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ಕರ್ನಾಟಕ ಪ್ರೌಢ ಶಿಕ್ಷಣ ಪರೀಕ್ಷಾ ಮಂಡಳಿ, ಮಲ್ಲೇಶ್ವರಂ, ಬೆಂಗಳೂರು – 560 003

KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD, MALLESWARAM, BANGALORE – 560 003

ಎಸ್.ಎಸ್.ಎಲ್.ಸಿ. ಪರೀಕ್ಷೆ, ಜೂನ್ — 2018

S. S. L. C. EXAMINATION, JUNE, 2018

ಮಾದರಿ ಉತ್ತರಗಳು

MODEL ANSWERS

ದಿನಾಂಕ : 21. 06. 2018] ಸಂಕೇತ ಸಂಖ್ಯೆ : **81-E**

Date : 21. 06. 2018] CODE NO. : **81-E**

ವಿಷಯ : ಗಣಿತ

Subject : MATHEMATICS

(ಹೊಸ ಪಠ್ಯಕ್ರಮ / New Syllabus)

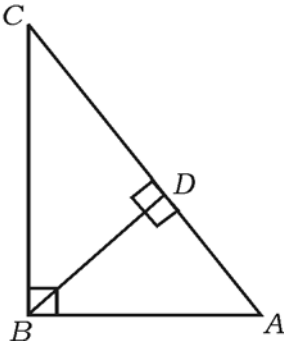
(ಪುನರಾವರ್ತಿತ ಖಾಸಗಿ ಅಭ್ಯರ್ಥಿ / Private Repeater)

(ಇಂಗ್ಲಿಷ್ ಭಾಷಾಂತರ / English Version)

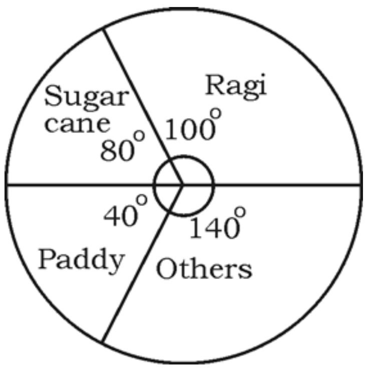
[ಗರಿಷ್ಠ ಅಂಕಗಳು : **100**

[Max. Marks : 100

Qn. Nos.	Ans. Key	Value Points	Marks allotted
I. 1.		A and B are two sets, such that $n(A) = 37$, $n(B) = 26$ and $n(A \cup B) = 51$; then $n(A \cap B)$ is (A) 12 (B) 63 (C) 14 (D) 25 *Ans.* :	
	(A)	12	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
2.		Geometric mean between $\frac{1}{2}$ and $\frac{1}{8}$ is	
		(A) 16 (B) $\frac{1}{16}$	
		(C) $\frac{1}{4}$ (D) 4	
		Ans. :	
	(C)	$\frac{1}{4}$	1
3.		HCF of any two prime numbers is	
		(A) a prime number (B) a composite number	
		(C) an odd number (D) an even number	
		Ans. :	
	(C)	an odd number	1
4.		If $f(x) = 2x^3 + 3x^2 - 11x + 6$ then the value of $f(-1)$ is	
		(A) 0 (B) – 10	
		(C) – 18 (D) 18	
		Ans. :	
	(D)	18.	1
5.		In $\triangle ABC$, $\angle ABC = 90°$, $BD \perp AC$ if $BD = 8$ cm and $AD = 4$ cm then the length of CD is	
			
		(A) 16 cm (B) 4 cm	
		(C) 64 cm (D) 12 cm	
		Ans. :	
	(A)	16 cm	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
6.		$\dfrac{\sin(90^\circ - \theta)}{\cos(90^\circ - \theta)}$ where 'θ' is acute, is equal to	
		(A) $\sec\theta$ (B) $\cot\theta$	
		(C) $\tan\theta$ (D) $\operatorname{cosec}\theta$	
		Ans. :	
	(B)	$\cot\theta$	1
7.		The co-ordinates of the mid-point of the line segment joining the points (2, 3) and (4, 7) are	
		(A) (– 3, – 5)	
		(B) (1, 2)	
		(C) (3, 5)	
		(D) (6, 10)	
		Ans. :	
	(C)	(3, 5)	1
8.		Formula used to find the surface area of a sphere whose radius 'r' units is	
		(A) πr^2 (B) $2\pi r^2$	
		(C) $3\pi r^2$ (D) $4\pi r^2$	
		Ans. :	
	(D)	$4\pi r^2$.	1

