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ಕರ್ನಾಟಕ ಪ್ರೌಢ ಶಿಕ್ಷಣ ಪರೀಕ್ಷಾ ಮಂಡಳಿ, ಮಲ್ಲೇಶ್ವರಂ, ಬೆಂಗಳೂರು – 560 003

KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD, MALLESWARAM, BANGALORE – 560 003

ಎಸ್.ಎಸ್.ಎಲ್.ಸಿ. ಪರೀಕ್ಷೆ, ಜೂನ್ — 2019

S. S. L. C. EXAMINATION, JUNE, 2019

ಮಾದರಿ ಉತ್ತರಗಳು

MODEL ANSWERS

ದಿನಾಂಕ : 21. 06. 2019] ಸಂಕೇತ ಸಂಖ್ಯೆ : **81-E**

Date : 21. 06. 2019] CODE NO. : **81-E**

ವಿಷಯ : ಗಣಿತ

Subject : MATHEMATICS

(ಹೊಸ ಪಠ್ಯಕ್ರಮ / New Syllabus)

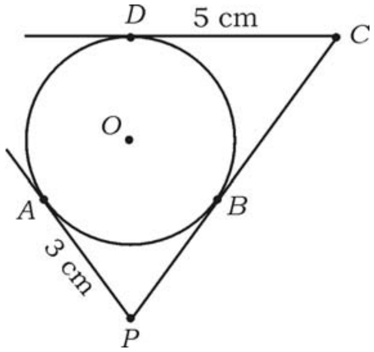
(ಪುನರಾವರ್ತಿತ ಖಾಸಗಿ ಅಭ್ಯರ್ಥಿ / Private Repeater)

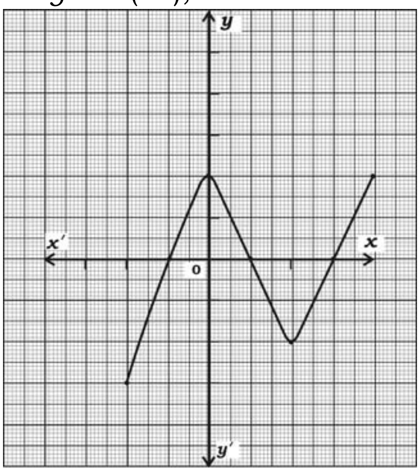
(ಇಂಗ್ಲಿಷ್ ಭಾಷಾಂತರ / English Version)

[ಗರಿಷ್ಠ ಅಂಕಗಳು : **100**

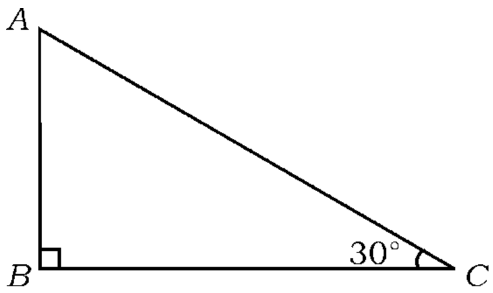
[Max. Marks : 100

Qn. Nos.	Ans. Key	Value Points	Marks allotted
I. 1.		If the n-th term of an arithmetic progression is $5n + 3$, then 3rd term of the arithmetic progression is	
		(A) 11 (B) 18	
		(C) 12 (D) 13	
		Ans. :	
	(B)	18	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
2.		In the following figure, *PA*, *PC* and *CD* are tangents drawn to a circle of centre *O*. If *AP* = 3 cm, *CD* = 5 cm, then the length of *PC* is	
			
		(A) 3 cm (B) 5 cm (C) 8 cm (D) 2 cm	
		Ans. :	
	(C)	8 cm	1
3.		If the lines drawn to the linear equations of the type $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident on each other, then the correct relation among the following is	
		(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (B) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	
		(C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$	
		Ans. :	
	(A)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	1
4.		The distance between the origin and co-ordinates of a point (*x*, *y*) is	
		(A) $x^2 + y^2$ (B) $\sqrt{x^2 - y^2}$	
		(C) $x^2 - y^2$ (D) $\sqrt{x^2 + y^2}$	
		Ans. :	
	(D)	$\sqrt{x^2 + y^2}$	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
5.		If the HCF of 72 and 120 is 24, then their LCM is (A) 36 (B) 720 (C) 360 (D) 72 *Ans.* :	
	(C)	360	1
6.		The value of $\sin 30° + \cos 60°$ is (A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{1}{4}$ (D) 1 *Ans.* :	
	(D)	1	1
7.		In the given graph of $y = P(x)$, the number of zeros are  (A) 4 (B) 3 (C) 2 (D) 7 *Ans.* :	
	(B)	3	1
8.		Faces of a cubical die numbered from 1 to 6 is rolled once. The probability of getting an odd number on the top face is (A) $\frac{3}{6}$ (B) $\frac{1}{6}$ (C) $\frac{2}{6}$ (D) $\frac{4}{6}$ *Ans.* :	
	(A)	$\frac{3}{6}$	1

Qn. Nos.	Value Points	Marks allotted
II.	Answer the following : $6 \times 1 = 6$	
	(Question Numbers 9 to 14, give full marks to direct answers)	
9.	Write the formula to find the sum of the first n terms of an Arithmetic progression, whose first term is a and the last term is a_n .	
	Ans. :	
	$S_n = \frac{n}{2}[a + a_n]$ OR $S_n = \frac{n}{2}[2a + (n-1)d]$	1
10.	If a pair of linear equations represented by lines has no solutions (inconsistent) then write what kinds of lines are these ?	
	Ans. :	
	Parallel lines	1
11.	Write the formula to find area of a sector of a circle, if angle at the centre is θ degree.	
	Ans. :	
	$\frac{\pi r^2}{360} \times \theta$ OR $\frac{\theta}{360} \times \pi r^2$	1
12.	Write 96 as the product of prime factors.	
	Ans. :	
	$3 \,\lvert\, 96$; $2 \,\lvert\, 32$; $2 \,\lvert\, 16$; $2 \,\lvert\, 8$; $2 \,\lvert\, 4$; $2 \,\lvert\, 2$; 1	
	∴ The product of prime factors are ½	
	$96 = 3 \times 2 \times 2 \times 2 \times 2 \times 2$ ½	
	$= 3 \times 2^5$	1

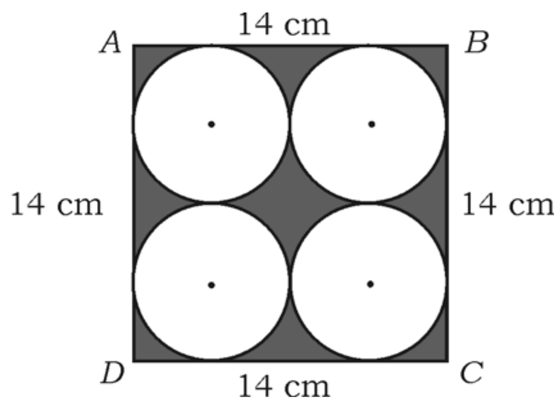
Qn. Nos.	Value Points		Marks allotted
13.	Find the degree of the polynomial $P(x) = x^3 + 2x^2 - 5x - 6$.		
	Ans. :		
	The degree of the polynomial is 3		1
14.	In a ΔABC, $\angle ABC = 90°$ and $\angle ACB = 30°$, then find $AB : AC$.		
			
	Ans. :		
	$AB : AC = \frac{AB}{AC}$		
	$\sin\theta = \frac{AB}{AC}$		
	$\sin 30° = \frac{AB}{AC}$	½	
	$\frac{1}{2} = \frac{AB}{AC}$ $\quad\therefore\quad AB : AC = 1 : 2$	½	1
III. 15.	Find the solution for the pair of linear equations :	2	
	$x + y = 14$		
	$x - y = 4$		
	Ans. :		
	Substitution method :		
	$x + y = 14 \Rightarrow y = 14 - x$ (ii)		
	$x - y = 4$ (i)		
	Substitute $y = 14 - x$ in (i)	½	

Qn. Nos.	Value Points		Marks allotted
	$x-(14-x)=4$		
	$x-14+x = 4$		
	$2x = 4+14$	½	
	$2x = 18 \Rightarrow x = \frac{18}{2} \Rightarrow x = 9$	½	
	substitute $x=9$ in (ii)		
	$y = 14-x$		
	$y = 14-9 \Rightarrow y = 5$	½	2
	Alternate method :		
	Elimination method :		
	$x + y = 14$ (i)		
	$x - y = 4$ (ii) [(i) – (ii)]	½	
	(–) (+) (–)		
	$2y = 10$		
	$y = \frac{10}{2} \Rightarrow y = 5$	½	
	Substitute $y = 5$ in (i)		
	$x+5 = 14$	½	
	$x = 14-5$		
	$x = 9$	½	2
	Alternate method :		
	Cross multiplication method :		
	$x+y-14 = 0$ $a_1 = 1$ $b_1 = 1$ $c_1 = -14$		
	$x-y-4 = 0$ $a_2 = 1$ $b_2 = -1$ $c_2 = -4$		

x		y	1
1	– 14	1	1
– 1	– 4	1	– 1

½

Qn. Nos.	Value Points		Marks allotted
	$\dfrac{x}{-4-14} = \dfrac{y}{-14+4} = \dfrac{1}{-1-1}$		
	$\dfrac{x}{-18} = \dfrac{y}{-10} = \dfrac{1}{-2}$	½	
	$\therefore \quad \dfrac{x}{-18} = \dfrac{1}{-2} \qquad \therefore \quad \dfrac{y}{-10} = \dfrac{1}{-2}$	½	
	$-2x = -8 \qquad -2y = -10$ $x = \dfrac{-18}{-2} \qquad y = \dfrac{-10}{-2}$	½	
	$x = 9 \qquad y = 5$		2
16.	*ABCD* is a square of side 14 cm. Four congruent circles are drawn in the square as shown in the figure. Calculate the area of the shaded region. [Circles touch each other externally and also sides of the square]	2	



