of this, one kilogram was spoilt. To get 20% profit, at what price a kilogram should the remaining be sold?

- Shine sold two tables at 9900 rupees each. He got 10% profit in one sale and 10% loss in the

Computer commerce

With the widespread use of computers, lots of buying and selling is done using the Internet. It is called *e-commerce*. There are many such online shops based in India also. The goods and their prices are shown on their websites. After selecting what we want, we can also pay from our bank accounts through the Net. The goods will be sent to us through courier services. Some such shops also give the facility of payment after receiving the goods.

Where are you two going at this time?

Oh! Just to the e-market.

other. Did the total sale result in a profit or loss? What percentage?

- A trader makes 20% profit in selling a washing machine at 12000 rupees. How much would he have invested in the machine? During new year sales, its price in reduced by 1200 rupees. Is the sale now at a profit or loss? What is the percent?

Increase and decrease

A shop decreases prices by 50% and then sells at 50% discount. Do we get the goods for free there?

If a trader raises price by 20% and then sells at 20% discount, does he make a profit or suffer a loss? What percent?

What if prices are raised by 25% and then a discount of 20% is given?

Brother! If I increase prices by 50% and sell at 50% less, is it a loss or gain?

Alas!

Discount

Haven't you seen such ads during festive seasons?

Many shops try to increase sales by reducing the prices. Such a decrease in price is called *discount*.

For example in a shop, a shirt normaly sold for Rs. 500 is given 20% discount means the customers can get it for 20% less than 500 rupees; that is

$$500 \times \frac{80}{100} = 400 \text{ rupees}$$

Here, the price shown on the shirt in called the *regular price*. The discount is often given as a percent of the regular price.

- George bought an almirah at 8% discount. The price reduction was 960 rupees. What is the regular price? How much did George actually pay for it?

  Discount is 5% of the regular price

  So, regular price $\times \frac{8}{100} = 960$ rupees

  From this, the regular price can be calculated as

  $960 \times \frac{100}{8} = 12000$ rupees.

  Subtracting the discount amount from this gives the amount George paid.

- The price of a gold sovereign (8 grams) is 22500 rupees. For jewellery, manufacturing charge is 6% of the price of gold. A shop offers 20% discount on this charge. How much money is needed to buy a bangle of one sovereign from here?

  Manufacturing charge is 6% of the price of gold.

  Manufacturing charge $= 22500 \times \frac{6}{100}$

  $= 1350$ rupees

Since 20% discount is given on this, only 80% of 1350 need be paid. So manufacturing charge after discount is

$1350 \times \frac{80}{100}$

Now to find the price of the bangle, we add this to the price of gold.

- A person bought khadi clothes on Gandhi Jayanthi day, when 30% rebate was given. He paid 3500 rupees. What is the regular price of clothes he bought?

  The reduction is 30%. So he paid only 70%.

Different discounts

In India, Khadi and other handloom cloth are sold at a discount of 10% through outlets approved by the government. During some special occasions, the discount may be as high as 30%. The money for this is paid by the government. It is called *rebate*.

In countries like USA, rebate means something different. Some manufacturers there pay back a certain percent of the price, if the end-user fills up a coupon or form attached to the bill and send it to them. It is this amount they call rebate.

That is, regular price $\times \dfrac{70}{100} = 3500$

From this can't you find the regular price?

- Look at two ads in a TV shop:

20 inch Rs. 11900/- 20% discount	21 inch Rs. 12900/- 20% discount

  - If one has only 10000 rupees with him, which of these TV's can he buy?

  - What is the difference in prices after discount?

- In a furniture shop, a cot and an almirah sold separately for Rs. 15000 and Rs. 25000, are sold together for Rs. 36000. What is the discount per-cent in this deal?

- Susan and Gayatri bought copies of an English-Malayalam dictionary each for 490 rupees. To get 20% discount, they decided to pay the bill together. When the salesman told them they would get 30% discount on purchase above 1000 rupees, each bought a picture book for 60 rupees and paid the bill together.

Book Fair

Upto Rs. 500
10% off.

Rs. 500 - 1000
20% off.

Above 1000
30% off.

Discount percent

A soap company makes an offer that when 4 soaps are bought together one is given free. What is the percent of discount in this deal?

Here we get 5 soaps for the price of 4 which means the discount is the price of one soap in the price of five soaps.

Now can you work out the percent?

- How much did they pay together? What is the share of each?

- If both had bought only the dictionaries and paid the bill together, how much would they have paid? What would be the share of each?

- If each had bought these two books separately, how much would each have spent?

- The bill below shows cloths bought from a khadi outlet. What is the total amount to be paid?

Khadi Cloth

Cotton ..... 30% rebate

Polyester .... 20% rebate

Silk .... 20% rebate

Khadi Vasthralayam

Bill No: 777   Date: ....

No.	Item	Number	Price	Rs.
1	Cotton dhoti	1	350	
2	Cotton shirt	1	550	
3.	Polyester shirt	1	450	
4	Silk saree	1	1500	

- A trader buys an electric fan for Rs. 2500. He raises the price by 40% and then offers 15% discount. At what price does he sells it?

- A gas stove is bought for Rs. 3600. What should be the price marked on it, if 20% profit is to be got, after 10% discount.

- A shop offers an electric iron free with a fridge. They sell the fridge for Rs. 9000 and the iron for Rs. 1000. If 20% profit is to be got on this deal, what should be the selling price of the fridge?

The iron is still cold!

No wonder! Didn't you get it free with the fridge?

History of interest

It was about 5000 years back that men collectively started agriculture in an extensive way. During those days, people often loaned grain and cattle to one another. Since seeds and cattle multiply naturally, much more than what was borrowed could be returned.

At that time, grain and cattle were used as money. Trouble started when metal coins began to be used as money. Metal do not grow out of metal as in the case of grain.

When crops are bad, prices rise and the farmers have to borrow money; when crops are good, prices fall and the farmers don't get money to pay back.

Interest

Have you seen ads like these in front of banks?

Co operative Bank Term Deposits
Interest up to 11%
for 45 days 6%

We approach banks for depositing money and getting loans.

Amal deposited 15000 rupees in a bank. After one year, he got 16500 rupees back.

Attractive Interest
Gold Loan 9%
Agricultural Loan 4%

How much more did he get?

This extra money is called *interest*.

What if we take out a loan from a bank?

Interest rate

Thomas took out 50000 rupees as loan from a bank. After one year, he had to pay back 52000 rupees.

How much is the interest?

What percent of the loan amount is this?

$$\frac{2000}{50000} \times 100 = 4\%$$

For one year, he paid back 4% more than the loan.

It is called the *rate* of interest

If 1500 rupees is the interest got in a year for a deposit of 15000 rupees, then the rate of interest is,

$$\frac{1500}{15000} \times 100 = 10\%$$

NANDINI BANK
One and a half rupees interest a month for 100 rupees.

K. S. BANK
3 rupees interest every 4 months for 50 rupees.

Which bank gives more interest?

Let's look at Nandini Bank.

One month's interest for 100 rupees is $1\frac{1}{2}$ rupees.

One year's interest for 100 rupees $12 \times 1\frac{1}{2} = 18$ rupees.

So, the rate of interest is 18%

What about K.S. Bank?

4 month's interest for 50 rupees is 3 rupees.

4 month's interest for 100 rupees is $3 \times 2 = 6$ rupees.

One year's interest for 100 rupees is $6 \times 3 = 18$ rupees.

So in this bank also, the rate of interest is 18%.

Compute the rate of interest in each case:

Amount	Time	Interest
Rs. 500	1 year	Rs. 30
Rs. 1000	4 months	Rs. 40
Rs. 200	2 months	Rs. 2
Rs. 2	1 month	Rs. 3
Rs. 5000	2 years	Rs. 1200

As time changes

The Co-operative Bank gives 9% interest for fixed deposits. Ravi deposited 30000 rupees. How much would he get after a year?

The interest for one year is 9% of the deposit.

That is,

$$30000 \times \frac{9}{100} = 2700 \text{ rupees}$$

So he gets 32700 rupees back.

Suppose he withdraws the amount only after two years.

Loans written off

Even in olden days, there was a practice of writing off agricultural loans. When a monetary system based on coins was introduced in ancient Egypt and Babylonia, the kings used to fix the exchange rates between coins and food grains to ensure that the fluctuation in crops do not affect the prices.

Loans to farmers were written off during periods of drought.

I got a good price for my crops.

And I got nothing but praise. I said farmers and the king heard famous.

He gets interest for two years; that is,

$$2 \times 2700 = 5400 \text{ rupees}$$

We can compute two years interest directly as,

$$30000 \times \frac{9}{100} \times 2 = 5400 \text{ rupees}$$

What about interest for three years?

Similarly, What is the interest for 2000 rupees at 8% for 4 years?

- Suma deposited 25000 rupees in a bank, which gives 11% interest. How much would she get back after 3 years?

We can directly compute the interest for 3 years.

$$25000 \times \frac{11}{100} \times 3 = 8250 \text{ rupees}$$

To find the amount she would get back, we need only add this interest to the deposit.

How much is it?

- Vijayan took out a loan of 50000 rupees at 12% interest. After two years, he paid back 25000 rupees. How much should he pay back after one more year?

Here some money is paid back after two years.

So let's first calculate the interest for two years.

$$50000 \times \frac{12}{100} \times 2 = 12000 \text{ rupees}$$

The actual amount to be paid after two years

$$50000 + 12000 = 62000 \text{ rupees}$$

Of this 25000 rupees was given. Remaining amount is

$$62000 - 25000 = 37000 \text{ rupees}$$

So the amount to be paid back after one more year is 37000 rupees and the interest on it. Can't you compute it?

- Babu deposited 25000 rupees in a bank, which gives 15% interest. How much would he get back after two years?

- Dileep took out a loan of 36000 rupees from a bank, which charges 10% interest. He decided to pay back the total amount including interest for 2 years, in 24 monthly instalments. How much should he pay every month?

- Johny deposited 60000 rupees in a bank, which gives 1 paise interest on one rupee every month. How much would he get back after two years?

- Sujith and Anish took out agricultural loans, 50000 rupees each from a bank at 4% interest. Sujith settled the debt after one year and took out another loan of 50000 rupees, which he settled the next year. Anish could not settle after one year and he was asked to pay 7% interest for the second year. How much did each give as interest?

- Rahul and Rajani opened accounts in a bank on the same day, each depositing 8000 rupees. After one year, Rahul withdrew the entire amount including interest and re-deposited it. After one more year, both closed their accounts. How much did each get? Why are the amounts different?

Changing times

In the olden days, there were protests against the very idea of interest. In some books of the fifth century AD in India, religious prohibitions against interest can be seen.

The famous Greek Philosopher Aristotle, of third century BC, also condemns the practice of interest. He calls it " the most hated form of money making"

After many years, around the second century AD, this was reduced to protest against excessive interest in most places.

Aristotle's comments on interest are interesting

Philosophers don't have any interest in money making
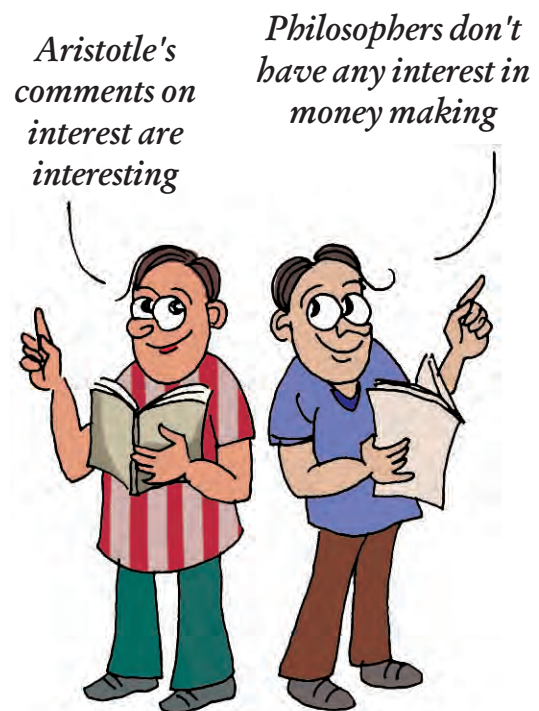
Looking   back

Achievements	On my own	With teacher's help	Must improve
• Using the concept of percent in solving practical problems related to investments, sale price, profit and loss.			
• Solving practical problems involving discount and rebate.			
• Critically examining commercial tactics, such as regular price and discount.			
• Calculating interests of specified amount for specified periods.			
• Finding out and interpreting the relations between interest, rate, original amount and period.			
• Converting interest specified in various ways to annual rates to solve problems.			
• Finding out suitable methods of solution and explaining them.			

11

Numbers and Algebra
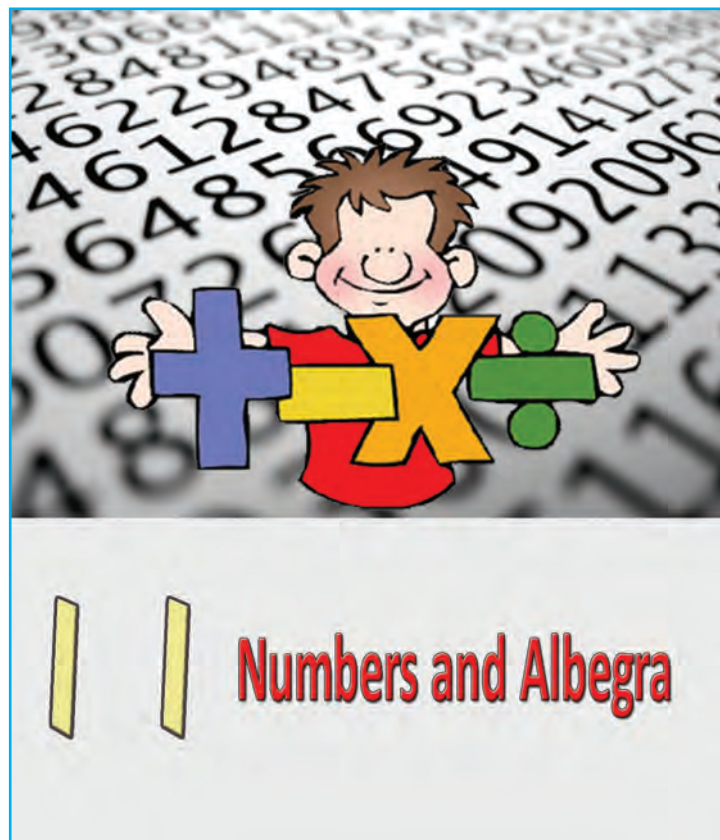
Odd and Even

Look at these sums:

$$1 + 2 = 3$$
$$2 + 3 = 5$$
$$3 + 4 = 7$$

All the sums are odd numbers, right?

Why is it that the sum of any two consecutive natural numbers is an odd number?

Let n be some natural number. Then the next natural number must be written $n + 1$.

What is their sum?

$$n + (n + 1) = 2n + 1$$

The number $2n + 1$ divided by 2 gives quotient n and remainder 1.

So, whatever be the natural number n, the number $2n + 1$ is an odd.

Thus we can see that the sum of any two consecutive natural numbers is an odd number.

Now look at these sums:

$$1 + 3 = 4$$
$$2 + 4 = 6$$
$$3 + 5 = 8$$

The sum of alternate natural numbers is an even number.

Can you explain this using algebra?

Number pyramid

Look at this picture:
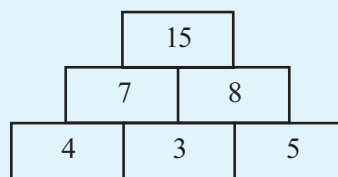
Do you see any relation beween the numbers?

Pair of adjacent numbers in the bottom row are added to get the two numbers in the middle row; and these two numbers are added to get the top most number.

Let's make such a pyramid starting with the numbers 1, 2, 3.
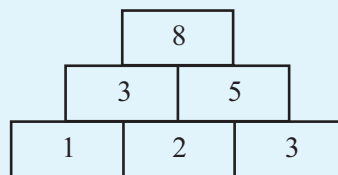
What if we start with 2, 3, 4?
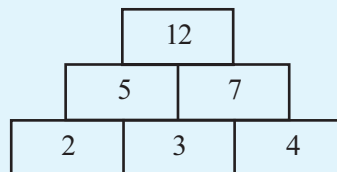
Try with other sets of three consecutive natural numbers.

Do the final numbers have anything in common?

What three consecutive numbers should we start with, to get 100 finally?

Algebra aid

Whatever consecutive natural numbers we start with, our number pyramid ends in a multiple of 4, right?

Why does this happen?

Let's take the first of these numbers as x.