Qn. Nos.	Value Points	Marks allotted
II.	Answer the following : $6 \times 1 = 6$	
	(Question Numbers 9 to 14, give full marks to direct answers)	
9.	A boy has 2 pants and 4 shirts. How many different pairs of a pant and a shirt can he dress up with ?	
	Ans. :	
	Number of ways of pairing a pant and a shirt = $2 \times 4 = 8$	1
10.	Write sample space for the random experiment 'tossing two fair coins simultaneously once'.	
	Ans. :	
	$S = \{HH, TT, HT, TH\}$	1
11.	The given pie chart shows the annual agricultural yield of different crops in a certain place. If the total production is 3600 tons, what is the yield of Ragi in tons ? 	
	Ans. :	
	Yield of Ragi = $\cancel{3600}^{100} \times \frac{10\cancel{0}}{36\cancel{0}}$ ½	
	= 1000 ½	1

Qn. Nos.	Value Points		Marks allotted
12.	If $(x + 3)$ is one of the factor of $f (x) = x^2 + 5x + 6$, find the other factor.		
	Ans. :		
	Method 1 : Factor method		
	$x^2 + 5x + 6$ (6 → 3, 2)		
	$= x^2 + 3x + 2x + 6$		
	$= x (x + 3) + 2 (x + 3)$	½	
	$= (x + 3) (x + 2)$		
	The other factor is $(x + 2)$	½	1
	Method 2 : Division method		
	$x + 3)\ x^2 + 5x + 6\ (x + 2$	½	
	$x^2 + 3x$		
	(–)		
	$2x + 6$		
	$2x + 6$		
	(–) (–)		
	0		
	The other factor is $(x + 2)$	½	1
13.	What are concentric circles ?		
	Ans. :		
	Circles having the same centre but different radii are called concentric circles.		1
14.	Two straight lines are perpendicular to each other. If the slope of one line is $\frac{1}{\sqrt{3}}$, find the slope of the other line.		
	Ans. :		
	$m_1 m_2 = -1$	½	
	$\frac{1}{\sqrt{3}} \times m_2 = -1$		
	$\therefore \quad m_2 = -\sqrt{3}$		
	Slope of the other line $= -\sqrt{3}$.	½	1

Qn. Nos.	Value Points		Marks allotted
III. 15.	If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$ are the subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, verify $(A \cap B)^{l} = A^{l} \cup B^{l}$.		
	Ans. :		
	$A \cap B = \{2, 3\}$	½	
	$(A \cap B)^{l} = U - (A \cap B)$		
	$= \{1, 4, 5, 6, 7, 8\}$ ... i)	½	
	$A^{l} = \{4, 5, 6, 7, 8\}$		
	$B^{l} = \{1, 6, 7, 8\}$	½	
	$A^{l} \cup B^{l} = \{1, 4, 5, 6, 7, 8\}$ ... ii)		
	From (i) and (ii)		
	$(A \cap B)^{l} = A^{l} \cup B^{l}$	½	2
16.	Find the sum of infinite terms of the geometric series $2 + \frac{2}{3} + \frac{2}{9} + \ldots\ldots$.		
	Ans. :		
	$a = 2, \quad r = \frac{1}{3}, \quad S_{\infty} = ?$		
	$S_{\infty} = \frac{a}{1-r}$	½	
	$= \frac{2}{1-\frac{1}{3}}$	½	
	$= \frac{2}{\frac{3-1}{3}}$		
	$= \frac{2}{\frac{2}{3}}$	½	
	$= \cancel{2} \times \frac{3}{\cancel{2}}$		
	$= 3$	½	2

Qn. Nos.	Value Points		Marks allotted
17.	Prove that $2+\sqrt{3}$ is an irrational number.		
	Ans. :		
	Let us assume $2+\sqrt{3}$ is a rational number.		
	$\Rightarrow \quad 2+\sqrt{3} = \frac{p}{q}$ where $p,\ q \in z,\ \ q \neq 0$	½	
	$\Rightarrow \quad \sqrt{3} = \frac{p-2q}{q}$		
	$\Rightarrow \quad \sqrt{3}$ is a rational number		
	$\because \quad \frac{p-2q}{q}$ is rational.	½	
	But $\sqrt{3}$ is not a rational number. This leads to a contradiction.	½	
	$\therefore$ Our assumption that $2+\sqrt{3}$ is a rational number is wrong.		
	$\therefore \quad 2+\sqrt{3}$ is an irrational number.	½	2
18.	Find the number of diagonals that can be drawn in an octagon.		
	Ans. :		
	An octagon has 8 vertices $\quad \therefore \quad n = 8$		
	$\therefore$ Total number of sides and diagonals $= {}^{8}C_{2}$	½	
	${}^{n}C_{2} = \frac{n(n-1)}{2} \quad \Rightarrow \quad {}^{8}C_{2} = \frac{8(8-1)}{2}$	½	
	$= 4 \times 7$		
	$= 28$	½	
	28 lines includes 8 sides.		
	$\therefore$ Number of diagonals $= 28 - 8$		
	$= 20$	½	2