Ans. :

	Area of square *ABCD* = 14×14 = $196\ cm^2$	½	
	Diameter of each circle = $\dfrac{14}{2}$ cm = 7 cm		
	So, radius of each circle = $\dfrac{7}{2}$ = 3·5 cm		
	$\therefore$ Area of one circle = πr^2		

Qn. Nos.	Value Points		Marks allotted
	$= \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$	½	
	$= 38{\cdot}5 \text{ cm}^2$		
	$\therefore$ Area of four circle $= 4 \times 38{\cdot}5$		
	$= 154 \text{ cm}^2$	½	
	Hence, area of shaded region $= (196 - 154) = 42 \text{ cm}^2$	½	2
17.	Find the distance between the points (2, 3) and (4, 1).	2	
	Ans. :		
	(2, 3) (4, 1) (x_1, y_1) (x_2, y_2)	½	
	Distance $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	½	
	$d = \sqrt{(4-2)^2 + (1-3)^2}$		
	$d = \sqrt{(2)^2 + (-2)^2}$	½	
	$d = \sqrt{4+4}$		
	$d = \sqrt{8}$		
	$d = 2\sqrt{2}$	½	2
18.	Find the area of a triangle whose vertices are (1, – 1), (– 4, 6) and (– 3, – 5).	2	
	Ans. :		
	(1, – 1) (– 4, 6) (– 3, – 5) (x_1, y_1) (x_2, y_2) (x_3, y_3)	½	
	Area of triangle $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$	½	

Qn. Nos.	Value Points		Marks allotted
	$= \frac{1}{2}[1(6-(-5))+(-4)(-5-(-1))+(-3)(-1-6)]$		
	$= \frac{1}{2}[11+16+21]$	½	
	$= \frac{1}{2} \times 48$		
	$= 24 \text{ cm}^2$	½	
	∴ Area of triangle is 24 cm^2.		2
19.	Prove that $5+\sqrt{3}$ is an irrational number.	2	
	Ans. :		
	Let us assume, to the contrary, that $5+\sqrt{3}$ is rational		
	Such that $5+\sqrt{3} = \frac{a}{b}$ $[a \neq b, \; b \neq 0]$	½	
	Therefore, $\frac{a}{b} - 5 = \sqrt{3}$		
	Rearranging the equation $\sqrt{3} = \frac{a}{b} - 5$		
	$\sqrt{3} = \frac{a-5b}{b}$	½	
	Since a and b are integers, we get $\frac{a}{b} - 5$ is rational, and so $\sqrt{3}$ is rational		
	But this contradicts the fact that $\sqrt{3}$ is irrational	½	
	This contradiction has arisen because of our incorrect assumption that $5+\sqrt{3}$ is rational		
	So, we conclude that $5+\sqrt{3}$ is irrational number.	½	2

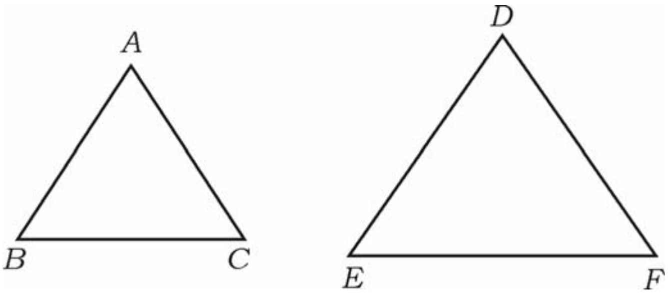
Qn. Nos.	Value Points	Marks allotted

20. $\Delta\ ABC \sim \Delta\ DEF$ and their areas are 64 cm^2 and 100 cm^2 respectively. If $EF = 12$ cm then find the measure of BC. 2

OR

A vertical pole of height 6 m casts a shadow 4 m long on the ground, and at the same time a tower on the same ground casts a shadow 28 m long. Find the height of the tower.

Ans. :



$\Delta\ ABC \sim \Delta\ DEF$

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

$$\therefore \quad \frac{\text{Area of } \Delta\ ABC}{\text{Area of } \Delta\ DEF} = \frac{BC^2}{EF^2}$$ ½

$$\frac{64}{100} = \frac{BC^2}{(12)^2}$$

$$\frac{64}{100} = \frac{BC^2}{144}$$ ½

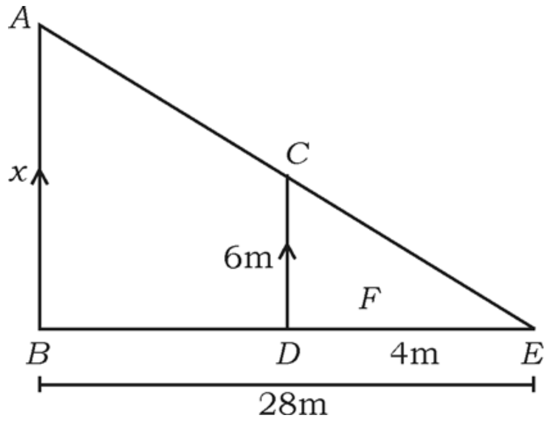
$$\frac{64 \times 144}{100} = BC^2$$

$$\frac{8 \times 12}{10} = BC$$ ½

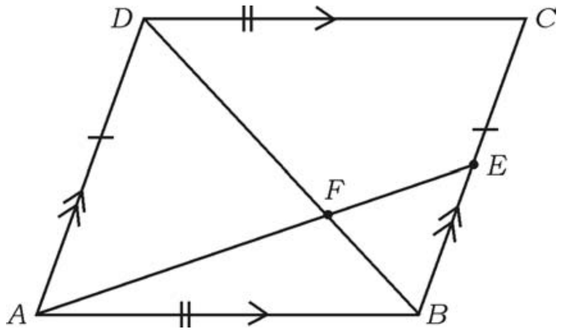
$$9{\cdot}6 = BC$$

$$\therefore \quad BC = 9{\cdot}6 \text{ cm}$$ ½ 2

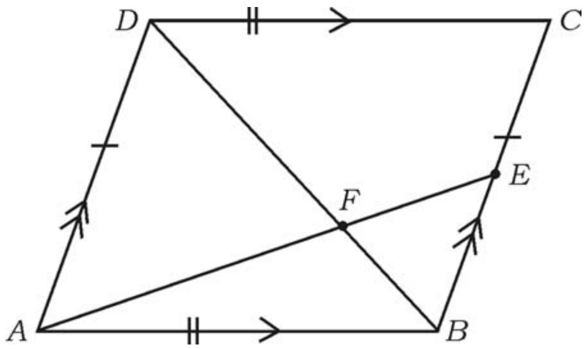
OR

Qn. Nos.	Value Points		Marks allotted
		½	
	In the Δ *ABE* and Δ *DCE*		
	i) $\angle ABE = \angle CDE$ (∵ 90°)		
	ii) $\angle E = \angle E$ (Common angle)		
	∴ Δ *ABE* ~ Δ *DCE*	½	
	$\frac{DE}{BE} = \frac{CD}{AB}$		
	$\frac{4}{28} = \frac{6}{AB}$	½	
	$4 \times AB = 28 \times 6$		
	$AB = \frac{28 \times 6}{4} \Rightarrow AB = x = 42$ m	½	2
	Alternate method :		
	AB || *CD*, according to the Thales theorem (Corollaries)		
	$\frac{DE}{BE} = \frac{CD}{AB}$	½	
	$\frac{4}{28} = \frac{6}{AB}$	½	
	$4 \times AB = 6 \times 28$		
	$AB = \frac{28 \times 6}{4} \Rightarrow 42$	½	
	∴ $AB = x = 42$ m	½	2

Qn. Nos.	Value Points	Marks allotted
21.	The diagonal *BD* of parallelogram *ABCD* intersects *AE* at *F* as shown in the figure, *E* is any point on *BC*, then prove that $DF \times EF = FB \times FA$. 2	



Ans. :