Then the bottom row has $x, x + 1, x + 2$
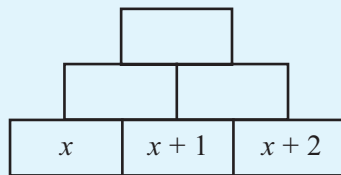
What then are the numbers in the middle row?

$$x + (x + 1) \quad = \quad 2x + 1$$
$$(x + 1) + (x + 2) \quad = \quad 2x + 3$$
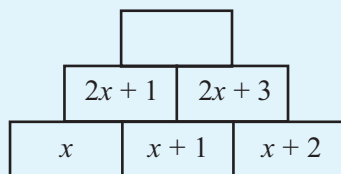
So, what is the topmost number?

$$(2x + 1) + (2x + 3) \quad = \quad 4x + 4$$
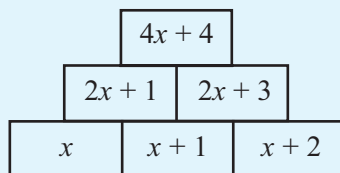
Let's write $4x + 4$ in a slightly different way:

$$4x + 4 \quad = \quad 4(x + 1)$$

Thus, whatever three consecutive number we start with, we end up with four times the middle number (Did you note this before?).

So to finish with 100, we should start with 24, 25, 26.

Now suppose we start with alternative numbers instead of consecutive numbers?

And three numbers skipping two each time?

Try these!

Number properties

We need algebra to show that certain properties are true for all numbers. For example, to prove that the sum of any two consecutive natural numbers is an odd number, we must first note that if a natural number is denoted by n, then the next natural number is $n + 1$ and that their sum is $2n + 1$; also we must show that whatever be the natural number n, the number $2n + 1$ is an odd number.

In many other sciences, if a certain fact is verified in a number of different contexts, then it is accepted as a general principle. This is not enough in mathematics. We must also show why it must be true in all cases. For numbers such reasoning is explained using algebra.

There are several instances where some facts are true for several numbers and then fail. For example if we divide 2^2 by 2, 2^3 by 3, 2^4 by 4 and so on, the remainder is not 3. Generally speaking, if we take any number less than 4700063497 as n, the number 2^n divided by n will not leave remainder 3, but if we take 4700063497 as n we do get 3 as remainder.

Here a fact is true in more than four hundred seventy crore instances, but is not true in general.

Suppose you add twice a number and thrice the same number. Then what you get, my dear friends, is five times the number!

$2x + 3x = 5x$

Translation

Three numbers

We have seen that the sum of two consecutive counting numbers is an odd number. What about the sum of three consecutive natural numbers?

$$1 + 2 + 3 = 6$$
$$2 + 3 + 4 = 9$$
$$3 + 4 + 5 = 12$$

All these are multiples of 3. Is this true, whatever number we start with?

If we write the first number as n, then the next numbers are $n + 1$ and $n + 2$.

Their sum is

$$n + (n + 1) + (n + 2) = 3n + 3$$
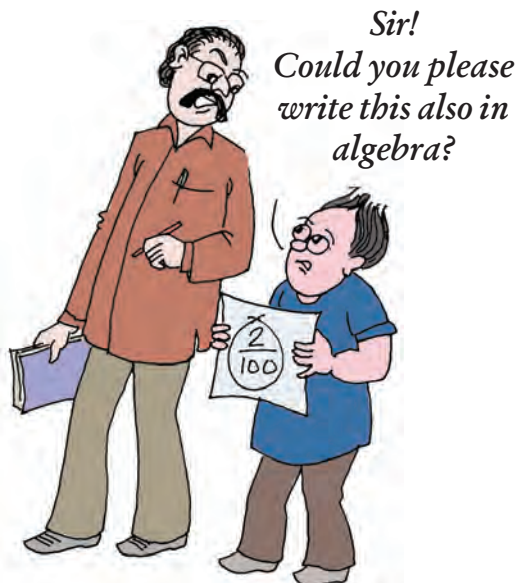
We can write

$$3n + 3 = 3(n + 1)$$

So we can see that sum is a multiple of 3.

Here we can note another thing.

The sum is three times the middle number.

Thus we get a more precise principle

The sum of any three consecutive counting numbers is three times the middle number

Is the sum of four consecutive counting numbers a multiple of four?

*Sir!
Could you please write this also in algebra?*

Can you complete these pyramids?

$x - 1$	x	$x + 1$

| $x - y$ | x | $x + y$ |
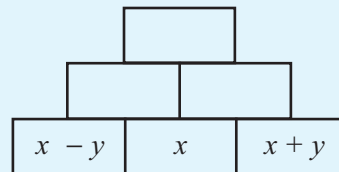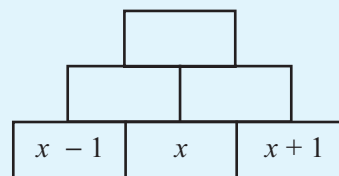
Can you write the speciality of the last pyramid in ordinary language?

Another pyramid

Here is a bigger pyramid:

	10	
10	1		10
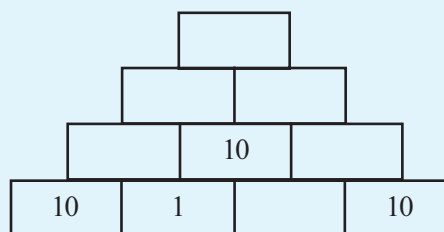
Can you write the other numbers?

What number should be written in the remaining space of the bottom row?

1 added to that number should be 10.

Now can't you write the remaining numbers?

Didn't you get 50 at the top?

Next fill up this pyramid:

	10	
10	2		10
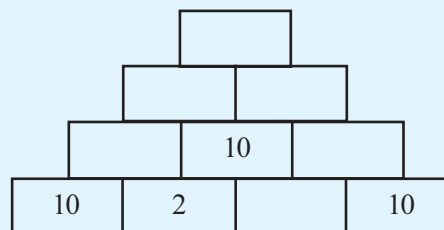
The topmost number is still 50, right?

Try with other numbers in the place of 2. Why do we always end up with 50?

Let's use algebra and take the second number in the bottom row as x.

Then what should be the next number?
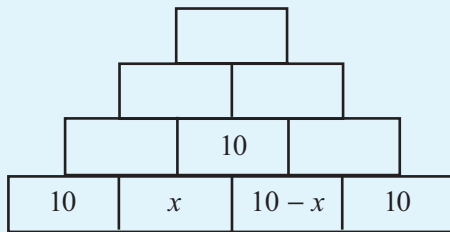
Now can't you fill up the next two row up?

Didn't you get $20 + x$, $30 - x$ in the third row?

So the topmost number is

$$(20 + x) + (30 - x) = 50$$

Now let's start with 9's in the place of 10's.

Take this,
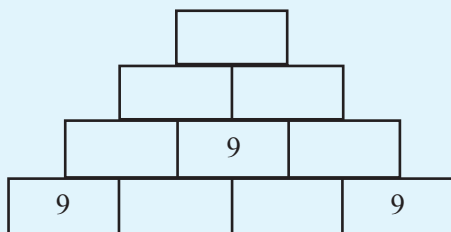
Take any number less than 9 as the second number in the bottom row and fill in the pyramid (why less than 9?)

Compare with your friends.

All get 45, right?

Now if we start with 11 in the place of 9, can you say what number we would end up with, whatever number (less than 11) we next take?

Why do we always get 5 times the beginning number?
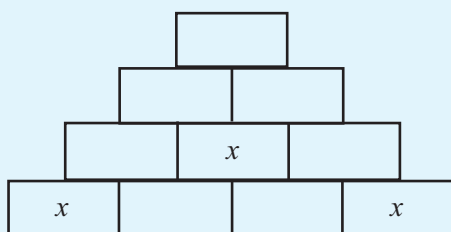
Another way

There is a another way to see that the sum of any three consecutive natural numbers is three times the middle number.

If the middle number is taken as n, then the first number is $n - 1$, and the last number is $n + 1$.

And their sum is

$$(n–1) + n + (n + 1) = 3n$$

Here the convenience is that the sum of $n - 1$ and $n + 1$ can be easily seen to be $2n$.

Now if we take any five consecutive a natural numbers and write the middle number as n, then the five numbers can be written as

$$n - 2, \quad n - 1, \quad n, \quad n + 1, \quad n + 2$$

To find the sum of all these we first note that

$$(n - 2) + (n + 2) = 2n$$
$$(n - 1) + (n + 1) = 2n$$

Then the sum of all five can be easily found as

$(n - 2) + (n - 1) + n + (n + 1) + (n + 2)$

$= (n - 2) + (n + 2) + (n - 1) + (n + 1) + n$

$= 2n + 2n + n$

$= 5n$

And this shows that the sum is five times the middle number.

What can you say about the sum of any seven consecutive natural numbers?

General forms

The even numbers 2, 4, 6, 8, .. are all multiples of 2; that is, they are got by multiplying the natural numbers 1, 2, 3, 4, … by n. Thus for any natural number n, the number $2n$ is even; on other hand, any even number is of the form $2n$.

If we subtract 1 from the even numbers 2, 4, 6, 8, …, we get the odd numbers. Generally speaking, the odd numbers are got by multiplying natural numbers by 2 and subtracting 1. Using algebra, the natural number n multiplied by 2 is written $2n$ and subtracting 1 gives $2n - 1$. Thus for any natural number n, the number $2n - 1$ is odd. On the other hand, any odd number can be written in the form $2n - 1$.

For any natural number n, the number $2n + 1$ is also odd. But if we take 1, 2, 3, … as n, we don't get the number 1 from $2n + 1$. To get all odd numbers from it, we should take 0, 1, 2, … as n.

Let's write the beginning number as x.
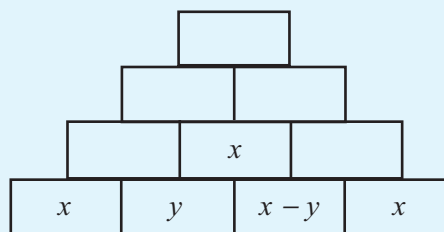
Let's now take the next number as y:

What are the numbers in the next row up?

And in the next row after that ?

Didn't you get $2x + y$, $3x - y$? So the final number is
$$(2x + y) + (3x - y) = 5x$$
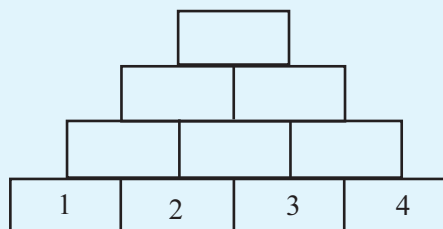
- Fill in the pyramid below:

  Try with other set of four consecutive natural numbers. What is the relation between the starting number and the final number? What is the relation between the middle two numbers in the bottom row and the top most number? Explain these relations using algebra.

- How about starting with five numbers?

  Explain the relation between the middle number in the
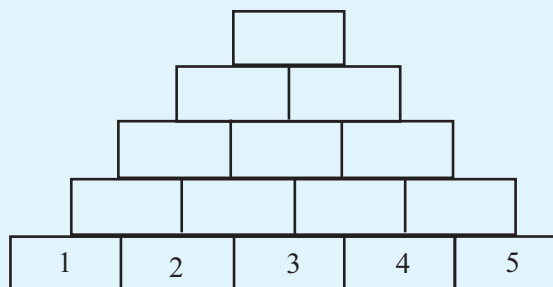
  bottom row and the topmost number using algebra.

- In the pyramids above, start with numbers skipping one at a time, two at a time and so on. Explain the various

common relations you find using algebra.

Playing with 11

Look at these numbers:

12, 23, 34, …

Starting with 12, we add 11, then add 11 again and so on.

If we continue like this, would we get 100?

Let's write and see:

12, 23, 34, 45, 56, 67, 78, 89, 100.

If we continue further, would we get 1000?

It is not easy to write them all.

Let's have another look at the numbers.

12 is 1 more than 11.

23 is 1 more than 22.

34 is 1 more than 33.

So all our numbers are got by adding 1 to multiples of 11.

In other words, they all leave remainder 1 on dividing by 11.

Now we can easily check whether 1000 is in this set.

The remainder on dividing 1000 by 11 is not 1, so 1000 is not among these numbers.

Next check whether 10000 is among them.

What about 100000?

We first noted that all these numbers were got by starting with 12 and adding 11 repeatedly.

As seen now, this entire number pattern could be written as a single operation.

Multiply all natural numbers by 11 and add 1 to each.

How do we state this using algebra?

In $11n + 1$, take n as 1, 2, 3, … in order .

(In algebra, natural numbers are usually denoted by letters

Some other sums

If we add two even numbers, we get an even number; but if we add two odd numbers, still we get an even number. Why is this so?

Let's use algebra. We can write any two even numbers as $2m, 2n$. Their sum is

$$2m + 2n = 2\,(m + n)$$

From this we see that the sum is also a multiple of 2; that is, an even number.

What about two odd numbers?

If we write them as $2m - 1, 2n - 1$ then their sum is

$$(2m - 1) + (2n - 1) = 2m + 2n - 2$$
$$= 2(m + n - 1)$$

This again is a multiple of 2, or in other words an even number.

What if we add three even number instead of two? How about four even numbers?

What can you say about the sum of three odd numbers? What about four odd numbers?

The sum of two odd numbers is even.

It sure is odd!

Numbers and letters

When we use algebra to state general properties of numbers, we should also specify the type of numbers indicated by the letter.

For example, when we say that numbers of the form $2n - 1$ are odd, we should also say that the n here indicates a natural number.

If in $2n - 1$ we take $1\frac{1}{2}$ as n, then we get

$$2n - 1 = (2 \times 1\frac{1}{2}) - 1 = 2$$

Which is an even number.

Look at this! When Sir agreed to write letters instead of numbers, I never thought it'd be the fifteenth letter!

Oh!
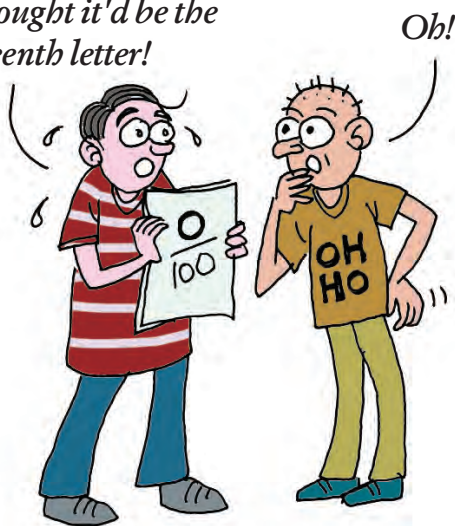
like n, m, p, k; not a rule, just a convention).

Now instead of 12, let's start with 21 and add 11 again and again:

$$21, 32, 43, \ ...$$

Can we write this pattern also in algebra?

We can write them as

$$11 + 10, \ \ 22 + 10, \ \ 33 + 10, ...$$

That is,

In $11n + 10$, take n as 1, 2, 3, … in order.

If we continue this, which of the numbers 100, 1000, 10000 and 100000 do we get?

(What remainder do they leave on division by 11?)

Now let's look at these patterns together:

12	23	34	45	...
21	32	43	54	...

What if we add the top and bottom numbers in order?

$$33 \quad 55 \quad 77 \quad 99 ...$$

Why do we get only multiples of 11? Let's check using algebra.

Any number in the first pattern can be written as $11n + 1$; the number in the same position in the second pattern is $11n + 10$ (same n as the first)

What is their sum?

$$\begin{aligned}(11n + 1) + (11n + 10) &= 22n + 11 \\ &= 11(2n + 1)\end{aligned}$$

Now don't you see why we get only multiples of 11?

Look at the sums again. Why do we get only odd multiple of 11?

Look at the algebraic form of the sum. When we take n as the natural numbers 1, 2, 3, … in it, what sort of numbers do we get as $2n + 1$?

Here we see different general forms such as $11n + 1$, $11n + 10$, $2n + 1$. Each indicates an operation. For example;

$11n + 1$ means multiply the number written as n by 11 and add 1.

Such general form indicating arithmetical operations are called *algebraic expressions*.

For example, all the numbers 12, 23, 34, … got by adding 11 repeatedly to 1 can be represented by the single algebraic expression $11n + 1$.

- Find the algebraic expression for the numbers got by adding 10 repeatedly to 1

- Find the algebraic expression for the numbers got by adding 10 repeatedly to 9

- Add the numbers in the same position of the first two patterns. Why do we get only multiple of 10? Do we get all multiples of 10 like this?

Two-digit numbers

We can write the multiples 10, 20, 30, .. of 10 in the common form $10n$. In it, we can take any natural number as n.

Suppose we need only the two-digit numbers among them. Then we need only take the numbers from 1 to 9. We write

$$10n \quad (n = 1, 2, 3, 4, 5, 6, 7, 8, 9)$$

Or in a shorter from

$$10n \quad (n = 1, 2, 3, ..., 9)$$

Similarly the numbers 11, 21, 31, … got by adding 1 to multiples of 10 can be shortened to the general form $10n + 1$. In this we can take any natural number as n.