Qn. Nos.	Value Points		Marks allotted
	Alternate method :		
	Number of diagonals in a polygon of n sides $= \dfrac{n(n-3)}{2}$	½	
	In an octagon $n = 8$		
	$\therefore$ Number of diagonals $= \dfrac{\overset{4}{\cancel{8}}(8-3)}{\cancel{2}}$	½	
	$= 4 \times 5$	½	
	$= 20$	½	2
	Any other correct alternate method may be given marks.		
19.	Find the sum of all two digit natural numbers which are divisible by 5.		
	Ans. :		
	Two-digit numbers which are divisible by 5 = 10, 15, 20, ... 95		
	Sum of all two-digit numbers = 10 + 15 + 20 + ... + 95		
	$a = 10, \quad d = 5, \quad T_n = 95$		
	$\therefore \quad T_n = a + (n-1)d$ $95 = 10 + (n-1)5$ $(n-1) = \dfrac{85}{5}$ $(n-1) = 17 \qquad \therefore \quad n = 18$	1	
	Method 1 :		
	Sum of n natural numbers $S_n = \dfrac{n}{2}[2a + (n-1)d]$ $S_{18} = \dfrac{18}{2}[2 \times 10 + (18-1)5]$ $= 9(20 + 85)$ $= 9 \times 105$	1	
	$S_{18} = 945$		2
	OR		

Qn. Nos.	Value Points		Marks allotted
	$n = 18, \quad a = 10, \quad l = 95$ $\therefore \quad S_n = \frac{n(a+l)}{2}$ $S_{18} = \frac{\overset{9}{\cancel{18}}(10+95)}{\cancel{2}} = 9 \times 105 = 945.$	1	
	Alternate method :		
	$= 5(2+3+4+ \dots + 19)$	½	
	$= 5(\Sigma 19 - 1)$	½	
	$= 5(190 - 1)$	½	
	$= 5 \times 189 = 945.$	½	2
20.	Find how many 4 digit numbers can be formed by using the digits 1, 2, 3, 4, 5 without repetition ? How many of these are less than 2000 ?		
	OR		
	If $2({}^{n}P_2) + 50 = {}^{2n}P_2$, find the value of *n*.		
	Ans. :		
	Number of 4-digit numbers $= {}^{5}P_4 = 5 \times 4 \times 3 \times 2$ $= 120$	½	
	4-digit numbers which are less than 2000		

Thousand's place	Hundred's place	Ten's place	Unit place
${}^{1}P_1$	${}^{4}P_1$	${}^{3}P_1$	${}^{2}P_1$

(1)

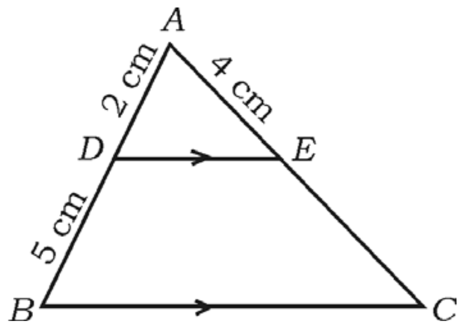
	$= 1 \times 4 \times 3 \times 2$		
	$= 24$ numbers.	½	
	OR		

Qn. Nos.	Value Points		Marks allotted
	4-digit numbers which are less than 2000 = $1 \times {}^{4}P_{3}$	½	
	$= 1 \times 4 \times 3 \times 2$	½	
	$= 24$	½	2
	OR		
	$2\left({}^{n}P_{2}\right) + 50 = {}^{2n}P_{2}$		
	$2n(n-1) + 50 = 2n(2n-1)$	½	
	$2n^2 - \cancel{2n} + 50 = 4n^2 - \cancel{2n}$	½	
	$4n^2 - 2n^2 = 50$		
	$2n^2 = 50$	½	
	$n^2 = 25$ $\quad\therefore\quad n = \pm 5$		
	$\therefore\quad n = 5$	½	2
21.	Two unbiased dice whose faces are numbered 1 to 6 are rolled once. Find the probability of getting a sum equal to 7 on their top faces.		
	Ans. :		
	Total number of possible outcomes = $6 \times 6 = 36$		
	$\therefore\quad n(s) = 36$	½	
	Event of getting a sum equal to 7 = $A = \left\{ \begin{matrix} (1,6)\ (2,5)\ (3,4) \\ (4,3)\ (5,2)\ (6,1) \end{matrix} \right\}$		
	$n(A) = 6$	½	
	Probability of getting the event A = $P(A) = \dfrac{n(A)}{n(S)}$	½	
	$= \dfrac{6}{36}$	½	
	$= \dfrac{1}{6}$		2