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In the $\Delta\, AFD$ and $\Delta\, BFE$

i) $\angle AFD = \angle BFE$ (vertical opposite angles)

ii) $\angle ADF = \angle EFB$

iii) $\angle DAF = \angle BEF$ ($\because$ $AD \parallel BC$ alternate angles) ½

$\therefore \quad \Delta\, AFD \sim \Delta\, BFE$

$$\frac{FA}{EF} = \frac{DF}{FB}$$ ½

$FA \times FB = EF \times DF$

$DF \times EF = FB \times FA$ ½ 2

Qn. Nos.	Value Points		Marks allotted
22.	Sum and product of the zeroes of a quadratic polynomial $P(x) = ax^2 + bx - 4$ are $\frac{1}{4}$ and -1 respectively. Then find the values of a and b.	2	
	OR		
	Find the quotient and remainder when $P(x) = 2x^2 + 3x + 1$ is divided by $g(x) = x + 2$.		
	Ans. :		
	$P(x) = ax^2 + bx - 4$ $\quad\therefore\quad c = -4$		
	$\alpha + \beta = \frac{1}{4}$ $\quad \alpha \times \beta = -1$	½	
	$\frac{1}{4} = \frac{-b}{a}$ $\quad -1 = \frac{c}{a} = \frac{-4}{a}$		
	$a = -4b \rightarrow$ (i) $\quad -a = -4$	½	
	$a = 4$	½	
	Substitute $a = 4$ in (i)		
	$4 = -4b$		
	$\frac{4}{-4} = b \quad \Rightarrow \quad b = -1$	½	2
	OR		

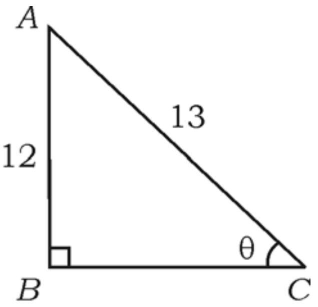
Qn. Nos.	Value Points		Marks allotted
	$p(x) = 2x^2 + 3x + 1 \qquad g(x) = x + 2$		
	$$\begin{array}{r\|l} & 2x - 1 \\ \hline x+2 & 2x^2 + 3x + 1 \\ & 2x^2 + 4x \\ & (-) \quad (-) \\ \hline & -x + 1 \\ & -x - 2 \\ & (+) \quad (+) \\ \hline & +3 \end{array}$$	1	
	$\therefore$ Quotient $q(x) = 2x - 1$	½	
	Remainder $r(x) = 3$	½	2
23.	Find the value of k, in which one of its zeros is -4 of the polynomial $P(x) = x^2 - x - (2k + 2)$.	2	
	Ans. :		
	$P(x) = x^2 - x - (2k + 2)$ Zeros of polynomial $= -4$		
	$0 = (-4)^2 - (-4) - (2k + 2)$	½	
	$0 = 16 + 4 - 2k - 2$	½	
	$0 = 18 - 2k$		
	$2k = 18$		
	$k = \frac{18}{2}$	½	
	$k = 9$	½	2

Qn. Nos.	Value Points	Marks allotted
24.	Solve the equation $x^2 - 3x - 10 = 0$ by using formula. 2	
	Ans. :	
	$x^2 - 3x - 10 = 0$	
	$ax^2 + bx + c = 0, \quad a = 1, \quad b = -3, \quad c = -10$	
	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ½	
	$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$ ½	
	$x = \dfrac{3 \pm \sqrt{9+40}}{2}$	
	$x = \dfrac{3 \pm \sqrt{49}}{2}$ ½	
	$x = \dfrac{3 \pm 7}{2}$	
	$x = \dfrac{3+7}{2}$ | $x = \dfrac{3-7}{2}$	
	$x = \dfrac{10}{2}$ | $x = \dfrac{-4}{2}$ ½	
	$x = 5$ | $x = -2$	2
25.	If $\operatorname{cosec} \theta = \dfrac{13}{12}$, then find the value of $\cos \theta$. 2	
	Ans. :	
	$\operatorname{cosec} \theta = \dfrac{13}{12}$ $\left(\therefore \ \operatorname{cosec} \theta = \dfrac{1}{\sin \theta} \right)$	

Qn. Nos.	Value Points	Marks allotted

$\frac{1}{\sin\theta} = \frac{13}{12}$ ½

$\sin\theta = \frac{12}{13}$



$\sin\theta = \frac{AB}{AC} = \frac{12}{13}$ $\lfloor B = 90°$, AC diagonal ½

$AC^2 = AB^2 + BC^2$

$(13)^2 = (12)^2 + BC^2$

$\therefore \quad \cos A = \frac{BC}{AC}$ $169 = 144 + BC^2$ ½

$25 = BC^2$

$\cos A = \frac{5}{13}$ $BC = 5$ ½ **2**

26. Show that $(\tan A \times \sin A) + \cos A = \sec A$. 2

Ans. :

L.H.S. $= [\tan A \times \sin A] + \cos A$

$= \frac{\sin A}{\cos A} \times \sin A + \cos A$ ½

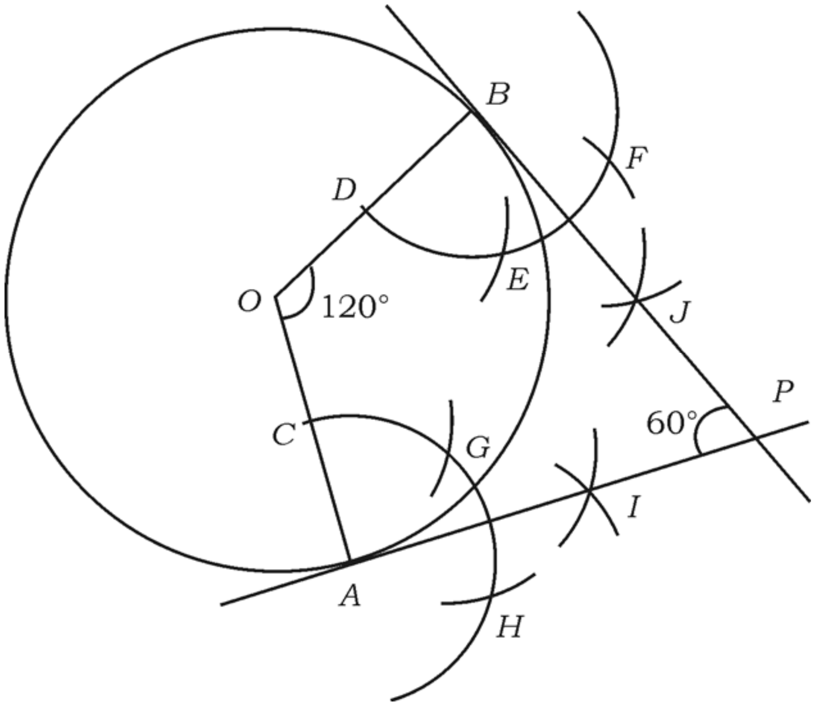
$= \frac{\sin^2 A}{\cos A} + \cos A$ ½

$= \frac{\sin^2 A + \cos^2 A}{\cos A} \Rightarrow \frac{1}{\cos A}$ ½

$\Rightarrow \sec A =$ R.H.S. ½ **2**

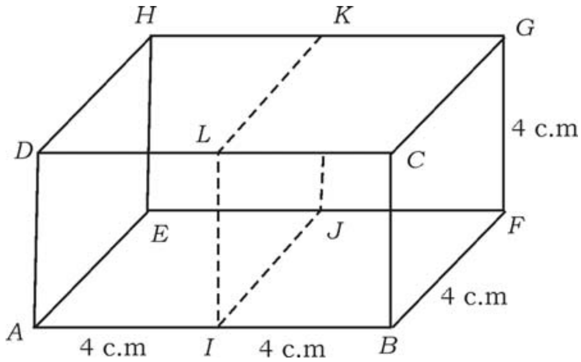
27. Draw a pair of tangents to a circle of radius 3·5 cm which are inclined to each other at an angle of 60°. 2

Ans. :

Qn. Nos.	Value Points	Marks allotted
	$\begin{array}{r} 180^\circ \\ -\ 60^\circ \\ \hline 120^\circ \end{array}$	
		
	i) Circle — ½	
	ii) Marking of angle between radius — ½	
	iii) For two tangents — ½ + ½	2
28.	A box contains 90 discs, which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears a perfect square number. 2	
	Ans. :	
	Sample space = S = { 1, 2, 3, 4, 5, ... 90 }	
	$\therefore \quad n(s) = 90$ ½	
	Event A = { A perfect square number }	
	A = { 1, 4, 9, 16, 25, 36, 49, 64, 81 } ½	
	$n(A) = 9$	

Qn. Nos.	Value Points	Marks allotted
	∴ Probability of the event $P(A) = \frac{n(A)}{n(S)}$ ½ $P(A) = \frac{9}{90}$ ½	2
29.	A metallic sphere of radius 9 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder. 2 *Ans.* : Radius of sphere = 9 cm Radius of cylinder = 6 cm ∴ Height of cylinder = ? Volume of sphere = Volume of cylinder $\frac{4}{3}\pi r^3 = \pi r^2 h$ ½ $\frac{4}{3} \times \frac{22}{7} \times 9 \times 9 \times 9 = \frac{22}{7} \times 6 \times 6 \times h$ ½ $\frac{4 \times 9 \times 9 \times 9}{3 \times 6 \times 6} = h$ ½ 27 cm = h ∴ Height of cylinder is 27 cm. ½	2
30.	The faces of two cubes of volume 64 cm^3 each are joined together to form a cuboid. Find the total surface area of the cuboid. 2 *Ans.* :	

Qn. Nos.	Value Points	Marks allotted



Volume of square $= a^3$

$64 = a^3$

$\sqrt[3]{64} = a$ ½

$a = 4$ cm

$\therefore$ The total surface area of the cuboid

$= 2 (lb + bh + hl)$ ½

$= 2 ((8) (4) + (4) (4) + (4) (8))$

$= 2 (32 + 16 + 32)$ ½

$= 2 \times 80$

$= 160 \text{ cm}^2$ ½ — **2**

31. Find the sum of series 3 + 7 + 11 + ... up to 10 terms. 2

Ans. :