If we want only the two-digit number among these, we write

$$10n + 1 \quad (n = 1, 2, 3, ..., 9)$$

How do we write the general form of the numbers 12, 22,

Algebraic Forms

It is easy to multiply any number by 10: just attach a zero at the end:

$$18 \times 10 = 180$$
$$250 \times 10 = 2500$$

But in algebra, we only write

$$10 \times n = 10n$$

and not $n0$.

Similarly all numbers got by adding 1 to multiple of 10 end in 1. But we write their general algebraic form as $10n + 1$ and not as $n1$.

Instead of saying, "numbers got by adding 1 to multiple of 10", we can also say "numbers leaving remainder 1 on division by 10." Such numbers leave remainder 1 on division by 5 also. To see this, note that

$$10n + 1 = (5 \times 2n) + 1$$

If we write these numbers as $n1$, we cannot do such analysis.

Prove yourself!

How do I demonstrate my mathematical ability?

Two-digit numbers

The number written as 35 is a short form for the number got by multiplying 3 by 10 and adding 5.

In general, the written form of a two-digit number actually denotes the number got by multiplying the first digit by 10 and adding second.

But in algebra, the result of multiplying the single digit number m by 10 and adding the single digit number n is written as $10m + n$ and not as mn.

But multiplying a single digit number n by 10 and adding the same number n can be written

$$10n + n = 11n$$

And from this we also see that all numbers got like this are multiples of 11.

32, … got by adding 2 to multiples of 10? And the two-digit numbers among them?

Let's take a look at all the two-digit numbers got so far:

$10n$	:	10	20	30	40	50	60	70	80	90
$10n + 1$	:	11	21	31	41	51	61	71	81	91
$10n + 2$	:	12	22	32	42	52	62	72	82	92

If we want all two digit numbers, what all algebraic expressions should we write?

What is the general form of the algebraic expressions, $10n$, $10n + 1$, $10n + 2$,..., $10n + 9$?

We add various numbers to be the expression $10n$ (the first number added is 0)

We can denote all these numbers added by a single letter, So we can write them as $10n + m$; here we take the numbers from 0 to 9 as m.

In short, all two-digit numbers are of the form

$$10n + m \ (n = 1, 2, 3, ..., 9; m = 0, 1, 2, ..., 9)$$

For example if we take $n = 5, m = 3$ we get

$$10n + m = (10 \times 5) + 3 = 53$$

What if we take $n = 3, m = 5$?

Thus in the general expression $10n + m$ for two digit numbers n is the first (place ten) digit and m is the second (place one) digit.

Now take any two-digit number say 25. Reversing the digits we get 52. Adding the two gives 77.

What about the sum of 36 and 63?

Do the digits repeat always?

How about 28 and 82?

Do all these sums have anything in common?

Why are they all multiples of 11?

To understand such common properties, algebra can be used.

Any two-digit number can be written in the form $10m + n$.

Reversing the digits means interchanging their position and this gives $10m + n$. Their sum is

$$(10m + n) + (10n + m) = (10m + m) + (10n + n)$$
$$= 11m + 11n$$
$$= 11(m + n)$$

Now reverse any two digit number and instead of adding, subtract smaller from the larger. Do it with other numbers.

Are all the differences, multiples of the same number?

Why does this happen?

$$(10m + n) - (10n + m) = 10m + n - 10n - m$$
$$= 9m - 9n$$
$$= 9(m - n)$$

Now try these on your own:

- Take any two-digit number and add the digits. Subtract this sum from the number. Do this for several numbers and find a common property of all such differences.

- If we subtract from any two digit number the sum of its digits, we get a multiple of 9. Explain why this is so using algebra.

- Write the general algebraic from of all three-digit numbers.

- If the first, second, third digits (in the hundreds, tens and ones place) of a three-digit number are taken as m, n, p, how do we write the number? How do we write the number got on reversing the digits?

- If we reverse any three-digit number and subtract the smaller from the larger we get a multiple of 99. Explain this using algebra.

- From any three - digit number if we subtract the sum of its digits we get a multiple of 9. Explain this using algebra.

Three numbers again

Take any three consecutive natural numbers and add the first and last.

Is there any relation between this sum and the middle number?

Use algebra to prove that in any three consecutive natural numbers, the sum of the first and last is twice the middle one.

Is this true if we take three consecutive even numbers such as 2, 4, 6? How about consecutive odd numbers?

Next, is this true for any three consecutive multiples of 3, such as 3, 6, 9?

What about 1 added to multiples of 4 or any other number?

Prove all such conclusions using algebra.

Not a mathematical mini story, Sir! My new algebraic novel!

Looking back

Achievements	On my own	With teacher's help	Must improve
• Explaining number relations using algebra.			
• Finding out algebraic expressions denoting arithmetical operations and explaining them.			
• Describing general forms of numbers using algebra.			
• Proving number properties using algebra.			

12

Squares and Right triangles

Another Way

There is another way to cut and rejoin two squares of the same size to one large square:
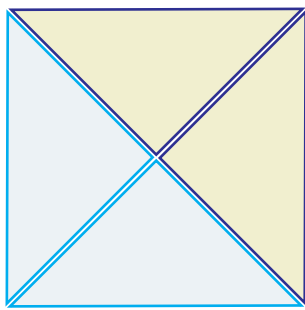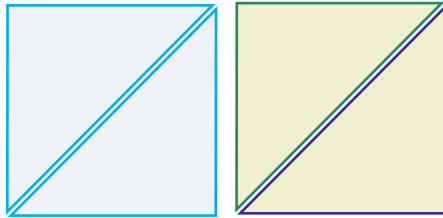
Is there any relation between the side of the large square and some length related to the smaller squares?

Doubling a square

Cut out two squares of the same size from thick paper:
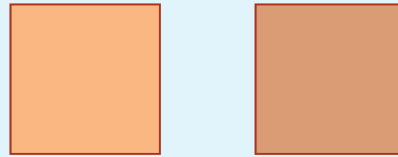
These are to be cut and the pieces arranged to form one large square.

It can be done like this.

First cut along the diagonals' of one square to make four triangles:
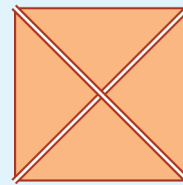
Flip all these triangles outwards:
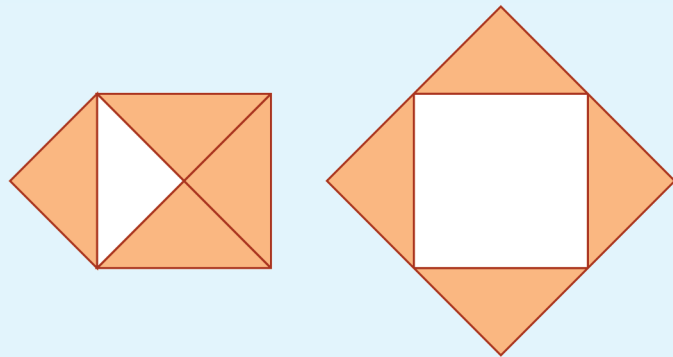
Now place the uncut square in the vacant spot in the middle:
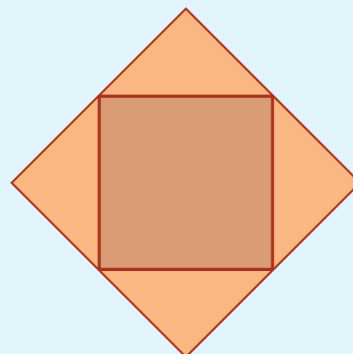
Why does the second square fit inside exactly?

Suppose the sides of the first two squares are all 2 centimetres long. What is the area of each of these?

So, what is the area of the final large square?

Now let's do this with two squares of area 9 square centimetres each. To make a square of area 9 square centimetres, how much should be the length of the sides?

Cut out two such squares and cut one of these as above to make a single large square. What is its area?

How do you make a square of area 50 square centimetres?

What about a square of area 32 square centimetres?

Making larger

We have seen that by cutting a square along the diagonals and placing the pieces around another square of the same size, we can make a square of double the size.

Now let's cut in a different way.

Cut out a square of side 5 centimetres.

Instead of joining opposite corners to draw diagonals, mark points 1 centimetre away from corners and join:
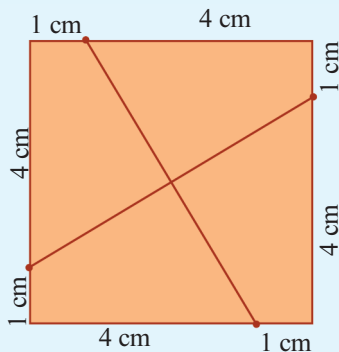
Cut along these lines and flip the four pieces outwards:

How to draw?

When we cut out two squares of the same size and rejoin to make one large square, the length of a side of this large square is length of the diagonal of the small square.

So, given a square, it is easy to draw a square of double the area.
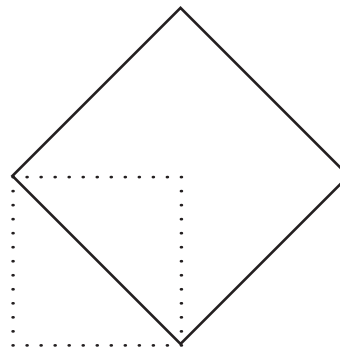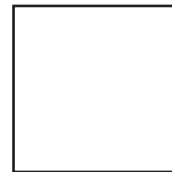
Using this idea, can you draw a square of area 50 square centimetres?

How about 32 square centimetres?

Parallel way

There is another way to draw a square of double the area of a given square.

First draw a diagonal:
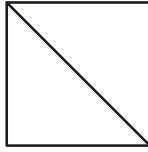
Next draw lines parallel to this diagonal through the other two corners:
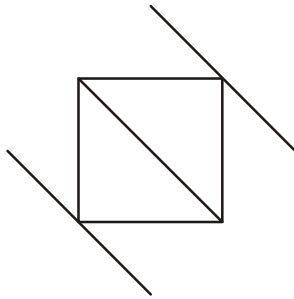
Draw the other diagonal also and draw lines parallel to it as before:
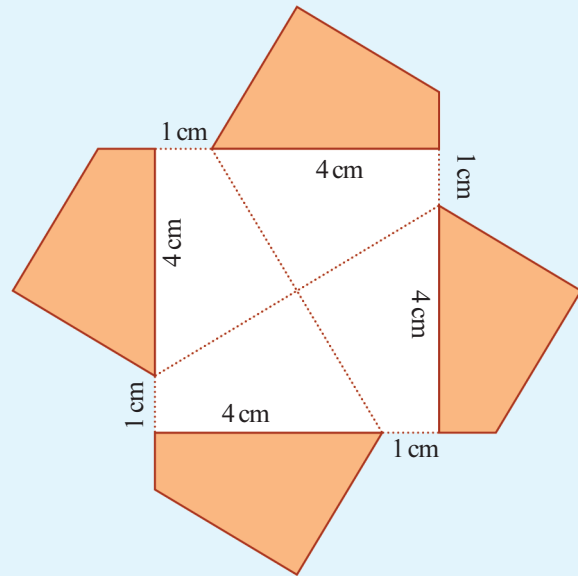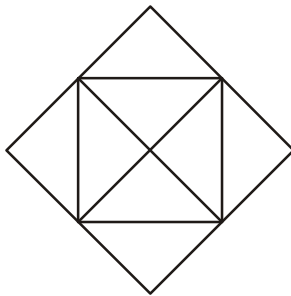
Slide the left piece a little downwards, the right piece towards the left, and the top piece left and down to make a large outer square with a small square hole inside as below:
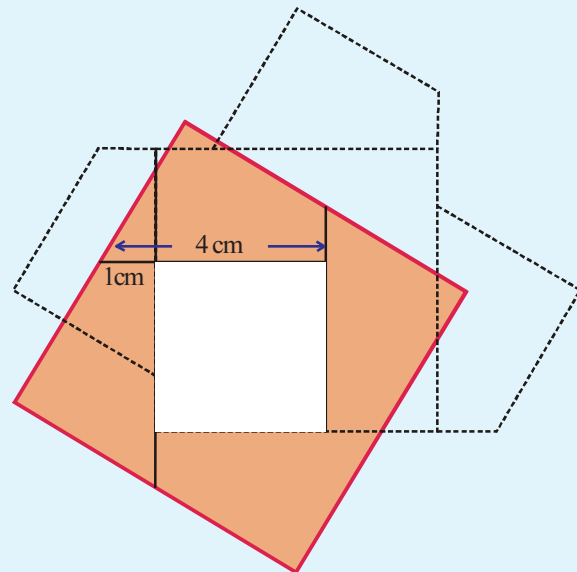
What is the length of the sides of the inner square?

So, if we cut out a square of side 3 centimetres, we can fit it exactly into the hole.
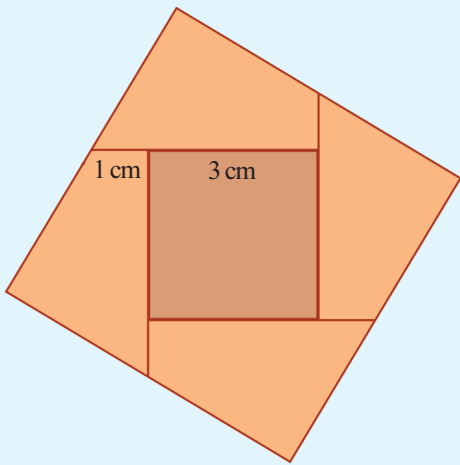
It was the first square that was cut into four pieces. So the sum of the areas of these four pieces is equal to the area of the first square; that is $5^2 = 25$ square centimetres.

What about the area of the second square placed inside?

So, the area of the final large square is

$$5^2 + 3^2 = 34 \text{ sq.cm.}$$

Now cut out another square of side 5 centimetres again and join points marked 2 centimetres off the corners:
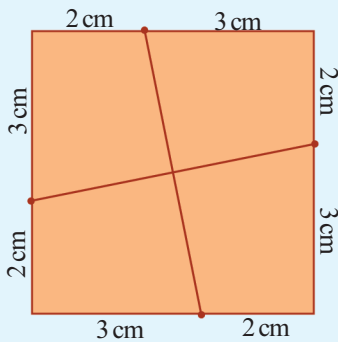
If we cut along these lines and flip the four pieces outwards it would be like this:

How to halve

To get a rectangle of half the area of a square, we need only cut horizontally or vertically along the middle:
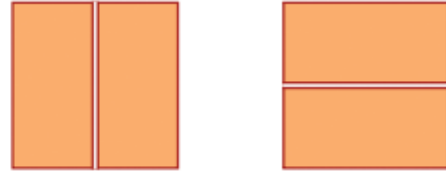
Suppose we want a square itself of half the area?

Fold all corners of the square into the centre:
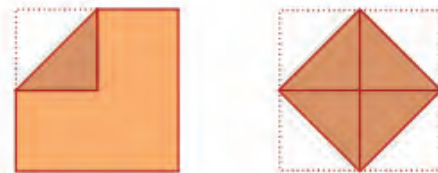
If we now unfold and cut along the creases, we get a square of half the area:
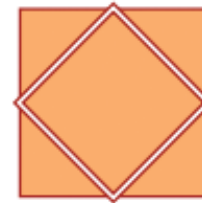
This 4 kilometre stretch to the school is full of potholes

I know that distance. I've been to school.