Qn. Nos.	Value Points		Marks allotted
22.	Rationalise the denominator and simplify : $\frac{3\sqrt{2}}{\sqrt{5}-\sqrt{2}}$.		
	Ans. :		
	Rationalising factor of $\sqrt{5}-\sqrt{2}$ is $\sqrt{5}+\sqrt{2}$		
	$= \frac{3\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$	½	
	$= \frac{3\sqrt{2}\,(\sqrt{5}+\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$	½	
	$= \frac{3\sqrt{10}+3(2)}{5-2}$	½	
	$= \frac{\cancel{3}(\sqrt{10}+2)}{\cancel{3}}$		
	$= \sqrt{10}+2$.	½	2
23.	Simplify $(\sqrt{75}-\sqrt{45})(\sqrt{20}+\sqrt{12})$.		
	Ans. :		
	$(\sqrt{75}-\sqrt{45})(\sqrt{20}+\sqrt{12})$		
	$= (\sqrt{25\times3}-\sqrt{9\times5})(\sqrt{4\times5}+\sqrt{4\times3})$	½	
	$= (5\sqrt{3}-3\sqrt{5})(2\sqrt{5}+2\sqrt{3})$		
	$= 5\sqrt{3}(2\sqrt{5}+2\sqrt{3})-3\sqrt{5}(2\sqrt{5}+2\sqrt{3})$	½	
	$= 10\sqrt{15}+10(3)-6(5)-6\sqrt{15}$	½	
	$= 10\sqrt{15}+30-30-6\sqrt{15}$		
	$= 4\sqrt{15}$.	½	2

Qn. Nos.	Value Points		Marks allotted
24.	Find the quotient and remainder by using synthetic division :		
	$(3x^3 - 2x^2 + 7x - 5) \div (x - 3)$		
	OR		
	Verify whether $(x - 2)$ is a factor of $f (x) = x^3 - 3x^2 + 6x - 20$ by using factor theorem.		
	Ans. :		
	3 | 3 −2 7 −5 0 9 21 84 3 7 28 | 79	1	
	$\therefore$ Quotient $= 3x^2 + 7x + 28$	½	
	Remainder $= 79$.	½	2
	OR		
	Let $f (x) = x^3 - 3x^2 + 6x - 20$		
	If $(x - 2)$ is a factor of $f (x)$,		
	then $f (2) = 0$	½	
	Now $f (2) = 2^3 - 3 (2)^2 + 6 (2) - 20$	½	
	$= 8 - 12 + 12 - 20$		
	$= - 12$		
	$\therefore f (2) \neq 0$	½	
	$\therefore x - 2$ is not a factor of $x^3 - 3x^2 + 6x - 20$.	½	2

Qn. Nos.	Value Points		Marks allotted
25.	In $\Delta\ ABC$, $DE \parallel BC$, if $AD = 2$ cm, $DB = 5$ cm and $AE = 4$ cm, find AC.		



Ans. :

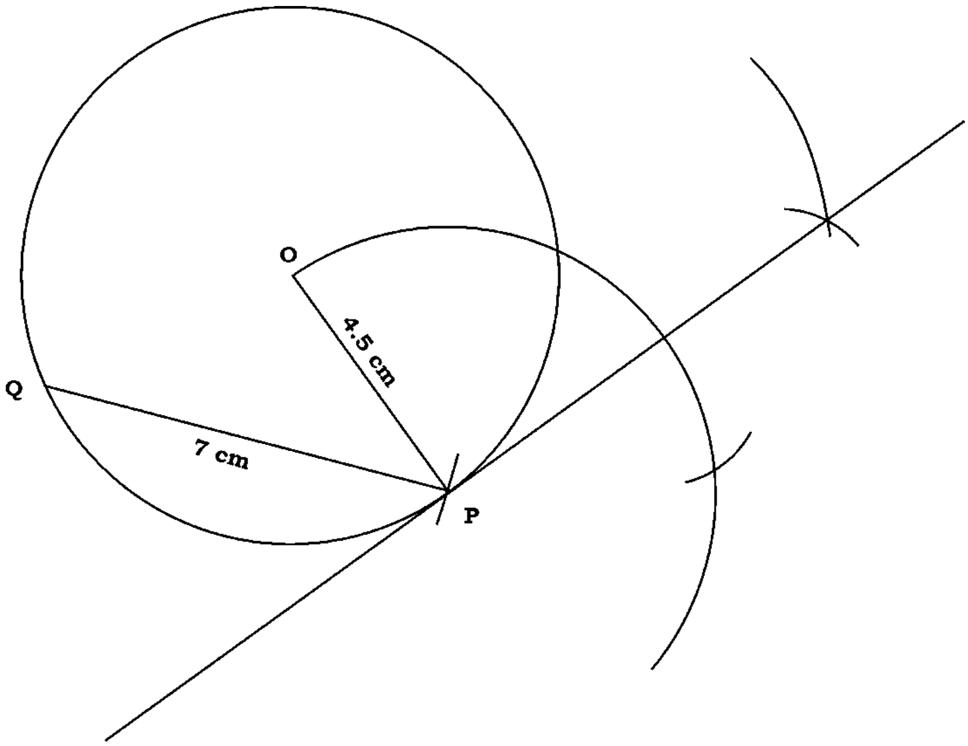
In $\Delta\ ABC$, $DE \parallel BC$

			Marks allotted
$\therefore \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$	BPT	½	
$\dfrac{2}{5} = \dfrac{4}{EC}$		½	
$EC = \dfrac{\cancel{4}^{2} \times 5}{\cancel{2}} = 10$ cm		½	
$\therefore \quad AC = AE + EC$			
$= 4 + 10$			
$= 14$ cm.		½	2