3 + 7 + 10 + ... up to 10 terms

$a = 3$ $d = 7 - 3$ $n = 10$ $S_{10} = ?$ ½

$d = 4$

$S_n = \frac{n}{2} [2a + (n - 1) d]$ ½

$S_{10} = \frac{10}{2} [2 \times 3 + (10 - 1) 4]$ ½

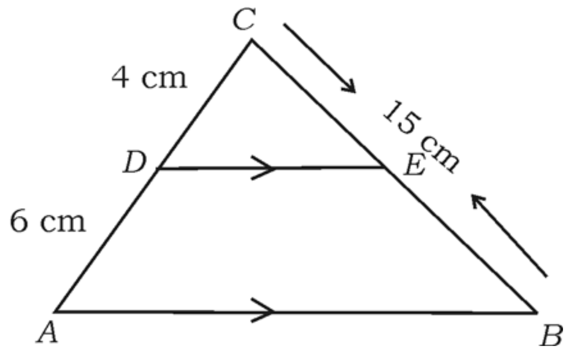
$S_{10} = 5 [6 + 36]$

$S_{10} = 5 \times 42$

$S_{10} = 210$ ½ — **2**

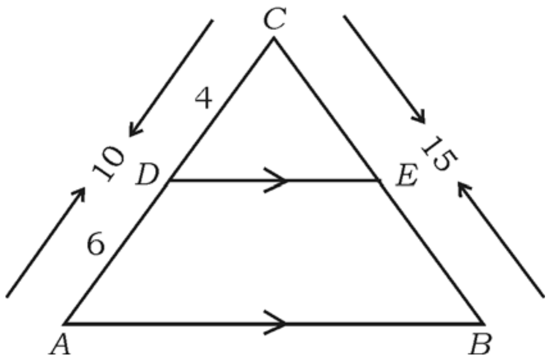
Qn. Nos.	Value Points	Marks allotted

32. In the given figure, $DE \parallel AB$, $AD = 6$ cm, $CD = 4$ cm and $BC = 15$ cm. Then find BE.



Ans. :

From the corollaries of Thales Theorem,



½

$$\frac{AD}{AC} = \frac{BE}{BC}$$

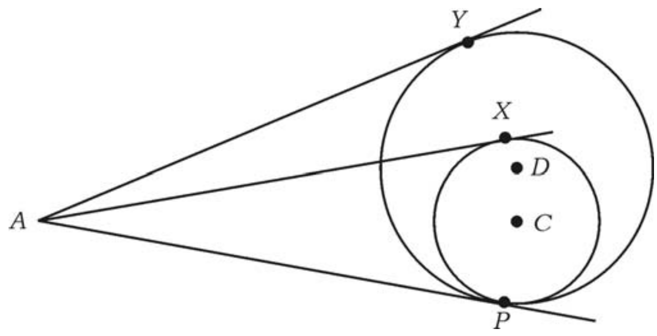
$$\frac{6}{10} = \frac{BE}{15}$$ ½

$$10 \times BE = 6 \times 15$$

$$BE = \frac{6 \times 15}{10}$$ ½

$BE = 9$ cm ½ **2**

Qn. Nos.	Value Points	Marks allotted
33.	In the figure, *AP, AX* and *AY* are the tangents drawn to the circles, show that $AY = AX$.	



Ans. :

Tangents drawn from *A* to the circle of centre *C* is

$AX = AP$ ... (i) ½

Tangents drawn from *A* to the circle of centre *D* is

$AY = AP$ ... (ii) ½

Compare (i) and (ii) ½

$AX = AP$

$AY = AP$

$\therefore$ $AX = AY$ ½ | 2

34\. The areas of two circles are 92 cm^2 and 62 cm^2 respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles. 2

Ans. :

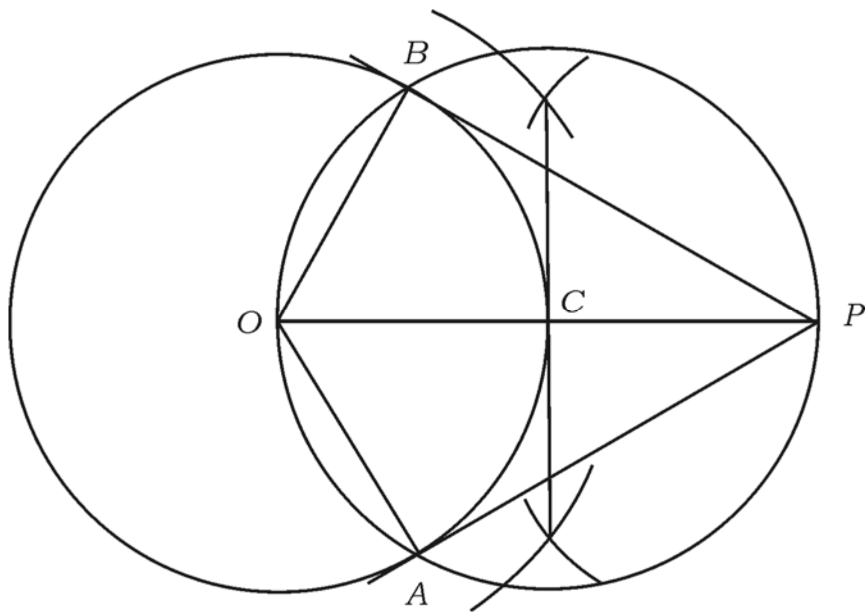
Area of 1st circle = 92 cm^2

Area of 2nd circle = 62 cm^2

$\therefore$ Total area of both circle = 92 + 62

Total area = 154 cm^2 ½

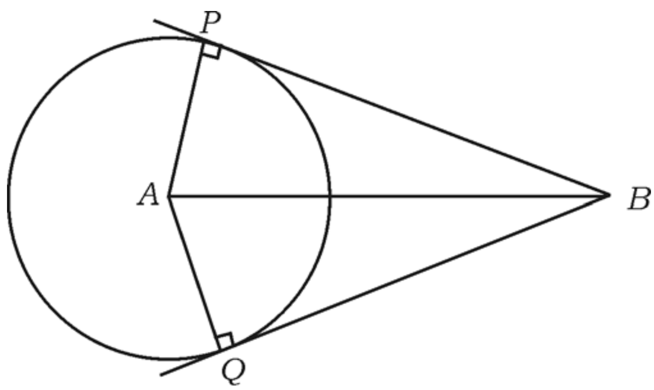
Qn. Nos.	Value Points		Marks allotted
	∴ Area of circle = $154\ cm^2$	½	
	$154 = \pi r^2$		
	$154 = \frac{22}{7} \times r^2$		
	$\frac{154 \times 7}{22} = r^2 \quad \Rightarrow \quad r^2 = 7 \times 7$	½	
	∴ $r = 7$ cm		
	∴ Radius of circle is 7 cm	½	2
35.	Draw a circle of radius 4 cm and construct two tangents to it from an external point 8 cm away from its centre.	2	
	Ans. :		



	i) Circle —	½	
	ii) Straight line $\overline{AB}$ (draw) —	½	
	iii) Drawing of perpendicular —	½	
	iv) Tangents —	½	2

Qn. Nos.	Value Points		Marks allotted
36.	Find the coordinates of the mid-point of the line segment joining the points (2, 3) and (4, 7).	2	
	Ans. :		
	(2, 3), (4, 7)		
	$(x_1, y_1), (x_2, y_2)$	½	
	By mid-point formula, coordinates of mid-point		
	$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	½	
	$= \left(\frac{2+4}{2}, \frac{3+7}{2} \right)$	½	
	$= \left(\frac{6}{2}, \frac{10}{2} \right)$		
	$= (3, 5)$	½	
	∴ Coordinates of mid-points are (3, 5)		2
37.	Find the roots of the equation $x^2 + 7x + 12 = 0$ by factorisation.	2	
	Ans. :		
	$x^2 + 7x + 12 = 0$ Last term = 12 = 4 × 3		
	Middle term = 7 = 4 + 3		
	$x^2 + 4x + 3x + 12 = 0$	½	
	$x (x + 4) + 3 (x + 4) = 0$	½	
	$(x + 3) (x + 4) = 0$		
	$x + 3 = 0$ $x + 4 = 0$	½	
	$x = -3$ $x = -4$	½	2

Qn. Nos.	Value Points		Marks allotted
38.	Find the nature of the roots of the equation $4x^2 - 4x + 1 = 0$.	2	
	Ans. :		
	$4x^2 - 4x + 1 = 0$		
	$ax^2 + bx + c = 0$		
	$a = 4, \quad b = -4, \quad c = 1$	½	
	Discriminant $\Delta = b^2 - 4ac$	½	
	$\Delta = (-4)^2 - 4(4)(1)$		
	$\Delta = 16 - 16$	½	
	$\Delta = 0$		
	$\therefore$ Nature of the roots are real and equal	½	2
39.	Evaluate : $\dfrac{\tan 65^\circ}{\cot 25^\circ} + \dfrac{\sin 25^\circ}{\cos 65^\circ}$.	2	
	Ans. :		
	$\dfrac{\tan 65^\circ}{\cot 25^\circ} + \dfrac{\sin 25^\circ}{\cos 65^\circ}$		
	$\cot A = \tan(90 - A)$ | $\sin A = \cos(90 - A)$	½	
	$\cot 25 = \tan(90 - 25)$ | $\sin 25^\circ = \cos(90 - 25)$		
	$\cot 25^\circ = \tan 65^\circ$ | $\sin 25^\circ = \cos 65^\circ$	½	
	$\therefore \quad \dfrac{\tan 65^\circ}{\tan 65^\circ} + \dfrac{\cos 65^\circ}{\cos 65^\circ}$	½	
	$= 1 + 1$		
	$= 2$	½	2