We don't know this distance
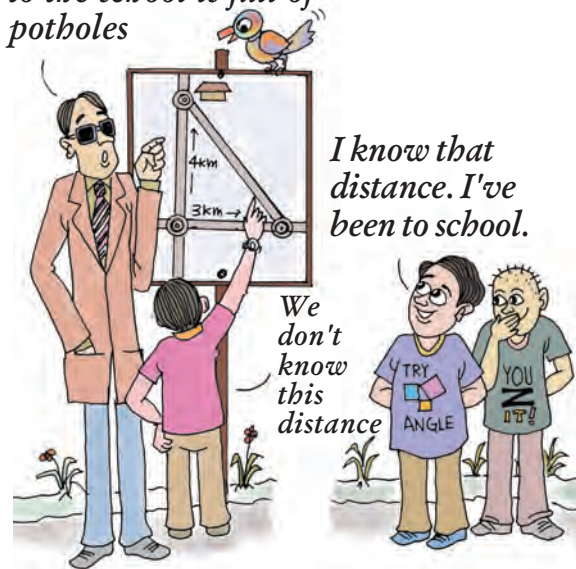
Three squares

We can also cut and join three squares of the same size to make one large square

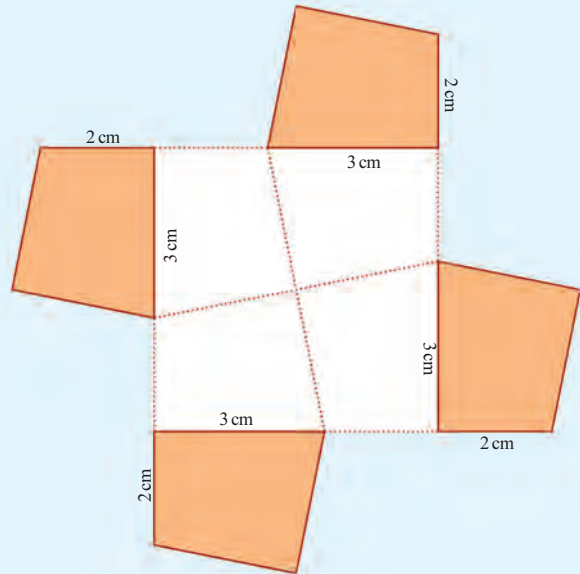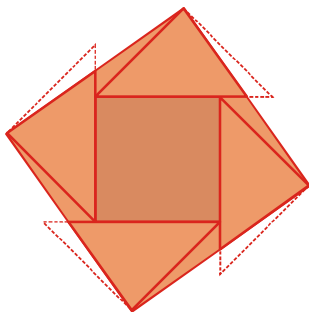First cut two squares along the diagonal:
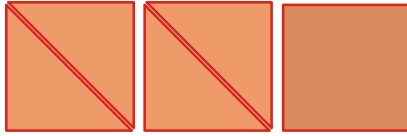
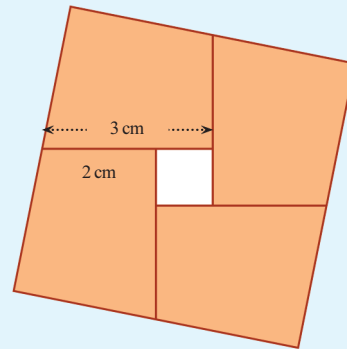Arrange the triangular pieces around the un-cut square like this:
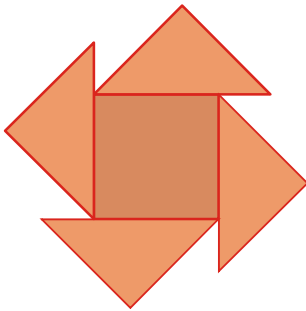
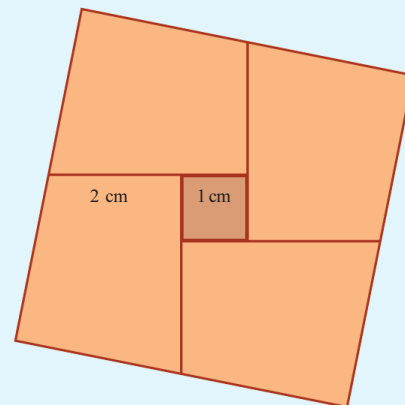Draw a square by joining the outer vertices of the triangles as shown below:
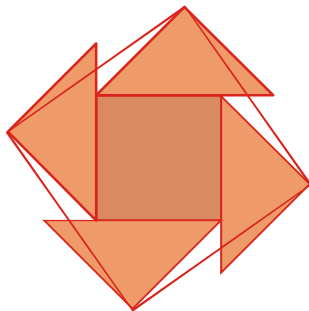
Cut out the four slender triangles putting out from the square and plug the gaps within:

Let's slide the pieces as before to make a square with a hole:

What should be the length of the sides of the square needed to plug this hole?

What is the area of the filled up large square?

$$5^2 + 1^2 = 26 \text{ sq.cm}$$

Again cut out a square of 5 centimetres and join points 1.5 centimetres off the corners. Cut out along these lines and rearrange the pieces as above.

What should be the length of the sides of the square to be placed inside?

What is the area of the filled up large square?

Two squares

We cut a square of side 5 centimetres in various ways and joined with another square to make larger squares of different sizes:
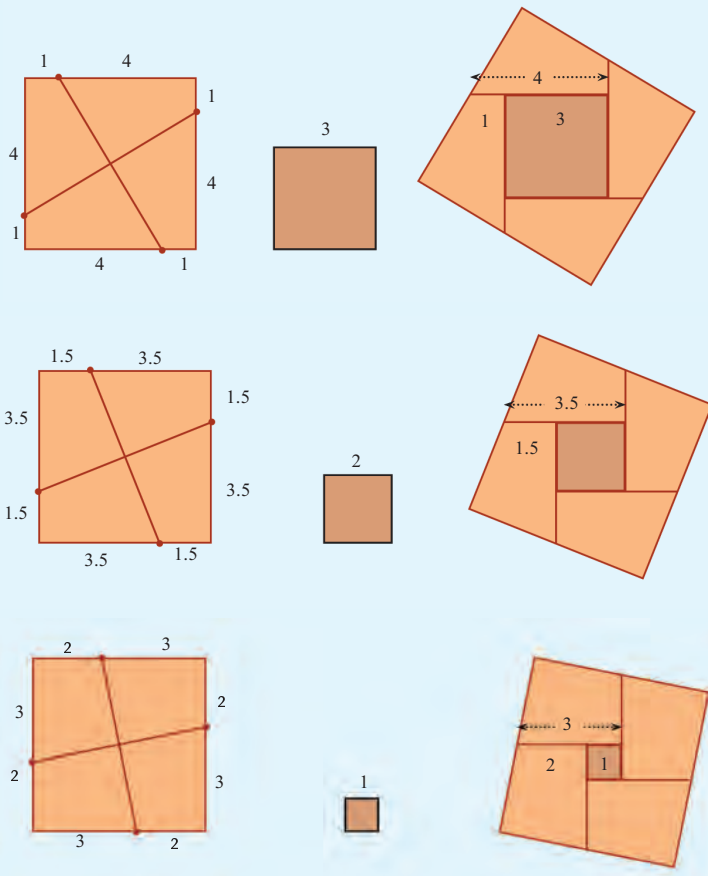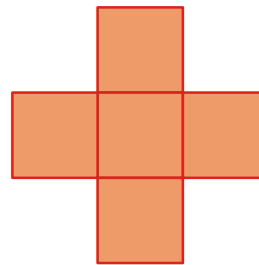
To cut the first square into pieces, we mark points at the same distance from the corners. What is the relation between this distance and the length of the sides of the second square cut out to plug the hole?
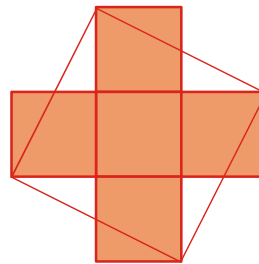
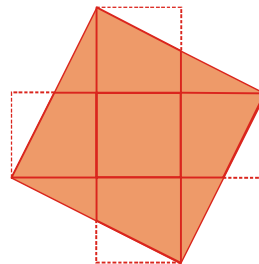Let's look again at the way we made the large square.

Place five squares of the same size like this:

Draw a square by joining some of the corners as below:

Now cut out the triangles outside the square and fill the gaps within:
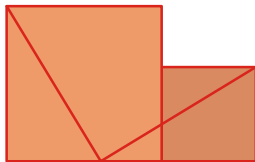
Have you seen such a picure in some other lesson?

Another cut

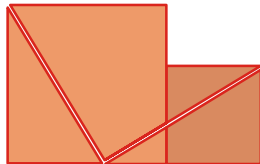There are other ways to cut two squares and rearrange to form a single large square.

Mark a side of the small square on one side of the larger square. Join this point with a corner of the square, as shown below:
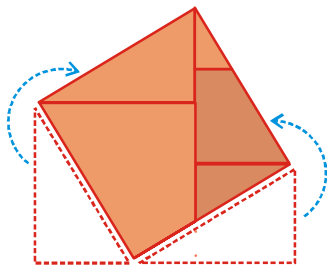
Now place the squares side by side and draw a line as below:
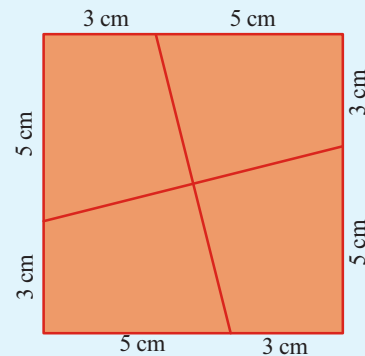
Cut along these lines:

Move the pieces below to the top as shown below, to make a square:

When we mark points, we subtract the distance from the side of the first square. When we slide the pieces into position, we again subtract this distance. Thus, the side of the hole is the side of the original square reduced twice by the distance between the corners and the marks.

So, suppose we cut a square of side 8 centimetres by marking points 3 centimetres from the corners?

If we cut along these lines and rearrange the pieces as before, the side of the square to fill the hole is.

$$8 - (2 \times 3) = 2 \text{ cm}$$
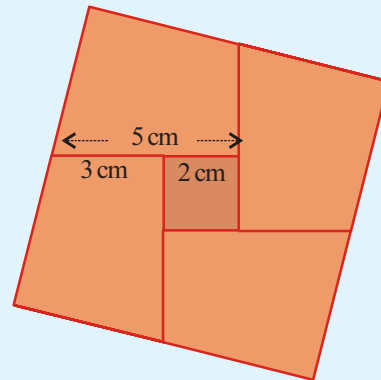
What is the area of this large square?

Another question

We want to cut a square of side 8 centimetres as before and fit a square of side 6 centimetres within to make a larger square. How should we cut the first square?
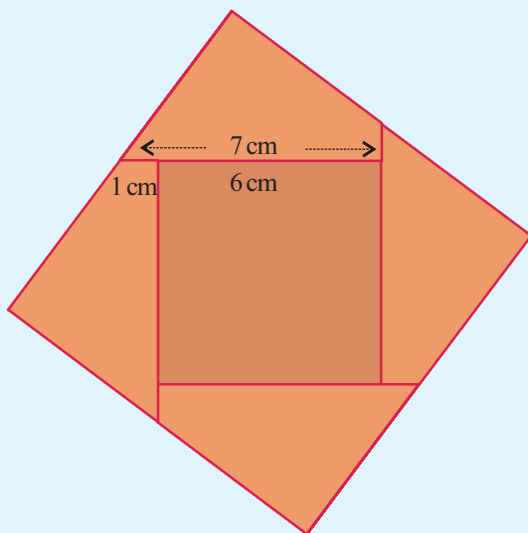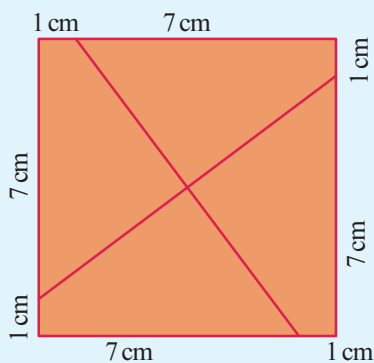
Twice the distance of the points to be marked on the first square from its corners subtracted from 8 centimeters should be 6 centimetres.

So, twice this distance is

$$8 - 6 = 2 \text{ cm}$$

So this distance is half of 2 centimetres; that is 1 centimetre.

Cut out a square of side 8 centimetres, cut into pieces like this and rearrange as before. Isn't the sides of the inner square to 6 centimetres?

Some history

Abu al-Wafa was a famous mathematician and astronomer who lived in Baghdad during the tenth century AD.

One of his books is "On Those Parts of Geometry Needed by Craftsmen." In it, he discusses various methods of joining small squares to make large squares and cutting large squares to make smaller squares.

One such discussion demonstrates that the method used by the craftsmen then to join three squares into one is not accurate and also gives the correct method. It is this method that is given under **Three squares** in this lesson.

What is the area of the final large square?

$$8^2 + 6^2 = 100 \text{ sq. cm}$$

Art and Geometry

Even before Abu al-Wafa's time, mosques in Persia were decorated with ornamental tiles on the floors and walls. Abu al-Wafa describes various geometric methods to cut and reassemble such tiles of different sizes.

Many beautiful geometric patterns can be seen in such tiles. The picture below shows such a work on a wall in the famous Janab Abbasi Mosque in Iran, built during the 17th century.

- Cut out a square of side 7 centimetres and another of side 3 centimetres. Cut the larger square suitably and join with the smaller square to make one large square.

  What is the area of this square?

- Cut out a square of side 8 centimetres. Cut this suitably and join with a suitable square to make a large square of area 80 square centimetres.

- A square of side 9 centimetres is to be cut and joined with another square to make a large square of area 117 square centimetres. What should be the side of the second square? At what distance from the corners of the first square should points be marked for cutting it?

Drawing Squares

Remember how we joined together squares of sides 5 centimetres and 8 centimetres and made a large square?

First we find half the difference of the sides.

$$(5 - 3) \div 2 = 1$$

Then we mark points on the large square, 1 centimetre away from the corners. Cutting along the lines joining those points and rearranging the pieces with the small square inside, we get a square of area $25 + 9 = 34$ square centimetres.
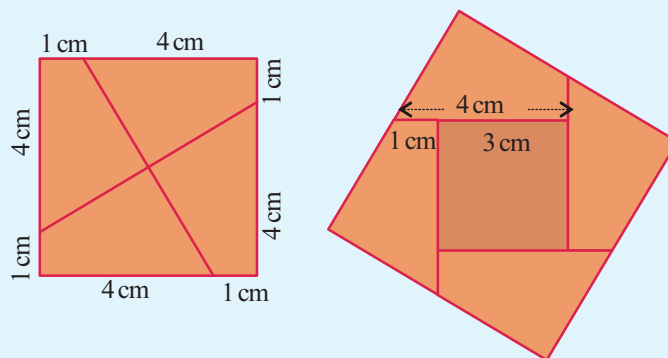
If we just want to draw a square of this area, instead of actually making it, we need only draw a line equal in length to the side of the square. Let's see how we do this.

Each of the four pieces got by cutting the square of side 5 centimetres have two sides of length 4 centimetres and 1 centimetre. By stacking the pieces, we can see that the other sides also are of equal lengths.
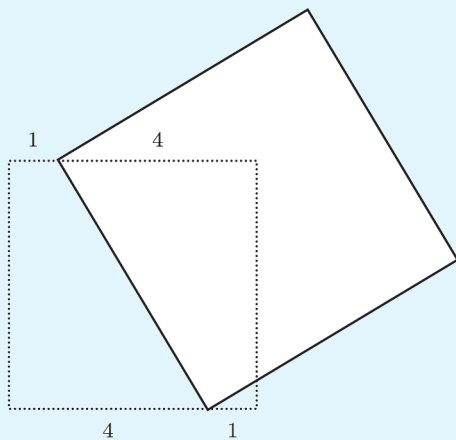
Now look at these pictures:
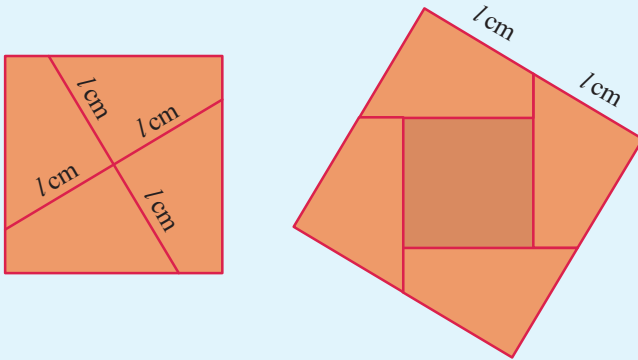
The length of the sides of the final large square is equal to the length of the lines drawn to cut the first square.

So, we have an easy method to draw a side of a square of area 34 square centimetres.
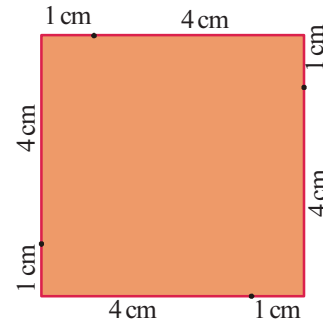
First draw a square of side 5 centimetres, mark two points 1 centimetre away from two opposite corners and join them:
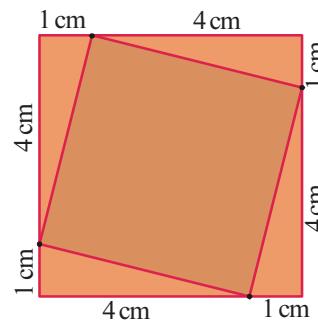

The square drawn with this line as a side has area 34 square centimetres.

Shrinking square

Draw a square of side 5 centimetres and mark points 1 centimetre from the corners as below:
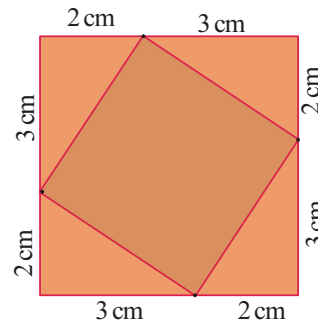

We get a square by joining these points:

What is its area?

We need only subtract the areas of four right triangles from the area of the large square.

$$25 - 4 \times \frac{1}{2} \times 4 \times 1 = 25 - 8 = 17 \text{ sq. cm}$$
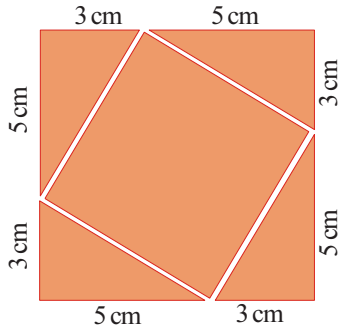
Suppose we join points 2 centimetres from the corners.