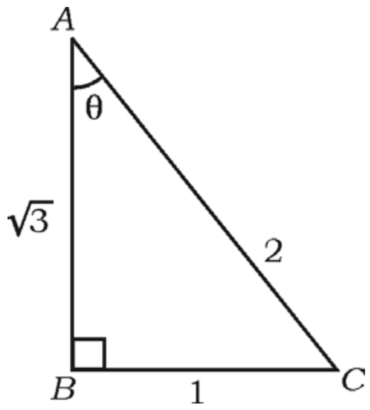
Alternate method :

In $\Delta\ ABC$, $DE \parallel BC$

			Marks allotted
$\therefore \quad \dfrac{AD}{AB} = \dfrac{AE}{AC}$	Cor. BPT	½	
$\dfrac{2}{2+5} = \dfrac{4}{AC}$		½	
$\therefore \quad AC = \dfrac{7 \times \cancel{4}^{2}}{\cancel{2}}$		½	
$= 14$ cm.		½	2

Qn. Nos.	Value Points	Marks allotted
26.	Draw a circle of radius 4·5 cm and a chord *PQ* of length 7 cm in it. Construct a tangent at *P*.	
	Ans. :	
	r = 4·5 cm Chord *PQ* = 7 cm	
		
	Circle — ½	
	Chord — ½	
	Tangent — 1	2
27.	Find the distance between the co-ordinates of the points (2, 4) and (8, 12) by using distance formula.	
	Ans. :	
	Coordinates of	
	(x_1 y_1)	
	Point *A* = (2, 4)	
	(x_2 y_2)	
	Point *B* = (8, 12) ½	

Qn. Nos.	Value Points		Marks allotted
	Distance between the points		
	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	½	
	$= \sqrt{(8-2)^2 + (12-4)^2}$		
	$= \sqrt{6^2 + 8^2}$	½	
	$= \sqrt{36 + 64}$		
	$= \sqrt{100}$		
	$= 10.$	½	2
28.	In a hockey match team *'A'* scored one goal less than twice the number of goals scored by team *'B'*. If the product of the number of goals scored by both the teams is 15, find the number of goals scored by each team.		
	Ans. :		
	Let the goals scored by team A be x		
	and goals scored by team B be y.		
	$\therefore \quad x = (2y - 1)$	½	
	Product of the goals scored by both teams = 15		
	$xy = 15$		
	$(2y - 1)\,y = 15$	½	
	$2y^2 - y - 15 = 0$ $(2)(-15) = -30$; -6, 5		
	$2y^2 - 6y + 5y - 15 = 0$		
	$2y(y-3) + 5(y-3) = 0$		
	$(y-3)(2y+5) = 0$		
	$\therefore \quad y = 3$	½	
	If $y = 3$, then $x = 2 \times 3 - 1 = 6 - 1 = 5$	½	
	$\therefore$ Goals scored by team A = 5		
	Goals scored by team B = 3.		2

Qn. Nos.	Value Points	Marks allotted
29.	In the given ⊿ *ABC*, 'θ' is acute. Write the values of the following trigonometric ratios related to θ :	
	(a) sin θ	
	(b) cos θ	
	(c) cosec θ	
	(d) sec θ.	
		
	Ans. :	
	a) $\sin\theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{BC}{AC} = \frac{1}{2}$ ½	
	b) $\cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ ½	
	c) $\text{cosec}\,\theta = \frac{1}{\sin\theta} = 2$ ½	
	d) $\sec\theta = \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}}$. ½	2
	Direct answers may be given marks.	

Qn. Nos.	Value Points		Marks allotted
30.	Draw a plan by using the information given below : (Scale 20 metres = 1 cm)		

	Metre to C	
	140	
80 to D	90	
	60	60 to B
30 to E	20	
	From A	

Ans. :