Qn. Nos.	Value Points	Marks allotted
40.	If two coins are tossed together simultaneously, find the probability of getting at least one head. 2	
	Ans. :	
	Sample space $S = \{ (H, H), (T, T) (H, T) (T, H) \}$	
	$n(S) = 4$ ½	
	Event $A = \{$ At least one head $\}$	
	$A = \{ (H, T) \ (T, H) \ (H, H) \}$ ½	
	$n(A) = 3$	
	$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$ ½	
	$P(A) = \frac{3}{4}$ ½	2
IV. 41.	Prove that "the lengths of tangents drawn from an external point to a circle are equal". 3	
	OR	
	Two concentric circles of radii 5 cm and 3 cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.	
	Ans. :	
		
	½	

Qn. Nos.	Value Points	Marks allotted

Data : A is the centre of the circle, B is an external point,

BP and BQ are tangents. ½

To prove : $BP = BQ$ ½

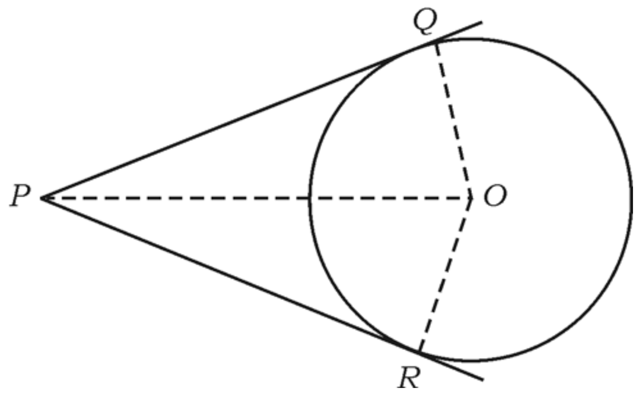
Construction : Join AB, AQ, and AP.

Proof :

Statement	*Reason*
In ΔAPB and ΔAQB $\angle APB = \angle AQB = 90°$	Radius drawn at the point of contact is perpendicular to the tangent 1
$hyp\ AB = hyp\ AB$	Common side
$AP = AQ$	Radii of the same circle
$\therefore \Delta APB \cong \Delta AQB$	RHS theorem
$\therefore BP = BQ$	CPCT ½

3

Alternate method :

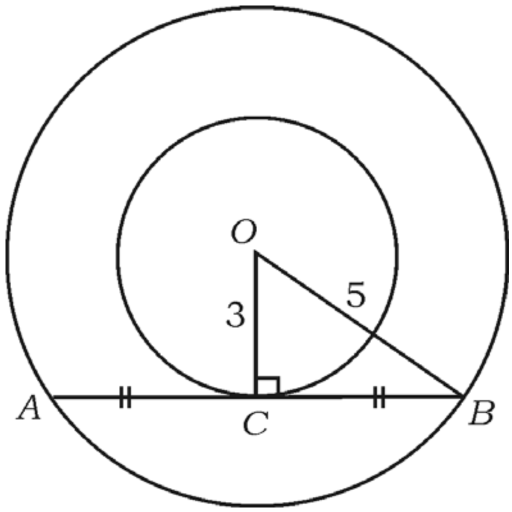


½

In a circle of centre O, a point P laying outside the circle and two tangents PQ, PR on the circle from P.

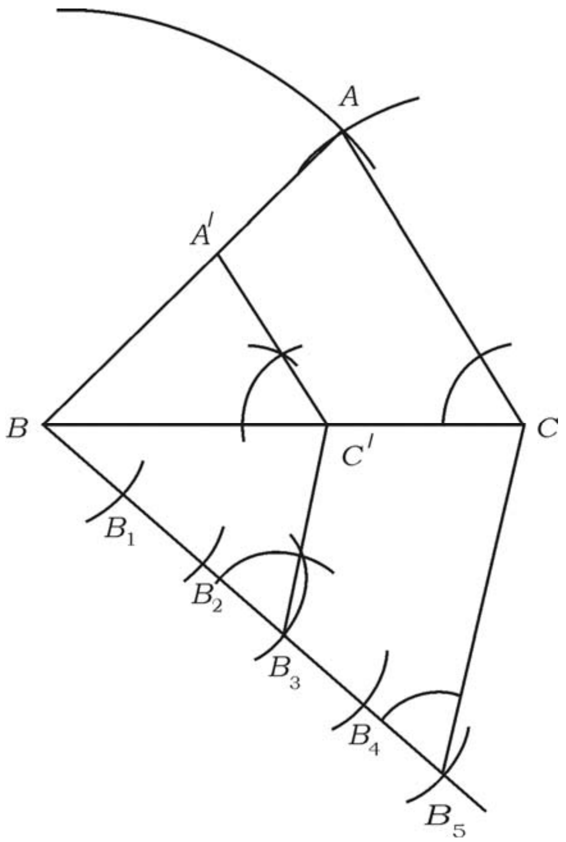
We are required to prove that $PQ = PR$ ½

For this we join OP, OQ and OR, then $\angle OQP$ and $\angle ORP$ are right angles (because these are angles between radii and tangents) ½

Qn. Nos.	Value Points	Marks allotted
	Now in right angled $\Delta\, OQP$ and $\Delta\, ORP$	
	$OQ = OR$ (Radii of the same circle)	
	$OP = OP$ (Common side) 1	
	$\Delta\, OQP = \Delta\, ORP$ (RHS)	
	$\therefore\quad PQ = PR$ (*CPCT*) ½	
	Hence proved.	
	OR	
	 ½	
	In the diagram *OC* is radius, *AB* is tangent	
	In the $\Delta\, OCB$, $\lfloor C = 90°$, *OB* is diagonal	
	$OB^2 = OC^2 + CB^2$ ½	
	$(5)^2 = (3)^2 + BC^2$ ½	
	$25 = 9 + BC^2$	
	$25 - 9 = BC^2$	
	$16 = BC^2$	
	$BC = \sqrt{16} = 4$ cm ½	

Qn. Nos.	Value Points	Marks allotted
	$BC = AC$ Length of chord $AB = AC + BC$	
	4 cm $= AC$ $= 4 + 4$ ½	
	$AB = 8$ cm	
	$\therefore$ Length of the chord $AB = 8$ cm ½	3
42.	Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the given triangle. 3	

Ans. :



i)	$\Delta\ ABC$ construction	1½
ii)	Drawing an acute angle line and division	½
iii)	Drawing $B_3 C^l \parallel B_5 C$	½
iv)	Drawing $A^l C^l \parallel AC$	½

Marks allotted: 3

[Note : Any given side of the triangle may be taken as base]

Qn. Nos.	Value Points	Marks allotted

43. Find the mode for the following data in the frequency distribution table : 3

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
Number of families	7	8	2	2	1

OR

Find the median for the following data in the frequency distribution table :

Weight (in kg)	15-20	20-25	25-30	30-35	35-40
Number of students	2	3	6	4	5

Ans. :

Family size	*No. of families*
1 — 3	7
3 — 5	8
5 — 7	2
7 — 9	2
9 — 11	1
	N = 20

Maximum class frequency is 8

$\therefore$ Mode class is 3 – 5

Lower limit of modal class $l = 3$

Class size $h = 2$

Frequency of the modal class $f_1 = 8$ 1

Frequency of class proceeding the modal class $f_0 = 7$

Frequency of class succeeding the modal class $f_2 = 2$

Qn. Nos.	Value Points		Marks allotted
	∴ Mode $= L + \left[\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$	½	
	$= 3 + \left[\dfrac{8-7}{(2 \times 8) - 7 - 2} \right] \times 2$	½	
	$= 3 + \left[\dfrac{1}{16 - 7 - 2} \right] \times 2$	½	
	$= 3 + \dfrac{2}{7}$		
	$= 3 + 0{\cdot}28$		
	$= 3{\cdot}28$		
	∴ Mode of the data is $3{\cdot}28$	½	3

OR

Weight (in kg)	*No. of students*	*C.f.*
15-20	2	2
20-25	3	5
25-30	6	11
30-35	4	15
35-40	5	20
	N = 20	

$\dfrac{N}{2} = \dfrac{20}{2} = 10$

∴ Median class is [25 – 30]

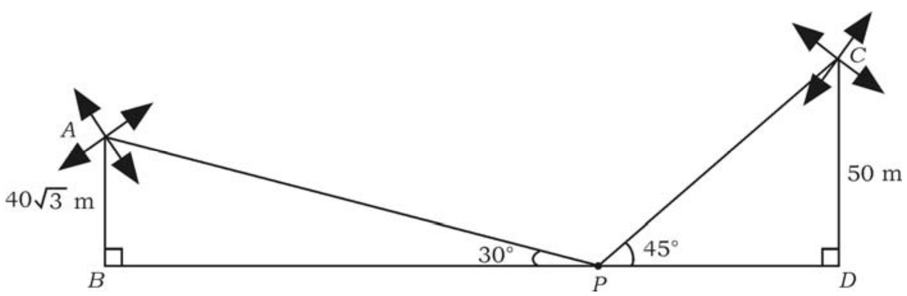
Lower limit of the median class $l = 25$ — 1