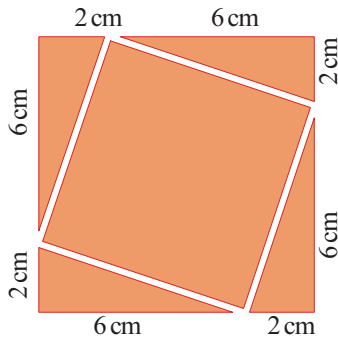What is the area of the small square?

Cutting off

From a square of side 8 centimetres, if we cut off four right triangles as shown below, we get a square of area 34 square centimetres:

What if we cut off triangles as in the picture below?

What is the area of the remaining square?

Can we cut out a square of area 50 square centimetres from the large square?

How about a square of area $44\frac{1}{2}$ square centimetres?

Instead of shifting 1 centimetre from each of the two corners we can shift $2 \times 1 = 2$ centimetres from one corner.
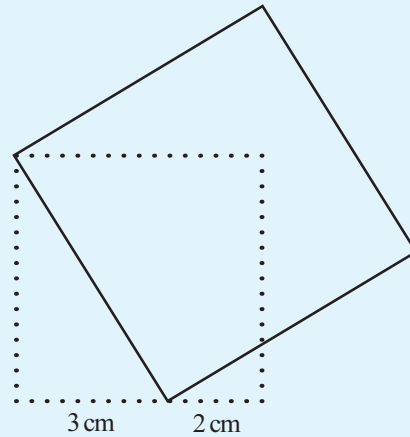
Suppose we want to join a square of side 5 centimetres and another of side 1 centimetre and draw square of area $25 + 1 = 26$ square centimetres. For this, we mark two points $(5 - 1) \div 2 = 2$ centimeters away from two opposite corners of the larger square and join them:
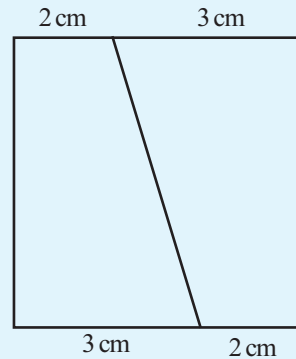
Then we draw a square with this line as a side.

We can draw without taking half of $5 - 1 = 4$. Instead of marking 2 centimetres on two sides, we mark 4 centimetres on one side.
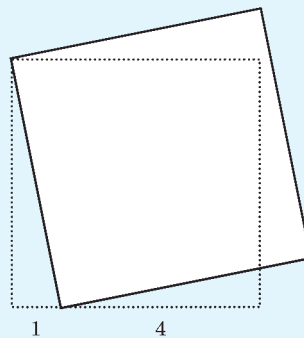
Let's have another look at the two squares we have drawn.

$$5^2 + 3^2 = 34$$

5

3

$$5^2 + 1^2 = 26$$

5

1
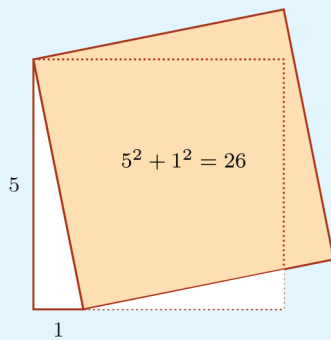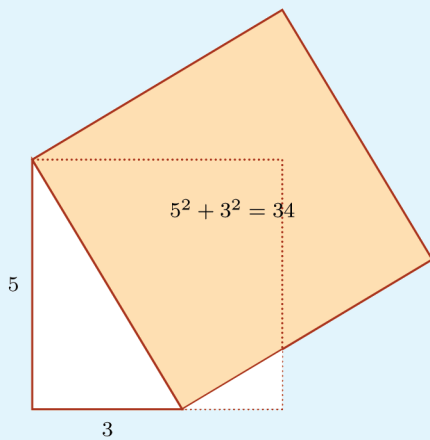
In each figure, the side of the square is the longest side of a right triangle.

What about its area?

The sum of the areas of squares drawn on the other two sides of the triangle.

$$5^2 + 3^2 = 34$$

5^2

3^2
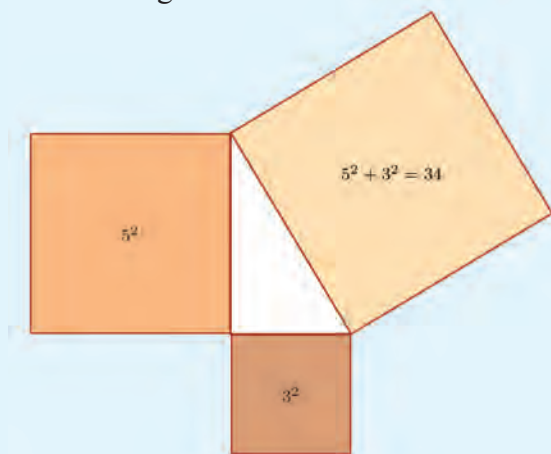
Right triangles

We have seen two ways of drawing a square of area 34 square centimetres:

34 sq.cm

5 cm

3 cm

3 cm   5 cm

5 cm

3 cm

34 sq.cm

3 cm

5 cm

3 cm   5 cm
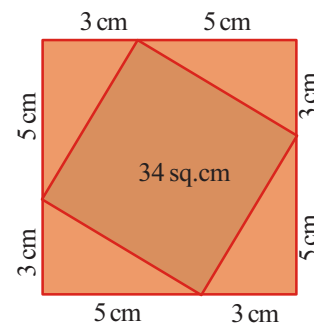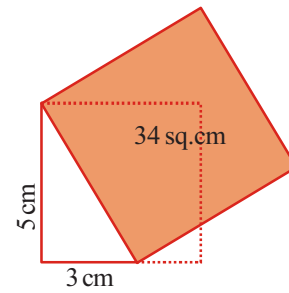
In both pictures, a side of such a square is the longest side of a right triangle

What is the relation between the lengths of the perpendicular sides of the triangle and area of the square?

Using Geogebra, we can verify the relation between the areas of squares drawn on the three sides of a right triangle.

Draw a line AB and the perpendicular through A as below.
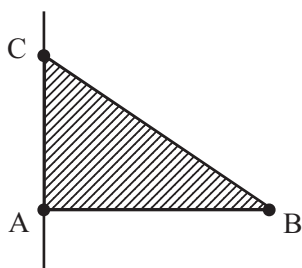
Mark a point C on the perpendicular.

Now we can slide the line AC.

Using the **Polygon** tool, draw triangle ABC. Then using the **Regular Polygon** tool, we can draw squares on the sides AB, BC and AC, By selecting **Area** tool and clicking within the squares, we can see their areas.
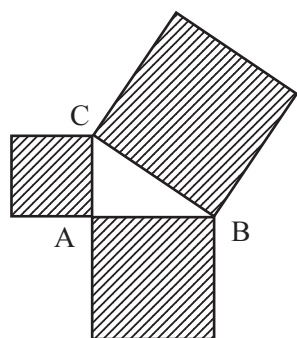
What is the relation between the areas? Drag the vertices of the triangle and check. Does the relation between areas change? To make the area of the largest square 25, what should be the sides of the smaller squares?

Suppose we want the largest square to have area 41?

Now draw a right triangle and squares on all three sides on thick paper:
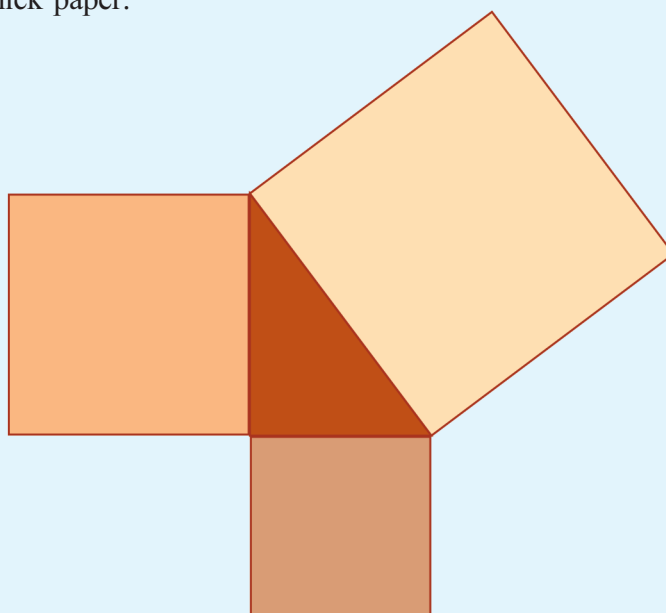
In the medium size square, mark the point of intersection of diagonals; and through this point, mark lines parallel to the perpendicular sides of the largest square:
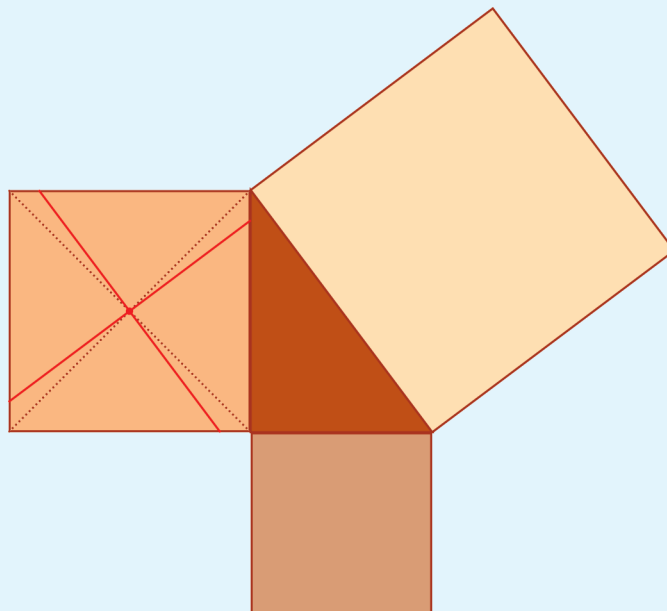
Now cut along these lines to split the square into four pieces. Cut out the smallest square also. Arrange all these inside the largest square as below:
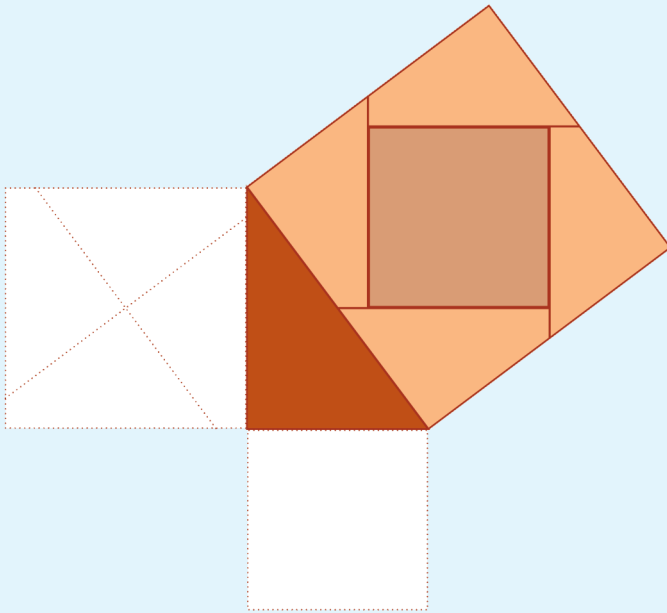
What do we see from all this?

> The area of the square drawn on the largest side of a right triangle is equal to the sum of the areas of the squares drawn on the other two sides.

This is known as Pythagoras Theorem, in honour of the philosopher Pythagoras, who lived in Greece.

Using this, we can draw a square of area 10 square centimetres. We have

$$10 = 3^2 + 1^2$$

So according to Pythagoras Theorem, we need only draw a right triangle of perpendicular sides 3 centimetres and 1 centimetre and then draw a square on the longest side.

3 cm

1 cm
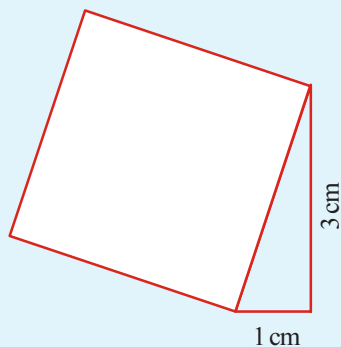
How about a square of area 7 square centimetres?

Pythagoras

We don't know much about Pythagoras, a famous mathematician of ancient times. He was born around 570 BC in the island of Samos in Greece.

History says, in his youth he studied in Egypt and later came back to Greece to start a school. He taught that:

"The real nature of things could be understood only through Mathematics"

The picture shows a statue of Pythagoras at Samos, his place of birth.

Indian Math

Certain geometric texts of ancient India are collectively termed *sulvastura*.

They are written by various authors at different periods estimated to be written around 500 BC.

In the *Baudhayana sulvasutra*, method of doubling a square is given:

> By stretching a rope along the diagonal of a square, another square of double the area can be made.

In the *Katyana sulvasutra*, written around 200 BC, a more general construction is described:

> By stretching a rope along the diagonal of a rectangle, the sum of the areas of square make by the horiztontal and vertical sides.

The word *sulva* in Sanskrit means string or rope. The word *sutra* also means the same, but it also used for shortened statements of general principles.

Where did you learn this trick?

The Great Indian Rope Trick!
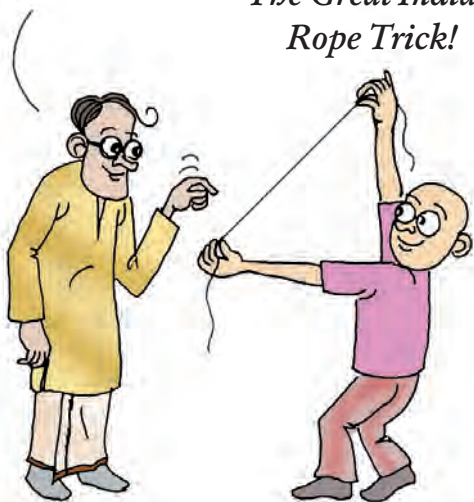
7 cannot be written as the sum of two perfect squares

But we have

$$7 = 4^2 - 3^2$$

So by Pythagoras Theorem, we need only draw a right triangle of longest side 4 centimetres and another side 3 centimetres.

How do we draw it ?

First draw a 3 centimetre long line and the perpendicular at one end:

3 cm
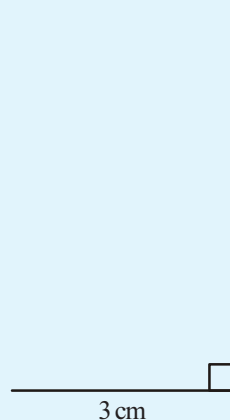
Now use a compass to mark a point on the perpendicular, at a distance of 4 centimetres from the other end of the first line.

4 cm

3 cm
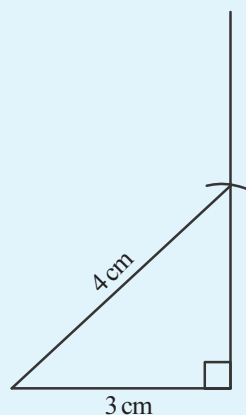
The area of the square in the vertical side of this triangle is $4^2 - 3^2 = 7$ square centimetres, according to Pythagoras Theorem.
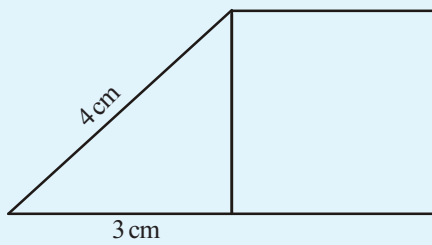
Can't you now draw squares of areas given below?

- 20 square centimetres
- 39 square centimetres
- 40 square centimetres
- 65 square centimetres

Squared relation

We can state Pythagoras Theorem as a relation between the lengths of the sides of a right triangle. The longest side of a right triangle is called its *hypotenuse*.

> The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

For example, if the length of the perpendicular sides of a right triangle are 3 centimetres and 4 centimetres, then the square of the hypotenuse is

$$3^2 + 4^2 = 25$$

and so the length of the hypotenuse is 5 centimetres.
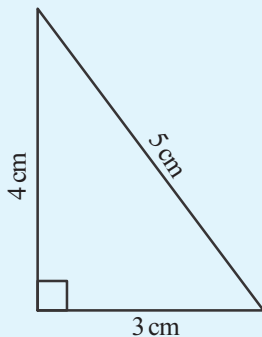
Look at this problem: What is the length of the diagonal of the rectangle below?
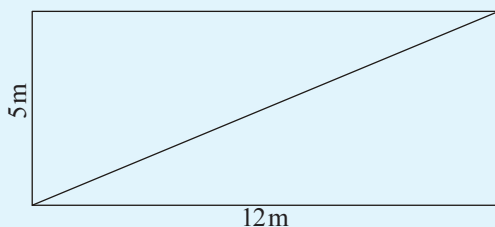
Pythagoras relation

The square of the longest side of a right triangle is equal to the sum of the squares of the other two sides

Putting it in reverse, if the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then it is a right triangle.