$20 \text{ m} = \frac{20}{20} = 1 \text{ cm}$

$60 \text{ m} = \frac{60}{20} = 3 \text{ cm}$

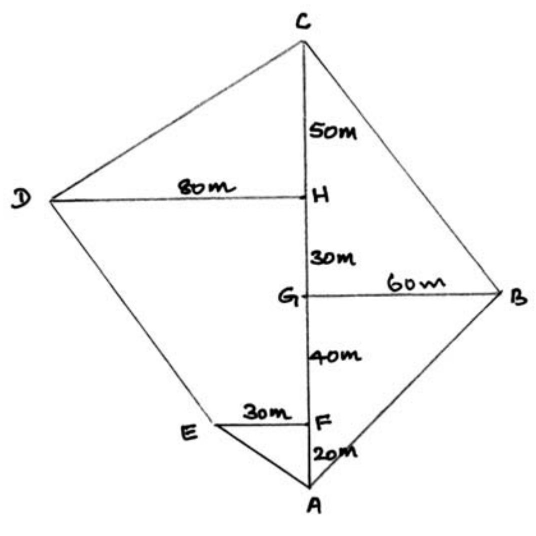
$90 \text{ m} = \frac{90}{20} = 4{\cdot}5 \text{ cm}$

$140 \text{ m} = \frac{140}{20} = 7 \text{ cm}$

$60 \text{ m} = \frac{60}{20} = 3 \text{ cm}$

$80 \text{ m} = \frac{80}{20} = 4 \text{ cm}$

$30 \text{ m} = \frac{30}{20} = 1{\cdot}5 \text{ cm}$ ½



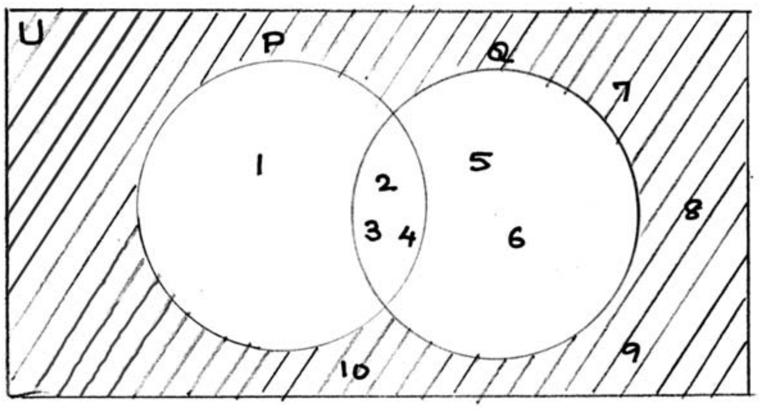
1½

2

Qn. Nos.	Value Points	Marks allotted

31. If $P = \{1, 2, 3, 4\}$, $Q = \{2, 3, 4, 5, 6\}$ are the subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then draw Venn diagram to represent $(P \cup Q)^{l}$.

Ans. :



(PUQ)'

2

32. Write the formula used to find the following :

(a) Sum of first 'n' natural numbers

(b) Harmonic mean between a and b ($a > b$).

Ans. :

a) $\Sigma n = \frac{n(n+1)}{2}$ 1

b) Harmonic mean $(H) = \frac{2ab}{a+b}$. 1

2

33. Write the values of the following :

(a) ${}^{100}P_0$

(b) ${}^{10}C_1$.

Ans. :

a) ${}^{100}P_0 = 1$ 1

b) ${}^{10}C_1 = 10$ 1

2

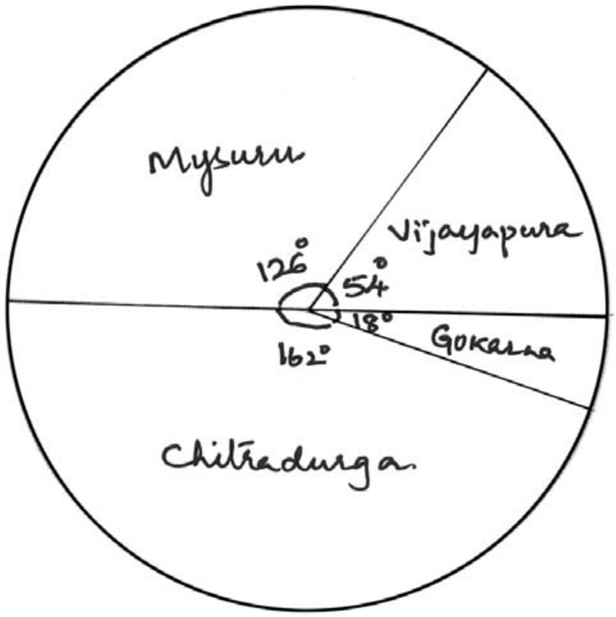
Qn. Nos.	Value Points	Marks allotted

34. Draw a pie chart to represent the survey carried out in the class regarding places of visit for excursion and the number of students who opted each place.