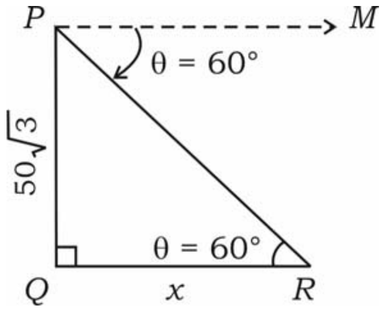
Cumulative frequency of class preceeding the median class $c.f. = 5$

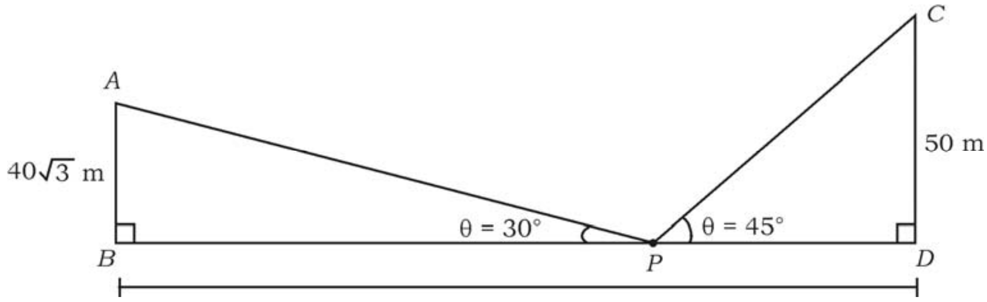
Frequency of median class $f = 6$

Class size $h = 5$

∴ Median $= L + \left[\dfrac{\dfrac{N}{2} - c.f.}{f} \right] \times h$ — ½

Qn. Nos.	Value Points	Marks allotted
	$= 25 + \left[\dfrac{10-5}{6}\right] \times 5$ ½	
	$= 25 + \left[\dfrac{5}{6}\right] \times 5$	
	$= 25 + 4 \cdot 16$ ½	
	$= 29 \cdot 16$	
	$\therefore$ Median of the data is 29·16 ½	3
44.	From the top of a vertical building of $50\sqrt{3}$ m height on a level ground the angle of depression of an object on the same ground is observed to be 60°. Find the distance of the object from the foot of the building. 3	
	OR	
	Two wind mills of height 50 m and $40\sqrt{3}$ m are on either side of the field. A person observes the top of the wind mills from a point in between them. The angle of elevation was found to be 45° and 30°. Find the distance between the wind mills.	
		
	Ans. :	

Qn. Nos.	Value Points		Marks allotted
		½	
	PQ represent the height of the building $\therefore \quad PQ = 50\sqrt{3}$ m *QR* be the distance between the building and the object $QR = x$ Angle of depression is 60° since $PM \parallel QR$ $\angle MPR = \angle PRQ$ $60° = \angle PRQ$	½	
	In $\Delta\, PQR$, $\angle PQR = 90°$, $\angle PRQ = 60°$ $\therefore \quad \tan\theta = \frac{PQ}{QR}$	½	
	$\tan 60° = \frac{50\sqrt{3}}{QR}$ (But $\tan 60° = \sqrt{3}$)	½	
	$\sqrt{3} = \frac{50\sqrt{3}}{QR}$	½	
	$QR = \frac{50\sqrt{3}}{\sqrt{3}}$ $QR = 50$ m ∴ The object is 50 m away from the foot of the building	½	3
	OR		

Qn. Nos.	Value Points		Marks allotted
			
	In ΔABD, $\tan\theta = \frac{AB}{BP}$	In ΔDPC, $\tan\theta = \frac{DC}{PD}$	1	
	$\tan 30° = \frac{40\sqrt{3}}{BP}$	$\tan 45° = \frac{50}{PD}$		
	$\frac{1}{\sqrt{3}} = \frac{40\sqrt{3}}{BP}$	$1 = \frac{50}{PD}$		
	$BP = 40\sqrt{3} \times \sqrt{3}$		
	$BP = 40 \times 3$	$PD = 50$ m	1	
	$BP = 120$ m		
	$\therefore$ Distance between the wind mills		
	$BD = BP + PD$		
	$BD = 120 + 50$	½	
	$BD = 170$ m		
	$\therefore$ The distance between the wind mills on either side of field is 170 m	½	3

Qn. Nos.	Value Points	Marks allotted

45. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield in kg/hectare	50-55	55-60	60-65	65-70	70-75	75-80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive. 3

Ans. :

Production yield (in kg/hac)	*No. of farms*	*c.f.*
More than 50	2	100
More than 55	8	98
More than 60	12	90
More than 65	24	78
More than 70	38	54
More than 75	16	16

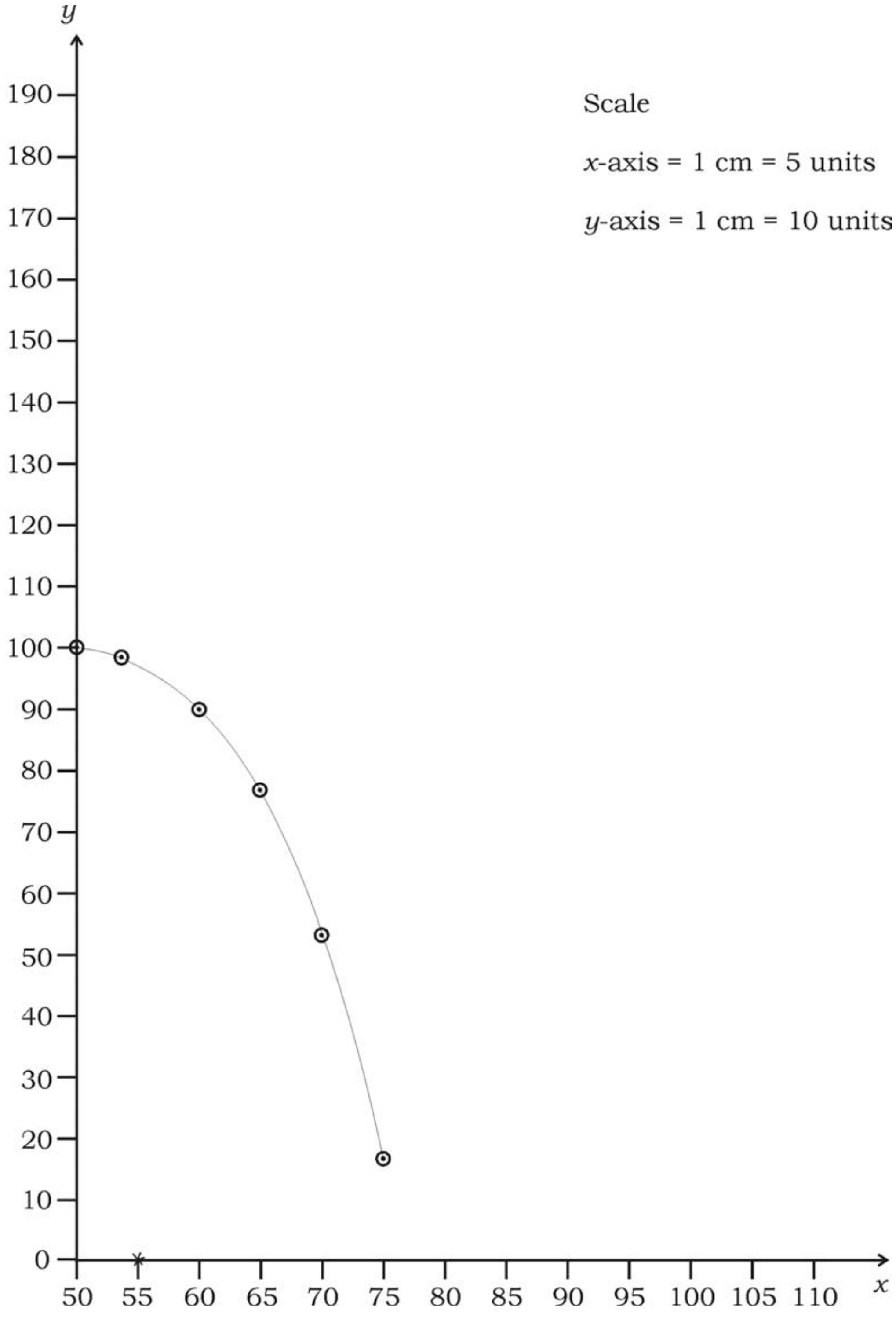
∴ Coordinate points are

(50, 100) (55, 98) (60, 90)