That is, the property of having the square of one side equal to the sum of the squares of the other two, is peculiar to right triangles.

For example, since $3^2 + 4^2 = 5^2$, a triangle with lengths of sides 3, 4, 5 is right. What if the lengths of sides are 6, 8, 10?

An area without boundary! What is it?

Pythagoras Theorem!

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Different uses

We can use Pythagoras Theorem to make one large square from two smaller ones and to draw squares of specified areas.

We can also use it to construct perpendiculars and to verify perpendicularity.

For example, look at these lines:
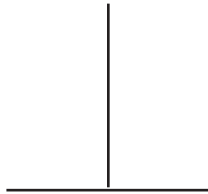
To check whether they are perpendicular to each other, mark a point on the horizontal line, 3 centimetres from the point where the lines meet; and a point on the vertical line, 4 centimetres up.

4 cm

3 cm
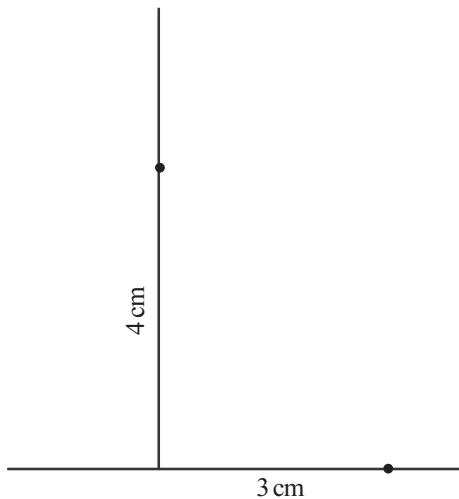
If the distance between these points is 5 centimetres, then the lines are perpendicular; otherwise not.

The diagonal of the rectangle is the hypotenuse of a right triangle :

5 m

12 m
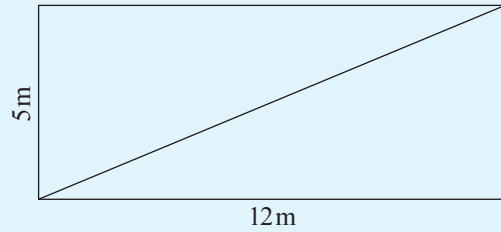
So, the square of the diagonal is

$$5^2 + 12^2 = 169$$

and the length of the diagonal is

$$\sqrt{169} = 13 \text{ m}$$

- What is the length of the fourth side of the quadrilateral shown below?

8 cm

15 cm

24 cm
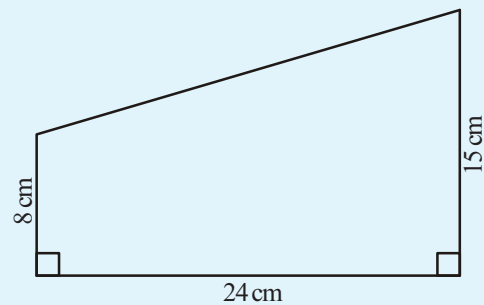
- What is the area of the triangle shown below?

25 cm   26 cm   7 cm
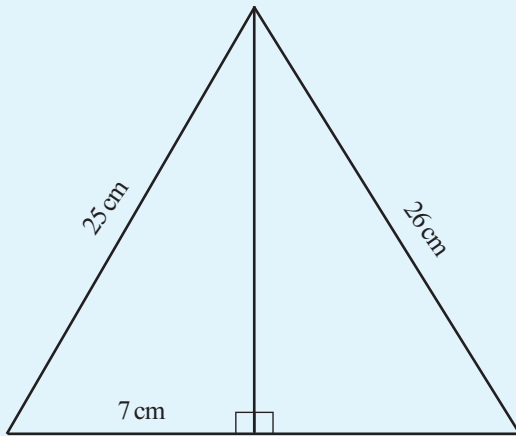
- The picture shows a plot of land in the form of a right angled triangle joined to a square.

40 m
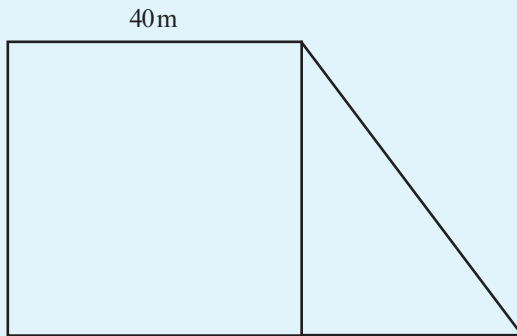
The total area of the plot is 2200 square metres. What is its perimeter?

Pythagorean triples

The sum of the squares of two natural numbers may not be the square of another natural number.

$$1^2 + 2^2 = 5$$

But we do have

$$3^2 + 4^2 = 25 = 5^2$$

$$5^2 + 12^2 = 169 = 13^2$$

$$8^2 + 15^2 = 289 = 17^2$$

and so on.

Any three natural numbers, with the sum of the squares of two of them equal to the square of the third, is called a Pythagorean triple.

Some examples of Pythagorean triples are

3, 4, 5

5, 12, 13

8, 15, 17

Can you find some more ?

*I don't need it.
Master ! I have learnt a
trick with this.*
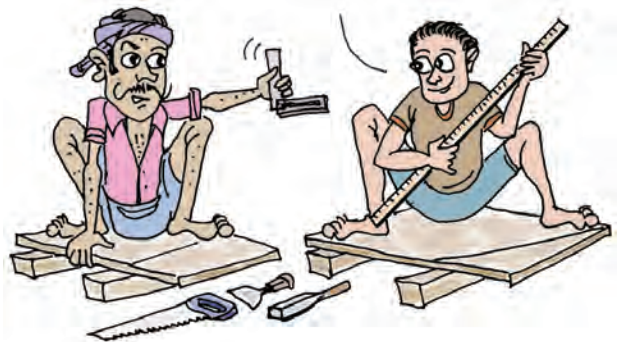
Looking   back

Achievements	On my own	With teacher's help	Must improve
• Making square of area equal to the sum of other two.			
• Logically justifying such a construction.			
• Describing the method to draw a square of specified area.			
• Logically justifying the relation between areas of squares drawn on sides of a right triangle.			
• Justifying Pythagoras Theorem.			
• Using Pythagoras theorem to solve problems.			
• Drawing geometric figures with precision and accuracy.			

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New Numbers

Negative temperature

Haven't you seen the newspapers and TV giving daily temperatures at various places in the country? In many parts of northern India, the temperature is below zero during winter.

The temperature at which water freezes to ice is taken as zero degree Celsius (0°C). Temperatures below this are taken as −1°C, −2°C and so on.

A column of mercury within a glass tube expands and rises when the temperature is increased, it contracts and falls when the temperature is low. Temperature is measured using such a device. In such thermometers used in cold countries, numbers below zero are also marked.

The thermometer in the picture shows a temperature between -20° C and -15° C.

Is it getting colder?

What does the thermometer say?

I think it has contracted a cold!

Colourful numbers

Neetu, Hari and Anvar are playing a game with number cards. There are 50 cards, each with a number from 1 to 5 on it. Every number is on 10 cards. Half the cards have black numbers, and the other half have red numbers.

To start the game, each player takes a black 5.

The remaining cards are shuffled and stacked face down. Now each player draws a card from this in turn. If the number drawn is black, it is added; if red, it is subtracted.

The game continues. The first to reach 10 or more wins.

They first drew these cards:

Neetu 2   Anvar   1   Hari   3

Let's write down their scores, according to the rules of the game:

Neetu	5	7
Anvar	5	6
Hari	5	2

In the second round, these were the cards they got:

Neetu 1   Anvar   3   Hari   3

How do we write the scores now?

Neetu	5	7	8
Anvar	5	6	3
Hari	5	2	

They got into an argument about Hari's score.

Hari said, since 3 cannot be subtracted from 2, his score should be written as 0.

Not so, said Anvar; Hari has lost and must quit the game; Neetu and himself will continue.

No, said Neetu; Hari can continue, but 1 will be subtracted from what he gets in the next round.

They all agreed and decided to write, "Subtract 1" in Hari's column.

But then, asked Anvar, why not shorten it as just −1? All agreed on this also.

Neetu	5	7	8
Anvar	5	6	3
Hari	5	2	−1

Hari was saved in the next round.

Neetu 4   Anvar 5   Hari 3

Can you write their scores now?

Neetu	5	7	8	4
Anvar	5	6	3	
Hari	5	2	−1	

Hari got 3; subtracting the earlier debt of 1, his scores now is 2.

What about Anvar?

5 cannot be subtracted from 3

As with Hari in the earlier round, they decided Anvar should subtract from the number he gets next.

Subtract how much?

"Subtract 2" can be written −2 as before.

Neetu	5	7	8	4
Anvar	5	6	3	−2
Hari	5	2	−1	2

In the fourth round, they got these cards:

Neetu 1   Anvar 3   Hari 3

Colder and Colder

The coldest place in India is Dras, a town in the Kargil district of Kashmir. Temperatures as low as −60 °C has been recorded there.

The coldest place in the world is the continent of Antartica.

The lowest temperature in the world was recorded here : −89 °C.

You could've said the temperature is now −89°C and can't get any colder.

That's just what I did! And that made him really hot!

I'M COOL
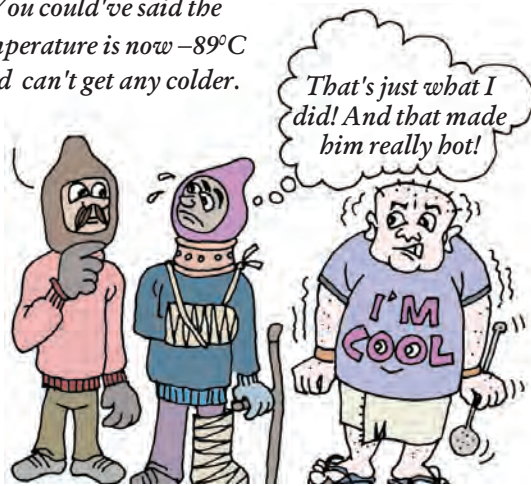
Limit of coldness

Considering the whole universe we know of, the lowest temperature is found in the star cluster named Boomerang Nebula, about fifty quadrillion (5×10^{16}) kilometres from the earth. It is $-272.15°C$.

This is the lowest temperature recorded in nature. Still lower temperatures are produced in labs.

According to the laws of physics, temperature of $-273.15°C$ or below is impossible, either in nature or in labs.

I'm the coolest kid in school.

You mean your temperature is $-273.15°C$

Can you write the scores now?

Neetu	5	7	8	4	
Anwar	5	6	3	−2	
Hari	5	2	−1	2	

Below Zero

In the card game when 3 had to be subtracted from 2, it was written as –1. Let's write this operation as

$$2 - 3 = -1$$

What does this mean?

2 subtracted from 2 gives 0. Here we have to subtract 3 and so has to subtract 1 again. This we write as –1. That is,

$$0 - 1 = -1$$

Similarly, how was 5 subtracted from 3?

3 subtracted from 3 is 0; what more should be subtracted?

$$0 - 2 = -2$$

Numbers written with a minus sign like these are called *negative numbers*.

Let's look at another problem.

An examination has 25 questions. Each correct answer gets 2 marks; for each wrong answer, 1 mark is taken away.

For example, if 19 answers are correct and 6 wrong, the total marks got is

$$(19 \times 2) - 6 = 32$$

What if it is the other way round?

The 6 correct answers get $(6 \times 2) = 12$ marks.

For the 19 wrong answers, 19 marks are reduced. So the total marks got is

$$12 - 19$$

How do we compute this?

12 subtracted from 12 gives 0, what more should be subtracted ?

$$19 - 12 = 7$$

Thus

$$12 - 19 = 0 - 7 = -7$$

Sometimes we need negative fractions also. Look at this problem.

An examination has 10 questions. Each correct answer gets 1 mark, and for each wrong answer, $\dfrac{1}{2}$ mark is reduced.

If one got only 3 answers right, what would be his total marks?

For the three correct answers, he would get 3 marks; and for the seven wrong answers, half of 7, that is $3\dfrac{1}{2}$, would be subtracted.

3 subtracted from 3 makes 0; and $\dfrac{1}{2}$ more should be reduced.

So total marks is

$$3 - 3\frac{1}{2} = 0 - \frac{1}{2} = -\frac{1}{2}$$

How much marks for someone getting just 1 question right?

$$1 - 4\frac{1}{2}$$

How do we compute this?

$$1 - 1 = 0$$

What more to be subtracted?

$$4\frac{1}{2} - 1 = 3\frac{1}{2}$$

So, we find

$$1 - 4\frac{1}{2} = 0 - 3\frac{1}{2} = -3\frac{1}{2}$$

When we start using negative numbers also, the numbers like 1, 2, $1\dfrac{1}{2}$ (which are not negative) are called *positive numbers*.

What about 0? It's neither positive, nor negative.

To subtract a larger positive number from a smaller positive number, we first go to zero and then subtract from zero. Can't we do this directly instead?

Negative Money

Even as early as the seventh century, negative numbers were used in India to denote debts in financial transactions. This practice continues to the day.

For example, many people use a prepaid scheme in their mobile phone connections, in which a certain amount of money is paid in advance. This gets reduced on usage. The subscriber can see at any time how much is left in the account. Even after the advance is exhausted, he can use the phone for some more time. The amount left will be shown as -2 rupees or -3 rupees then.

It means this amount will be deducted when the next advance is paid.

The king positively said no more advances. Your account is full of negatives!

Negative floors

To go from one floor to another in tall buildings, a lift is used.

It has a panel of numbered buttons and we can get to a floor by pressing its number. The picture shows such a panel:

Why are there negative numbers like -1 and -2 in it?

It means that the building has some floor below the ground level with the one just below the ground level named -1, the one below that -2 and so on.

Is this also called a lift?

Especially when it comes up!
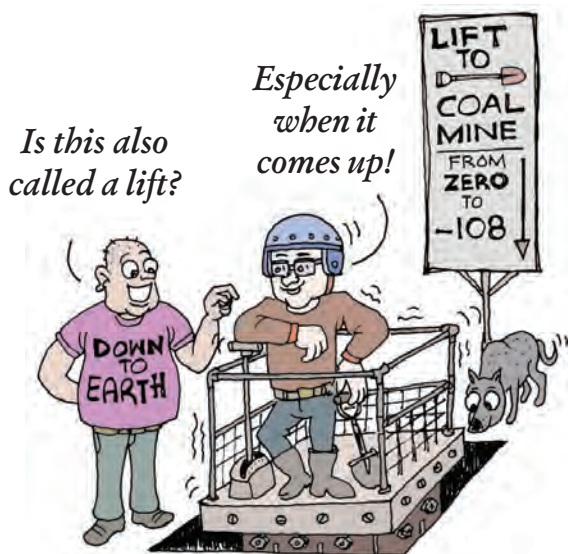
Let's have another look at the computations done so far:

$$2 - 3 \ = \ -1 \qquad\qquad 3 - 2 \ = \ 1$$
$$3 - 5 \ = \ -2 \qquad\qquad 5 - 3 \ = \ 2$$
$$12 - 19 \ = \ -7 \qquad\qquad 19 - 12 \ = \ 7$$
$$3 - 3\frac{1}{2} \ = \ -\frac{1}{2} \qquad\qquad 3\frac{1}{2} - 3 \ = \ \frac{1}{2}$$
$$1 - 4\frac{1}{2} \ = \ -3\frac{1}{2} \qquad\qquad 4\frac{1}{2} - 1 \ = \ 3\frac{1}{2}$$

What do we see from these?

> For positive numbers, the larger subtracted from the smaller is the negative of the smaller subtracted from the larger.

We can write this using algebra also:

> For any two positive numbers x, y if $x < y$ then
> $$x - y \ = \ -(y - x)$$

Now try these problems:

- $4 - 9$
- $14 - 29$
- $\dfrac{1}{2} - \dfrac{3}{4}$

- $5 - 10$
- $25 - 65$
- $\dfrac{1}{3} - \dfrac{1}{2}$

Addition and Subtraction

In the game of number cards when we say one's score is -2, it means 2 is to be subtracted from the next card he draws. So, if he draws a black 2 next, his score is
$$2 - 2 = 0$$

Adding 2 when the score is -2 can be written
$$-2 + 2$$

That is,
$$-2 + 2 = 2 - 2 = 0$$

In an exam of 10 questions, each correct answer is given 1 mark and for each wrong answer, 1 mark is subtracted.

If one gets the first 5 questions right and the next 5 wrong, what would be his total marks?

From the 5 marks for the correct answers, 5 marks are subtracted due to the wrong answers, so that total mark is 0.