Places	*Mysuru*	*Vijayapura*	*Gokorna*	*Chitradurga*
Number of students	14	6	2	18

Ans. :

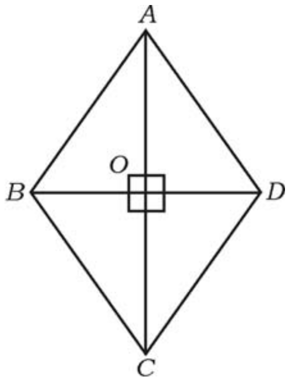
Places	*No. of students*	*Central angle*
Mysuru	14	$\frac{14}{\cancel{40}} \times \overset{9}{\cancel{360}} = 126^\circ$
Vijayapura	6	54°
Gokorna	2	18°
Chitradurga	18	162°
	40	

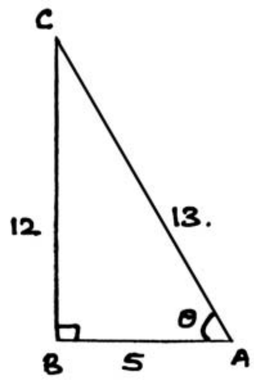


Calculation — ½

Pie chart — 1½ | 2

Qn. Nos.	Value Points		Marks allotted
35.	Find the product of $\sqrt[3]{2}$ and $\sqrt[4]{3}$.		
	Ans. :		
	LCM of the order of surds = 12	½	
	$\therefore \quad \sqrt[3]{2} \Rightarrow \sqrt[4\times 3]{2^4} = \sqrt[12]{16}$	½	
	$\sqrt[4]{3} \Rightarrow \sqrt[3\times 4]{3^3} = \sqrt[12]{27}$	½	
	$\therefore \quad \sqrt[3]{2} \times \sqrt[4]{3} = \sqrt[12]{16} \times \sqrt[12]{27}$		
	$= \sqrt[12]{16 \times 27}$		
	$= \sqrt[12]{432}.$	½	
	For any other alternative method give marks.		2
36.	Determine the nature of the roots of the equation $2x^2 - 5x - 1 = 0$.		
	Ans. :		
	$a = 2, \quad b = -5, \quad c = -1$	½	
	$\therefore \quad \Delta = b^2 - 4ac$		
	$= (-5)^2 - 4(2)(-1)$	½	
	$= 25 + 8$		
	$= 33.$	½	
	$\therefore \quad \Delta > 0$, Roots are real and distinct.	½	2

Qn. Nos.	Value Points		Marks allotted
37.	In Rhombus *ABCD*, prove that $4AB^2 = AC^2 + BD^2$.		
			
	Ans. :		
	In $\triangle AOB$, $A\hat{O}B = 90°$		
	$\therefore \quad AB^2 = AO^2 + BO^2$	½	
	But $AO = \frac{1}{2}AC$, $BO = \frac{1}{2}BD$	½	
	$\therefore \quad AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$		
	$= \frac{AC^2}{4} + \frac{BD^2}{4}$	½	
	$= \frac{AC^2 + BD^2}{4}$		
	$\therefore \quad 4AB^2 = AC^2 + BD^2$.	½	2
	Any correct alternate method may be given marks.		
38.	Find the remainder when $P(x) = x^3 + 3x^2 - 5x + 8$ is divided by $(x - 3)$ by remainder theorem.		
	Ans. :		
	By remainder theorem, the required remainder is $P(3)$	½	
	$\therefore \quad P(3) = (3)^3 + 3(3)^2 - 5(3) + 8$	½	
	$= 27 + 27 - 15 + 8$		
	$= 62 - 15$		
	$= 47$	½	
	$\therefore$ The remainder $P(3) = 47$.	½	2

Qn. Nos.	Value Points		Marks allotted
39.	Find the distance between origin and the point (– 8, 15).		
	Ans. :		
	Distance between origin and (x, y) = $\sqrt{x^2 + y^2}$	½	
	Here (x, y) = (– 8, 15)		
	$\therefore \quad d = \sqrt{(-8)^2 + 15^2}$	½	
	$= \sqrt{64 + 225}$		
	$= \sqrt{289}$	½	
	d = 17 units.	½	2
40.	If $\cos\theta = \dfrac{5}{13}$, find the value of $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.		
	Ans. :		
	Given $\cos\theta = \dfrac{5}{13} = \dfrac{\text{Adj}}{\text{Hyp}} = \dfrac{AB}{AC}$		
	In ⊿ ABC, $A\hat{B}C = 90°$ 		
	$\therefore \quad BC^2 = AC^2 - AB^2$		
	$\therefore \quad BC = \sqrt{13^2 - 5^2}$		
	$= \sqrt{169 - 25} = \sqrt{144} = 12$		
	$\therefore \quad \sin\theta = \dfrac{12}{13}$.		
	Figure —	½	
	Finding opp. side —	½	