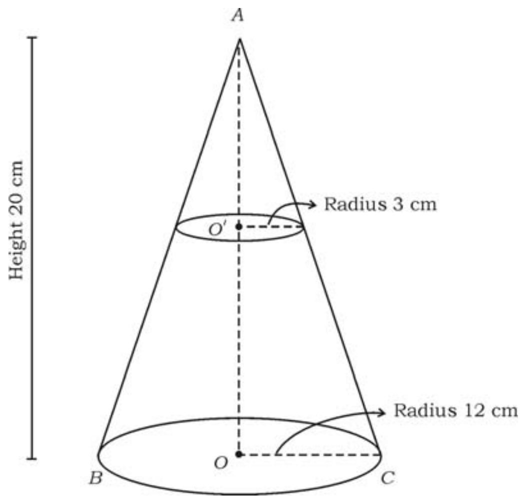
(65, 78) (70, 54) (75, 16)

Table — 1

Plotting the ogive — 2   3

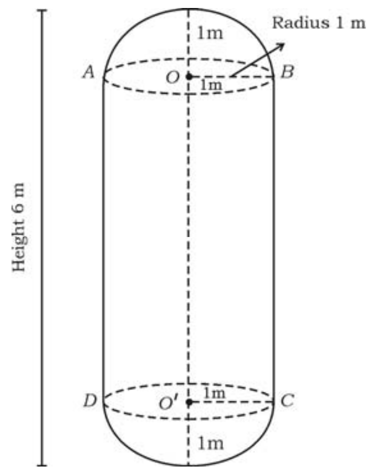
Qn. Nos.	Value Points	Marks allotted
		

Qn. Nos.	Value Points	Marks allotted
46.	A cone is having its base radius 12 cm and height 20 cm. If the top of this cone is cut into form of a small cone of base radius 3 cm is removed, then the remaining part of the solid cone become a frustum. Calculate the volume of the frustum. 3	



OR

A milk tank is in the shape of a cylinder with hemispheres of same radii attached to both ends of it as shown in figure. If the total height of the tank is 6 m and the radius is 1 m, calculate the maximum quantity of milk filled in the tank in litres. ($\pi = \frac{22}{7}$)

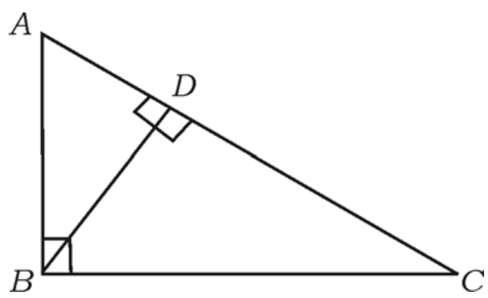


Qn. Nos.	Value Points		Marks allotted
	Ans. :		
	Given $r_1 = 12$ cm, $r_2 = 3$ cm, $h_1 = 20$ cm, $h_2 = ?$		
	We know $\frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow \frac{12}{3} = \frac{20}{h_2} \Rightarrow h_2 = 5$ cm	1	
	$\therefore$ Volume of the frustum		
	$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$	½	
	$= \frac{1}{3} \times \frac{22}{7} \times 15\left((12)^2 + (3)^2 + (12)(3)\right)$	½	
	$= \frac{110}{7} \times (144 + 9 + 36)$	½	
	$= \frac{110}{7} \times 189$		
	$= 2970 \text{ cm}^3$.		
	$\therefore$ Volume of Frustum is 2970 cm^3.	½	3
	OR		
	Radius of hemisphere $r = 1$ m		
	Radius of cylinder $r = 1$ m		
	Height of cylinder $h = 4$ m		
	Volume of solid = Volume of cylinder + 2 (volume of hemisphere)	½	
	$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$		
	$= \pi r^2 h + \frac{4}{3}\pi r^3$		
	$= \pi r^2\left[h + \frac{4}{3}r\right]$		
	$= \frac{22}{7} \times (1)^2\left[4 + \frac{4}{3}(1)\right]$	½	
	$= \frac{22}{7} \times \frac{16}{3} \text{ m}^3$	½	

Qn. Nos.	Value Points	Marks allotted
	$= \dfrac{352}{21} \times (100)^3 \ cm^3$ 1 m = 100 cm ½	
	$= \dfrac{352 \times 1000000}{21 \times 1000}$ litres ½	
	$= \dfrac{352000}{21}$	
	= 16,761·9 litres ½	
	∴ Capacity of milk tank is 16,761·9 litres	3
V. 47.	The sum of the fourth and eighth terms of an arithmetic progression is 24 and the sum of the sixth and tenth terms is 44. Find the first three terms of the Arithmetic progression. 4	
	Ans. :	
	$a_4 + a_8 = 24$	
	$a + 3d + a + 7d = 24$	
	$2a + 10d = 24$	
	$a + 5d = 12$ ... (i) 1	
	$a_6 + a_{10} = 44$	
	$a + 5d + a + 9d = 44$	
	$2a + 14d = 44$	
	$a + 7d = 22$ ... (ii) 1	
	(ii) — (i)	
	$a + 7d = 22$ Substitute $d = 5$ in (i) ½	
	$a + 5d = 12$	
	(–) (–) (–) $a + 5(5) = 12$	
	$2d = 10$ $a + 25 = 12$	
	$d = \dfrac{10}{2}$ $a = 12 - 25$	
	$\boxed{d = 5}$ $\boxed{a = -13}$ ½	

Qn. Nos.	Value Points		Marks allotted
	∴ Three terms of Arithmetic progression is		
	a, $a + d$ $a + 2d$		
	-13, $-13 + 5$, $-13 + 10$	½	
	-13, -8, -3	½	4
48.	Prove that "in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".	4	

Ans. :



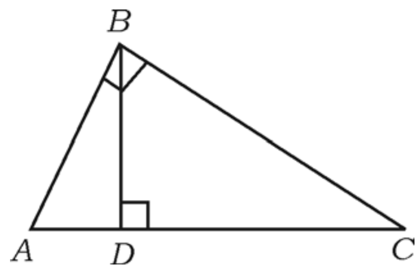
Data : In $\triangle ABC$, $\angle ABC = 90°$ ½

To prove : $AB^2 + BC^2 = AC^2$ ½

Construction : Draw $BD \perp AC$ ½

Proof :

Statement	*Reason*	
Compare $\triangle ABC$ and $\triangle ADB$		
$\angle ABC = \angle ADB = 90°$		
$\angle BAD$ is common	Data and construction	
∴ $\triangle ABC \sim \triangle ADB$	Equiangular triangles	
$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$	*AA* similarity	1

Qn. Nos.	Value Points			Marks allotted
	$\therefore \quad AB^2 = AC \times AD \quad \ldots$ (i)			
	Compare $\triangle ABC$ and $\triangle BDC$			
	$\angle ABC = \angle BDC = 90°$			
	$\angle ACB =$ is common	Data and construction		
	$\therefore \quad \triangle ABC \sim \triangle BDC$	Equiangular triangles	1	
	$\Rightarrow \quad \frac{BC}{DC} = \frac{AC}{BC}$			
	$BC^2 = AC \times DC \quad \ldots$ (ii)	AA similarity		
	(i) + (ii)			
	$AB^2 + BC^2 = AC \times AD + AC \times DC$			
	$= AC (AD + DC)$			
	$= AC \times AC$	$AD + DC = AC$	½	
	$AB^2 + BC^2 = AC^2$			4
	Alternate method :			
				
	In a $\triangle ABC$, $\angle ABC = 90°$		½	
	We need to prove that $AC^2 = AB^2 + BC^2$		½	
	Let us draw $BD \perp AC$		½	
	Now, $\Delta ADB \sim \Delta ABC$ (equiangular triangle)			
	So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)		1	

Qn. Nos.	Value Points		Marks allotted
	$\boxed{AD \times AC = AB^2}$ ... (i)		
	Also $\Delta\, BDC \sim \Delta\, ABC$ (equiangular triangle)		
	$\frac{CD}{BC} = \frac{BC}{AC}$		
	$CD \times AC = BC^2$ ... (ii)	1	
	(i) + (ii)		
	$AB^2 + BC^2 = AD \times AC + CD \times AC$		
	$AB^2 + BC^2 = AC\,(AD + DC)$		
	$AB^2 + BC^2 = AC \times AC$		
	$AB^2 + BC^2 = AC^2$	½	4
49.	Solve graphically :	4	
	$2x + y = 8$		
	$x - y = 1$		
	Ans. :		
	$2x + y = 8$		
	$y = 8 - 2x$		