If we do this in the order of the answers, we can write the total marks as $-5+5$. Thus

$$-5+5 = 5-5 = 0$$

What if the first 4 answers are wrong and the next 6 right? We can write it as $-4+6$. If from the 6 marks for the right answers, 4 marks are subtracted because of the wrong answers, it would give $6-4=2$. So,

$$-4+6 = 6-4 = 2$$

If the first 6 are wrong and the next 4 right?

The total marks can be written $-6+4$.

From the 4 marks for the right answers, 6 marks are subtracted because of the wrong answers, it would give $4-6=-2$. So,

$$-6+4 = 4-6 = -2$$

In an exam of 10 questions, each right answer gets 1 mark and $\dfrac{1}{2}$ mark is subtracted for each wrong answer.

If one gets only the last 3 answers right, what would be his total marks?

We have already seen that in this case, the total marks is

$3 - 3\dfrac{1}{2} = -\dfrac{1}{2}$. If marks are computed in the order of the

answers, we can say that the total marks is $-3\dfrac{1}{2} + 3$ also.

That is,

$$-3\dfrac{1}{2} + 3 = 3 - 3\dfrac{1}{2} = -\dfrac{1}{2}$$

Let's look at all these computations together:

$$-2 + 2 = 2 - 2 = 0$$
$$-5 + 5 = 5 - 5 = 0$$
$$-4 + 6 = 6 - 4 = 2$$

Changing direction

In discussing motion along a straight line, distances in one direction from a fixed point are often denoted by positive numbers and distances in the opposite direction by negative numbers.
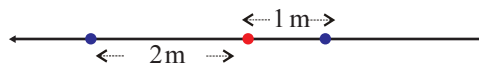
In the picture above, distances to the right of the red dot are taken positive and to the left are taken negative.

If from this point, 3 metres to the right and then 5 metres to the left is travelled, where would be the final point reached? To the left or right of the point? How far away from it? This we can write as,

$$3 - 5 = -2$$

What if 5 metres to the left is travelled first and then 3 metres to the right?

$$-5 + 3 = -2$$

5 metres to the left and then again 3 metres to the left?

So! No money for going left, isn't that right?

Oh! Is this what you meant by the right amount?

Speed Math

When an object is thrown up from the earth, it goes higher and higher with its speed decreasing every instant. When the speed becomes zero, it starts falling down. In the return journey, the speed increases, till it hits the ground.

If it is thrown directly upwards, the speed decreases by 9.8 m/s every second.

For example, if an object is thrown directly up with a speed of 49 m/s, then after 1 second, the speed becomes $49 - 9.8 = 39.2$ m/s;

After 2 seconds, $49 - (2 \times 9.8) = 29.4$ m/s.

After 5 seconds, the speed becomes.

$$49 - (5 \times 9.8) = 0$$

Thereafter, it falls down with speed increasing by 9.8 m/s every second.

What is the speed 7 seconds after throwing? Then, it has been falling for $7 - 5 = 2$ seconds. So the speed has increased by 2×9.8 m/s from 0; that is 19.6 m/s.

We can write these in shorter form using algebra. What is the speed t seconds after throwing?

If $t < 5$, then the speed is

$$49 - 9.8\,t \text{ m/s.}$$

And for $t > 5$? It has been falling for $t - 5$ seconds. So the speed is

$$(t - 5) \times 9.8 = 9.8t - 49 \text{ m/s.}$$

$$-6 + 4 = 4 - 6 = -2$$
$$-3\frac{1}{2} + 3 = 3 - 3\frac{1}{2} = -\frac{1}{2}$$

What do we see from these?

Adding to the negative of a positive number, a second positive number means subtracting the first number from the second number.

In the language of algebra, this can be put this way:

For any two positive numbers x and y
$$-x + y = y - x$$

Now try these problems:

- $-4 + 9$
- $-15 + 8$
- $-\frac{1}{2} + \frac{3}{4}$

- $-9 + 4$
- $-8 + 15$
- $-\frac{3}{4} + \frac{1}{2}$

Subtracting again

In an exam where 1 mark is subtracted for each wrong answer, if the first two answers are wrong, what would be the total marks then?

If the next answer also is wrong?

Since 3 answers are wrong, the marks then is –3.

We can look at this in another manner. With the first two wrong answers, marks is -2, with the next also wrong, 1 more mark should be subtracted, That is,

$$-2 - 1 = -3$$

What if the next two answers are also wrong?

5 answers are wrong; so -5 marks. Looking at it another way, 2 subtracted from -3; that is $-3 - 2$. We can write this as

$$-3 - 2 = -5$$

So, then how much $-5 - 3$?

-5 means 5 less then 0; if it is still 3 less, how much less than 0 altogether?

That is,

$$-5 - 3 = -(5 + 3) = -8$$

Can't we calculate $-5 - 7$ like this?

$$-5 - 7 = -(5 + 7) = -12$$

We can state this as a general principle:

> Subtracting a positive number from the negative of a positive number, we get the negative of the sum of these positive numbers.

How do we write it using algebra?

> For any positive numbers x and y,
>
> $$-x - y = -(x + y)$$

Do the following problems using this.

- $-1 - 1$
- $-7 - 8$
- $-\dfrac{1}{2} - \dfrac{1}{4}$

- $-2 - 2$
- $-8 - 7$
- $-2\dfrac{1}{2} - 1\dfrac{1}{2}$

- $8 - 12$
- $-10 - 4$

- $1\dfrac{1}{2} - 7\dfrac{1}{2}$
- $-25 - 3\dfrac{1}{2}$

- $-8 + 8$
- $-10 + 20$

- $-3\dfrac{1}{2} + 3\dfrac{1}{2}$
- $-20 + 40$

- $-7 + 4$
- $-4\dfrac{1}{2} + 5\dfrac{1}{2}$

- $-12\dfrac{1}{2} + \dfrac{1}{2}$

Negative Speeds

We wrote the speed of an object thrown up with a speed of 49 m/s using two algebraic statements.

$$\text{If } t < 5 \text{ then } v = 49 - 9.8\,t$$

$$\text{If } t > 5 \text{ then } v = 9.8\,t - 49$$

If we denote speeds upwards using positive numbers and speeds downwards using negative numbers, then we need only a single algebraic statement;

$$v = 49 - 9.8t$$

for the speed at any time.

For example, the speed 8 seconds after throwing is

$$49 - (9.8 \times 8) = -29.4 \text{ m/s}$$

Now I positively know that negative speed means faster!

Looking back

| Achievements	On my own	With teacher's help	Must improve
• Explaining negative numbers through contexts requiring subtraction of a larger number from a smaller number.			
• Discovering and describing contexts requiring the use of negative numbers.			
• Explaining the methods of adding a positive number to a negative number and subtracting a positive number from a negative number.			
• Using negative numbers in games and in other situations requiring scores.			

14

Pie Charts
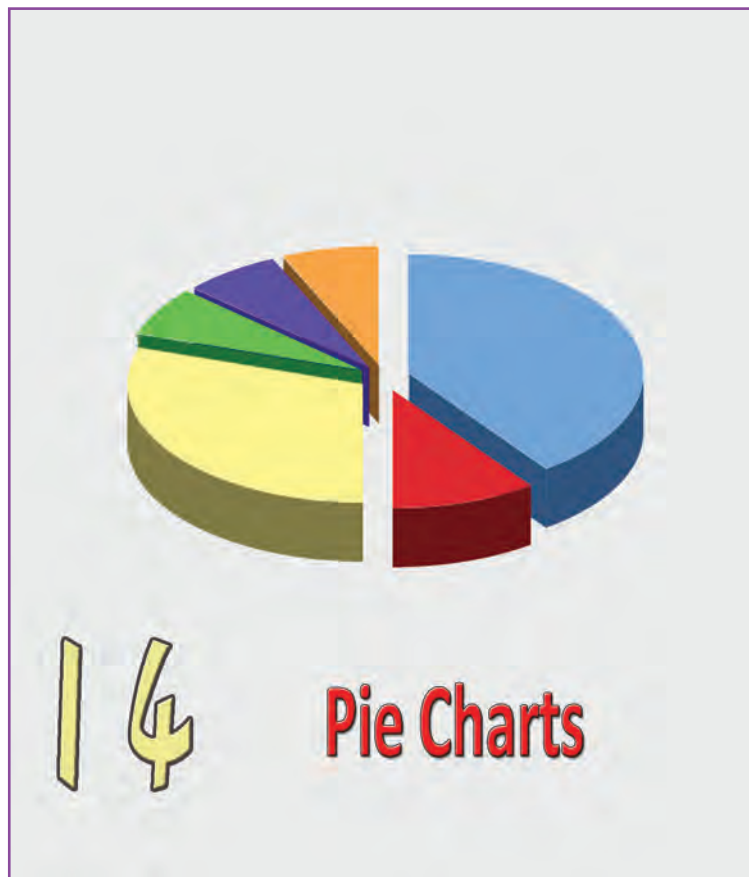
Pie charts

Pie charts are usually drawn to show a single numerical fact split into categories and to makes comparisons between these.

Such a picture is drawn with the size of each part depending on the number it represents.
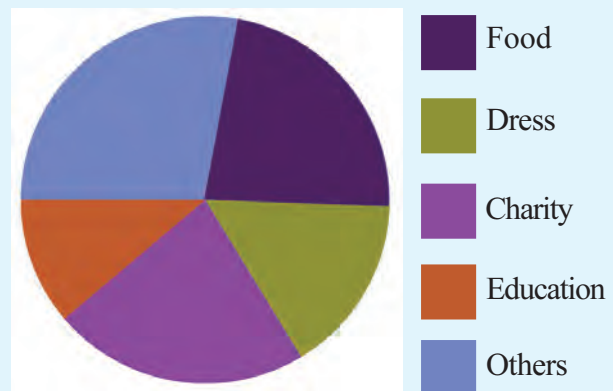
Election

The picture below shown the votes each candidate got in a school election.

Remya
Nazia
Maria
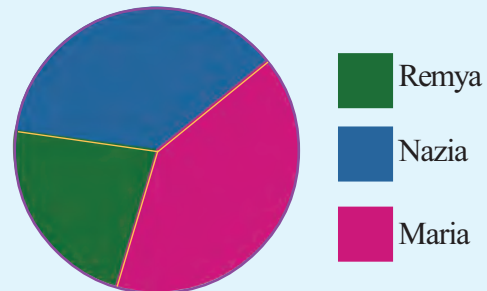
- Who won the election?
- What all information do you get from this picture?

Household expenses

Look at this picture showing various expenses at Fathima's home:

Food
Dress
Charity
Education
Others

For what, is the most amount spent?

And the least?

For what all are the same amount spent?

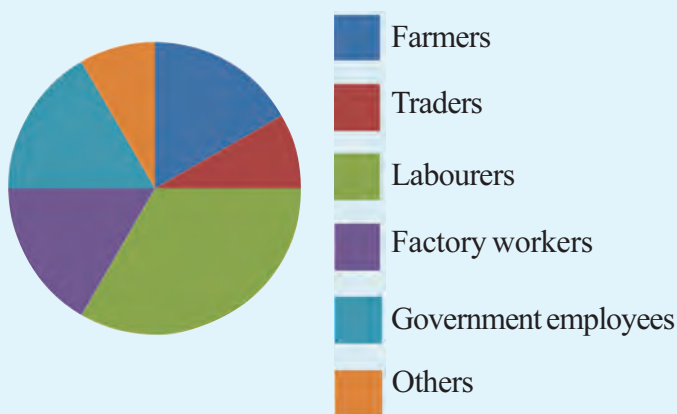How do we know that the same amount is spent on these?

-
-

What more do we get from the pictures?

-
-
-

A picture like this, which display information as parts of a circle is called *pie chart*.

Occupation

The pie chart below shows the various occupations of people in a panchayath.

- Farmers
- Traders
- Labourers
- Factory workers
- Government employees
- Others

- What is the occupation of the most number of people?
- Roughly, how many times the number of farmers is the number of labourers?
- Roughly what fraction of the total is the number of factory workers?
- Arrange the occupations in the order of the number of people in each.
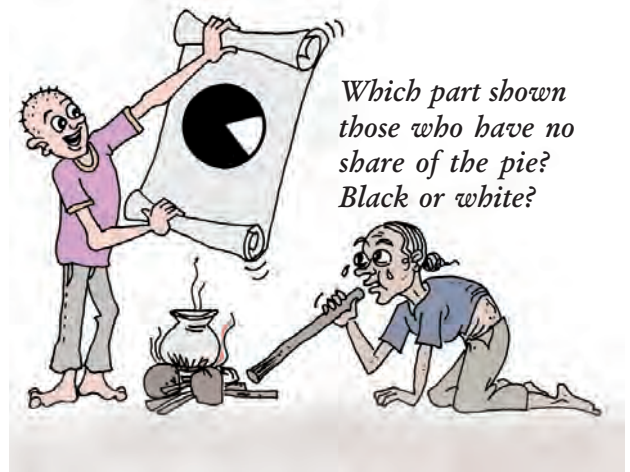
Make some more questions relating to this picture.

Food for thought

Pie is the name of a baked dish very much popular in England and USA.


It is divided and shared by cutting into slices as shown in the picture. The name pie chart originates from this.

*Look Ma! A pie chart of the have's and have - not's*

Which part shown those who have no share of the pie? Black or white?

Bar and Circle

The household expenses of Renu's family is shown as a bar graph and as a pie-chart below:

A Food
B Education
C Health
D Others
E Saving

Food
Education
Health
Others
Saving
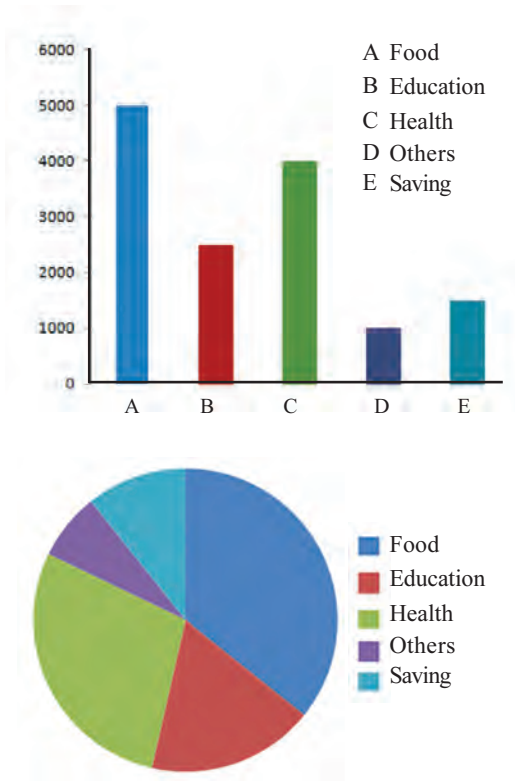
Look at the bar graph. We can easily see the amounts spent under each head and compare one another, but we cannot directly see what fraction of the total each is. That is easier to see in the pie chart. But the actual amount of each is not easy to see in it. Thus, each type of picture has certain advantages and defects.

Agriculture

The pie chart below shows how the total farmland in a panchayat is used for various crops.

- Rubber
- Plantain
- Tapioca
- Coconut
- Rice
- Vegetables
- Others
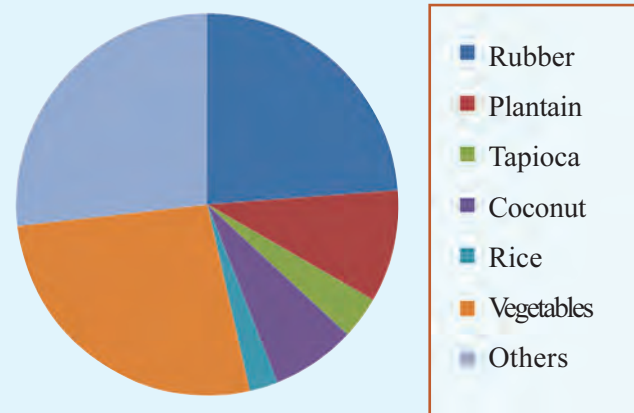
Based on this figure, answer the following questions.

- For which is the least land used?

- For which is the most land used?

- Roughly, what fraction of the total land is used for vegetables?
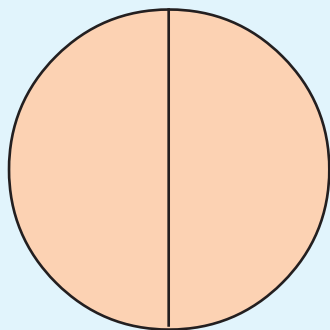
Let's draw!

It was decided to have a vegetable garden in the school. Amaranth in half the plot and beans and brinjal in equal parts of the other half. Let's draw a pie chart showing this;

First draw a circle.

Half the plot is for amaranth.

How do we show this in our picture?

How do we mark half the circle?

Now to show the parts for beans and brinjal one half of the circle must be halved again.

Can't you do it?

Also colour each part to tell these apart.

Travel math

There are 40 students in class 7 A of a school. 20 of them come in the school bus, 5 of them walk and 5 ride bicycles to school.

Let's draw a pie chart showing these.

What fraction of children take the school bus?

This we can mark in a circle as before.

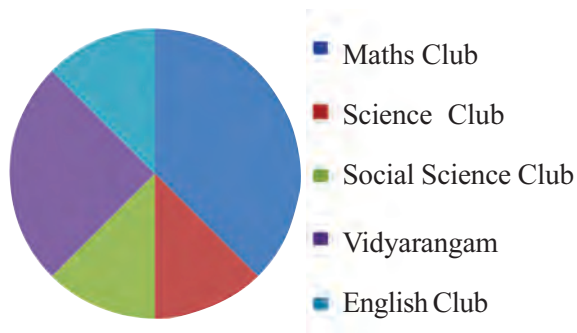What fraction of children ride bicycles?

How do we mark $\frac{1}{8}$ of the circle?

What angle should we draw for this?

$$\frac{1}{8} \text{ of } 360° = 45°$$

Let's make a table

All students of Class 7 in a school are in one of the various clubs. A pie chart showing the number of students in each club is shown below.

- Maths Club
- Science Club
- Social Science Club
- Vidyarangam
- English Club

There are 50 children in Vidyarangam. Make a table showing the number of children in each club.

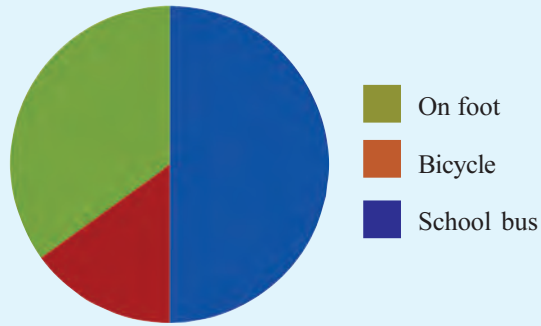*Dont worry! This is the pie chart showing full marks.*

The remaining part of the circle shows the number of children who walk to school.

- What fraction of the circle is this?
- How much degrees is its angle?

School clubs

In a school, there are 100 students in Class 7 and each one is a member of some club.

The table shows the number of students in various clubs.

Clubs	Numbers
Maths	30
Science	20
Social Science	25
English	10
Vidhyarangam	15

We want to draw a pie chart of this.

What all fractions of the circle should we mark to show the number of members of various clubs?

There are 100 students altogether.

Math club has 30 numbers.

To show this, we must mark $\dfrac{30}{100}$ of the circle.

What angle should we draw to get this?

Piechart with computers

Pie charts can be easily drawn using a computer.

Open **Libre Office Calc** program and type in the details as below:

Maths Club	30
Science Club	20
Social Science Club	25
Vidhyarangam	15
English Club	10

Click on any cell and choose Insert → chart → pie. This makes a pie-chart. Change the numbers and see what happens to the picture.

$$360º \times \frac{30}{100} = 108º$$

Similarly, what are the angles we have to draw to show the numbers in other clubs?

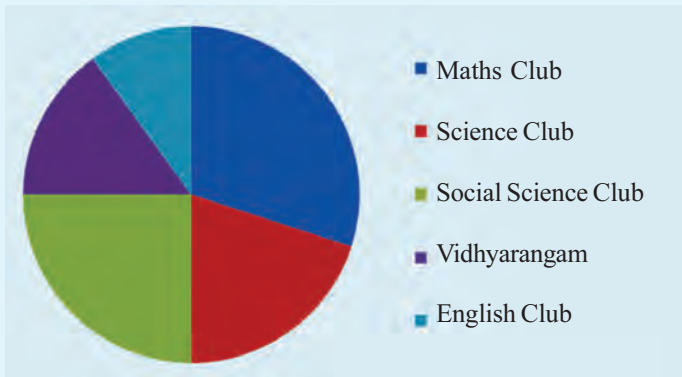|			
Science	: $360º \times \dfrac{20}{100}$ =	72°
Social Science	:	=
English	:	=
Vidhyarangam	:	=

Now we can draw the pie chart:

- Maths Club
- Science Club
- Social Science Club
- Vidhyarangam
- English Club

Grade chart

In Class 7 of a school, 25% got A grade, 45% got B, 20% got C and the remaining D. Let's draw a pie chart of this.

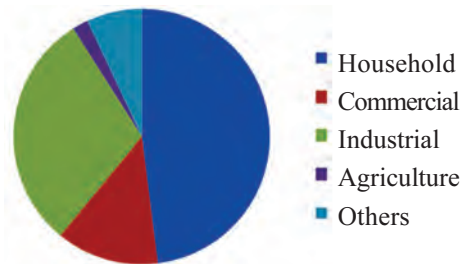Let's compute the fractions of circles to show those who got different grades:

25% got grade A.

We must use 25% of the circle to show this.

$$360º \times \frac{25}{100} = 90°$$

### Electricity

The pie chart below shows the details of electricity supplied by the Kerala State Electricity Board during 2011-2012.

- Household
- Commercial
- Industrial
- Agriculture
- Others

What all facts do we get from this?

The pie chart given below shows the revenue of Kerala State Electricity Board, during 2011-2012.

- Household
- Commercial
- Industrial
- Agriculture
- Others
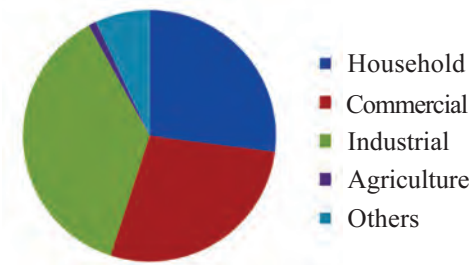
What all information do we get from this?

Compare the two charts.

40 % got B, and the angle is

$$= 360 \times \frac{45}{100} = 162°$$

Can't you compute the angles to be drawn to show those who got C and D grades, and draw the pie chart?

Collect such information about your class and other classes in your school and make pie charts. Display these in Maths lab.

Now draw pie charts of the following

- The final match of the school cricket tournament was between Ramanujan House and C.V. Raman House. The details of the scores are given below. Draw a pie chart of each:

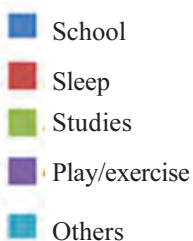| C.V.Raman House		Ramanujan House	
Batsman	**Runs**	**Batsman**	**Runs**
Jishnu	56	Ananthu	72
Abin	35	Thoufiq	36
Sachu	7	Abhilash	18
Ajmal	21	Others	18
Others	21	**Total**	**144**
Total	**140**		

Time management

Given below is the pie-chart showing how Aravind, a class 7 student spends his time:
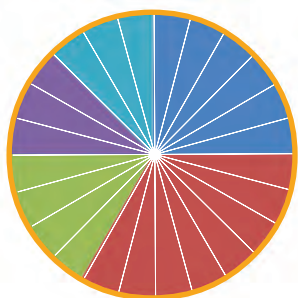
The circle is divided into 24 parts, each part indicating one hour.
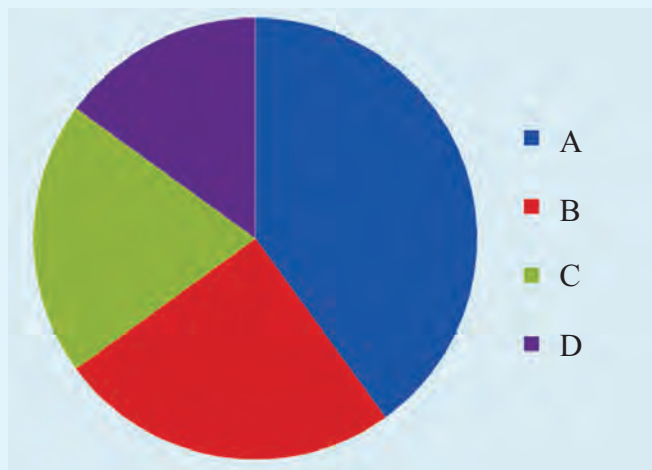
What each colour shows is given below.

- School
- Sleep
- Studies
- Play/exercise
- Others

Draw a bar graph showing these details.

- There are 1600 books in the school library, classified as shown below.

Short story	-	320
Poetry	-	192
Novel	-	384
Science	-	544
Biography	-	160

Draw a pie chart showing this

In a survey, the number of students who like different kinds of books were found as follows.

Short story	-	84
Poetry	-	36
Novel	-	48
Science	-	60
Biography	-	12

Draw a pie chart of this also. Compare the two pie-charts.

Are books bought according to the preference of the students?

- Collect various pictographs, bar graph and pie charts from various periodicals. Analyse and compare these.

- Make a pie chart of the number of students in various classes of your school, using computer

Change to circle

The bar graph below shows the number of girls in the three divisions of Class 7 in a school.
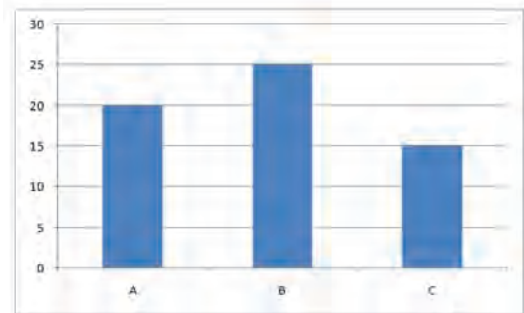
Draw a pie chart of this .

Looking   back

| Achievements	On my own	With teacher's help	Must improve
• Explaining and interpreting information in a pie chart.			
• Drawing pie chart of given information.			
• Explaining how a circle is to be divided into parts for drawing a pie chart.			
• Using computers to draw pie charts.			

Glossary

Linear pair (രേഖീയജോടി)

Parallel lines (സമാന്തരവരകൾ)

Parallelogram (സാമാന്തരികം)

Corresponding angles (സമാനകോണുകൾ)

Alternate angles (മറുകോണുകൾ)

Co-interior angles (ആന്തരസഹകോണുകൾ)

Co-exterior angles (ബാഹ്യസഹകോണുകൾ)

Rectangle (ചതുരം)

Triangle (ത്രികോണം)

Perpendicular (ലംബം)

Angle (കോൺ)

Trapezium (ലംബകം)

Algebra (ബീജഗണിതം)

Exponentiation (കൃതീകരണം)

Perfect numbers (അനഘസംഖ്യകൾ)

Factors (ഘടകങ്ങൾ)

Area (പരപ്പളവ്)

Right triangle (മട്ടത്രികോണം)

Square (സമചതുരം, വർഗം)

Square root (വർഗമൂലം)

Perfect squares (പൂർണവർഗങ്ങൾ)

Square numbers (സമചതുരസംഖ്യകൾ)

Triangular numbers (ത്രികോണസംഖ്യകൾ)

Digital root (അക്കത്തുക)

Speed (വേഗം)

Average speed (ശരാശരിവേഗം)

Circle (വൃത്തം)

Ratio (അംശബന്ധം)

Profit (ലാഭം)

Loss (നഷ്ടം)

Interest (പലിശ)

Hypotenuse (കർണം)

Negative numbers (ന്യൂനസംഖ്യകൾ)

Positive numbers (അധിസംഖ്യകൾ)

Pie chart (വൃത്തചിത്രം)

Line (വര)

Point (ബിന്ദു)

Side (വശം)

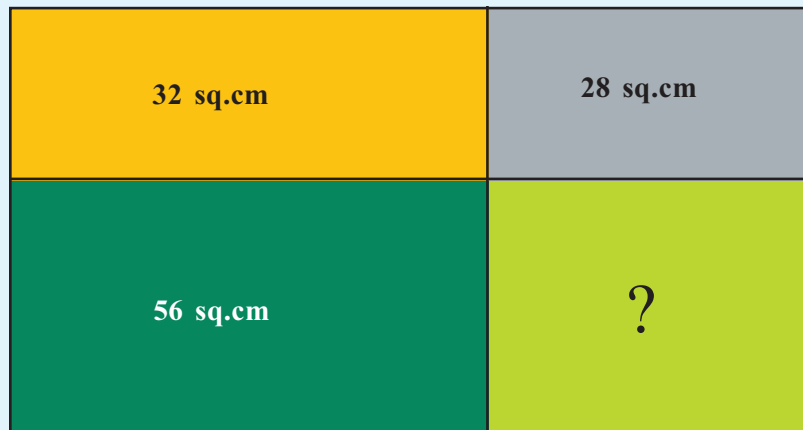Right angle (മട്ടകോൺ)

Setsquare (മട്ടം)

1. In the figure D, E, F are the midpoints of AB, BC, AC.

   G, H, I are midpoints of AD, DF, AF.

   J, K, L are midpoints of AG, GI, AI.

   The area of the shaded region is 21 sq.cm. What is the area of $\triangle ABC$?
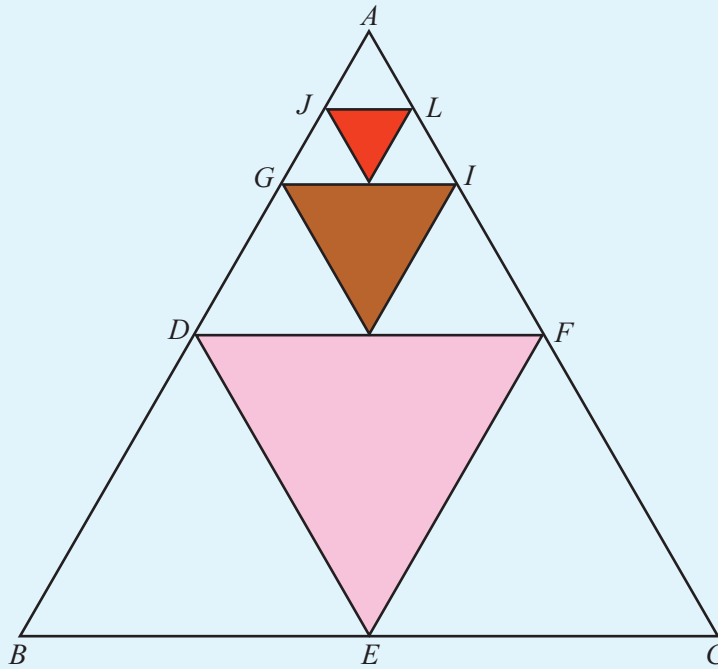
2. A large rectangle is split into four smaller rectangles. The area of the three smaller rectangles are given. Find the area of the fourth.

32 sq.cm	28 sq.cm
56 sq.cm	?

3. *ABCD*, *BDEF* are two rectangles. The area of *ABCD* is 50 sq.cm. What is the area of *BDEF*?
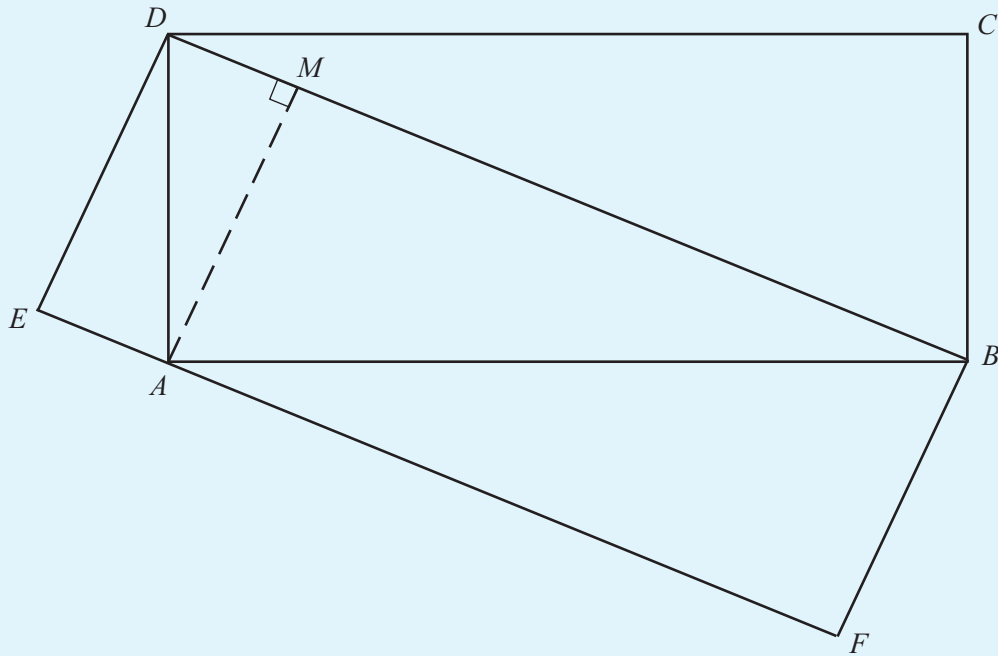
4. In the figure, *A*, *B*, *C*, *D* are squares. One side of *A* is 3 cm and *MN* = 20 cm. What fraction of the whole rectangle is the shaded rectangle?