x	1	2	3	4	5
y	6	4	2	0	– 2

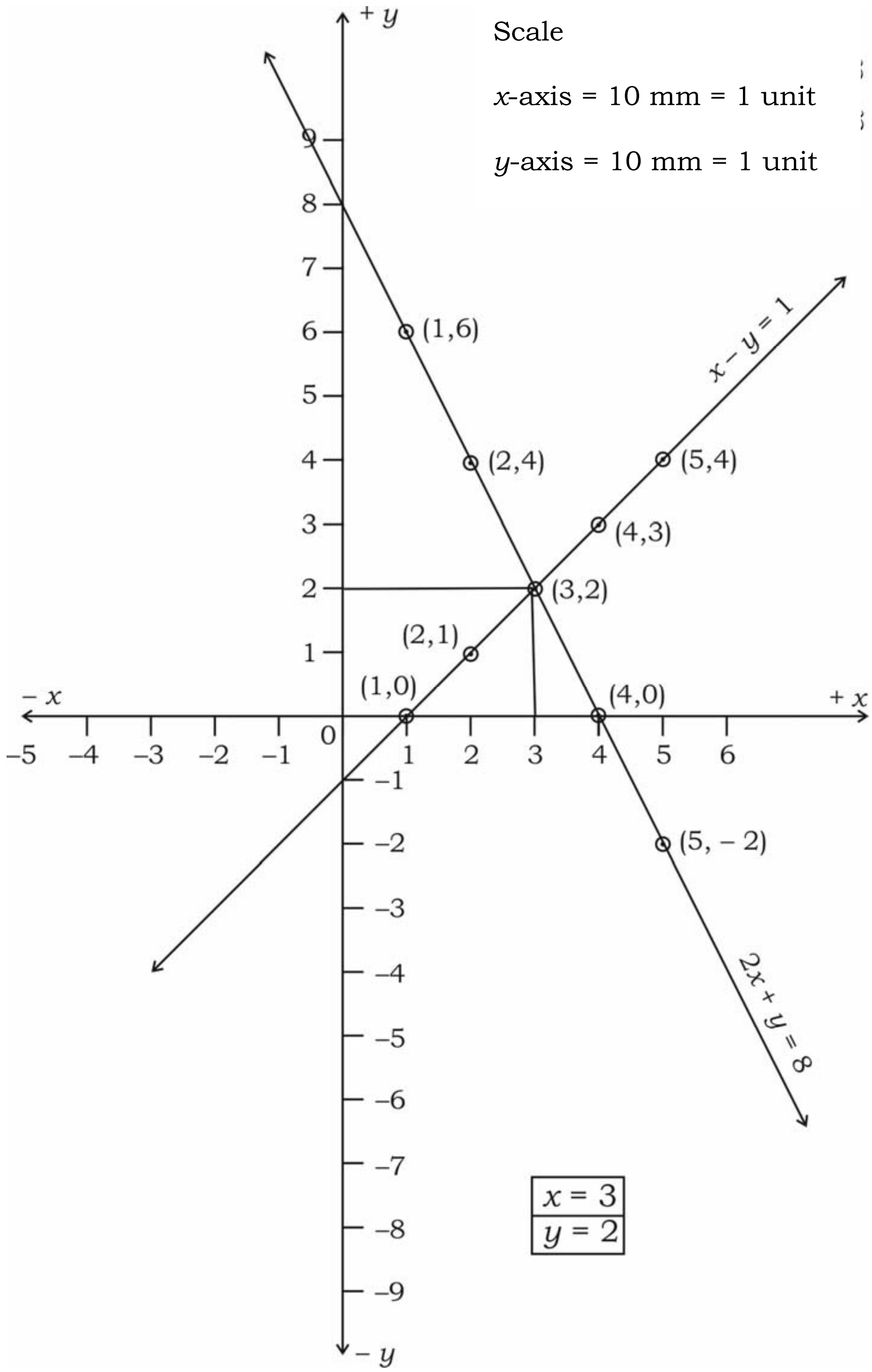
$x - y = 1$

$y = x - 1$

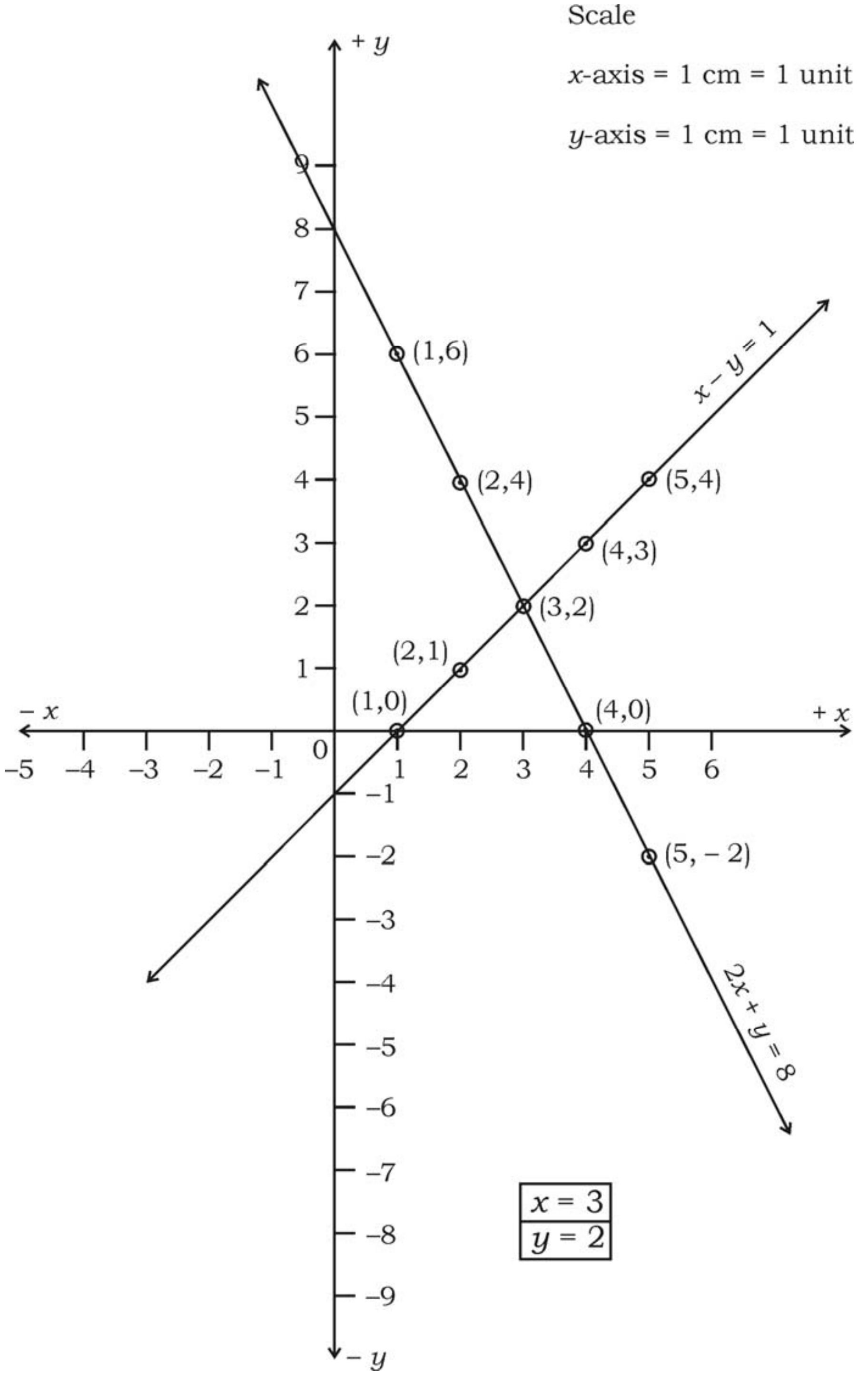
x	1	2	3	4	5
y	0	1	2	3	4

Note : Any *two* points for each equation may be given marks.

Qn. Nos.	Value Points		Marks allotted
	Tables —	2	
	Drawing the lines of two linear equations —	1	
	Identifying the values of x and y —	1	4



Qn. Nos.	Value Points	Marks allotted



50. The ages of two students *A* and *B* are 19 years and 15 years respectively. Find how many years it will take so that the products of their ages becomes equal to 480. 4

Qn. Nos.	Value Points		Marks allotted
	OR		
	If the quadratic equation $(b-c)\,x^2 + (c-a)\,x + (a-b) = 0$ has equal roots, then show that $2b = a + c$.		
	Ans. :		
	Let the required years are x		
	After x years the age of A is $= x + 19$		
	After x years the age of B is $= x + 15$	½	
	The product of their ages is 480	½	
	i.e. $(x+19)(x+15) = 480$		
	$x^2 + 19x + 15x + 285 = 480$		
	$x^2 + 19x + 15x + 285 - 480 = 0$	½	
	$x^2 + 34x - 195 = 0$	½	
	Last term : $-195 = +39 \times -5$		
	Middle term : $+34 = +39 - 5$	½	
	$\therefore\ x^2 + 39x - 5x - 195 = 0$		
	$x(x+39) - 5(x+39) = 0$	½	
	$(x-5)(x+39) = 0$		
	$x - 5 = 0$ $\quad$ $x + 39 = 0$		
	$x = +5$ $\quad$ $x = -39$	½	
	$\therefore$ After 5 years the product of their age is 480	½	4
	OR		
	$(b-c)\,x^2 + (c-a)\,x + (a-b) = 0$		
	$ax^2 + bx + c = 0$		
	$a = (b-c)$ $\quad$ $b = (c-a)$ $\quad$ $c = (a-b)$	½	

Qn. Nos.	Value Points	Marks allotted
	Roots are equal $\Delta = 0$ ½	
	Discriminant $\Delta = b^2 - 4ac$	
	$\therefore \quad 0 = b^2 - 4ac$	
	$b^2 - 4ac = 0$ ½	
	$(c-a)^2 - 4[(b-c)(a-b)] = 0$ ½	
	$c^2 - 2ac + a^2 - 4[ab - ac - b^2 + cb] = 0$ ½	
	$c^2 - 2ac + a^2 - 4ab + 4ac + 4b^2 - 4cb = 0$	
	$a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac = 0$ ½	
	$(a - 2b + c)^2 = 0$ ½	
	$a - 2b + c = 0$	
	$a + c = 2b$	
	$\therefore \quad \boxed{2b = a + c}$ ½	