Qn. Nos.	Value Points	Marks allotted
	$\therefore \quad \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{\dfrac{12}{13} + \dfrac{5}{13}}{\dfrac{12}{13} - \dfrac{5}{13}}$ — 1	
	$= \dfrac{17}{\cancel{13}} \times \dfrac{\cancel{13}}{7} = \dfrac{17}{7}$	2
	Any other alternate method may be given marks.	
IV. 41.	In a harmonic progression 5th term is $\frac{1}{12}$ and 11th term is $\frac{1}{15}$. Find its 25th term.	
	OR	
	If the third term of a geometric progression is 12 and its sixth term is 96, find the sum of first 9 terms.	
	Ans. :	
	$T_5 = \frac{1}{12}$ and $T_{11} = \frac{1}{15}$	
	Reciprocals of HP are in AP.	
	$\therefore \quad a + 4d = 12 \quad \ldots$ (i) — ½	
	$a + 10d = 15 \quad \ldots$ (ii) — ½	
	By solving (i) and (ii)	
	$\cancel{a} + 10d = 15$	
	$(-)\ \cancel{a} + 4d = 12$	
	$(-)$	
	$6d = 3$	
	$\therefore \quad d = \frac{3}{6} = \frac{1}{2}$ — ½	
	If $d = \frac{1}{2}$, then $a + \overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right) = 12$	
	$a + 2 = 12$	
	$\therefore \quad a = 10$ — ½	

Qn. Nos.	Value Points		Marks allotted
	If $a = 10$ and $d = \frac{1}{2}$ then		
	$T_n = \frac{1}{a+(n-1)d}$	½	
	$T_{25} = \frac{1}{10+(25-1)\frac{1}{2}}$		
	$= \frac{1}{10+\overset{12}{\cancel{24}}\times\frac{1}{\cancel{2}}}$		
	$\boxed{T_{25} = \frac{1}{22}}$	½	3
	Alternate method :		
	The corresponding T_5 and T_{11} of AP are		
	$T_5 = 12$ and $T_{11} = 15$		
	$\therefore \quad d = \frac{T_p - T_q}{p-q}$	½	
	$= \frac{T_5 - T_{11}}{5-11}$		
	$= \frac{12-15}{5-11} = \frac{-3}{-6} = \frac{1}{2}$	½	
	If $d = \frac{1}{2}$ then $a + \overset{2}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right) = 12$		
	$a + 2 = 12$		
	$\therefore \quad a = 10$	½	
	If $a = 10$ and $d = \frac{1}{2}$		
	$T_n = \frac{1}{a+(n-1)d}$	½	

Qn. Nos.	Value Points		Marks allotted
	$T_{25} = \dfrac{1}{10 + (25-1)\frac{1}{2}}$	½	
	$= \dfrac{1}{10 + \overset{12}{\cancel{24}} \times \frac{1}{\cancel{2}}}$		
	$T_{25} = \dfrac{1}{22}$	½	3
	OR		
	$T_3 = 12 \qquad \therefore \quad ar^2 = 12 \qquad \ldots \text{(i)}$	½	
	$T_6 = 96 \qquad \therefore \quad ar^5 = 96 \qquad \ldots \text{(ii)}$	½	
	$\therefore \quad \dfrac{\cancel{a}r^5}{\cancel{a}r^2} = \dfrac{\cancel{96}\,8}{\cancel{12}}$ OR $\left.\begin{array}{l} ar^2(r^3) = 96 \\ 12r^3 = 96 \\ r^3 = 8 \end{array}\right\}$		
	$r^3 = 8 \qquad \therefore \quad r = 2$	½	
	If $r = 2$ then $a(2)^2 = 12$		
	$4a = 12$		
	$\therefore \quad a = 3$	½	
	If $a = 3$ and $r = 2$, $n = 9$ then		
	$S_n = \dfrac{a(r^n - 1)}{r-1}$		
	$S_9 = \dfrac{3(2^9 - 1)}{2-1}$	½	
	$= 3(512 - 1)$		
	$= 3 \times 511$		
	$S_9 = 1533$	½	3

Qn. Nos.	Value Points	Marks allotted

42. Calculate the variance of the following data :

Class-interval	0-4	5-9	10-14	15-19	20-24
Frequency (f)	1	2	5	4	3

Ans. :

i) *Step deviation method* :

$A = 12$ $i = 5$

C.I.	f	x	$d = \frac{x - A}{i}$	d^2	fd	fd^2
0-4	1	2	− 2	4	− 2	4
5-9	2	7	− 1	1	− 2	2
10-14	5	12	0	0	0	0
15-19	4	17	1	1	4	4
20-24	3	22	2	4	6	12

1½

$N = 15$ $\sum fd = 6$ $\sum fd^2 = 22$

Variance $= \sigma^2 = \sum \frac{fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2 \times i^2$ ½

$= \frac{22}{15} - \left(\frac{6}{15} \right)^2 \times 5^2$

$= (1{\cdot}466 - 0{\cdot}16) \times 25$ ½

$= 1{\cdot}306 \times 25$

$= 32{\cdot}6$ ½ 3

Qn. Nos.	Value Points	Marks allotted

Direct method :

C.I.	f	x	fx	x^2	fx^2
0-4	1	2	2	4	4
5-9	2	7	14	49	98
10-14	5	12	60	144	720
15-19	4	17	68	289	1156
20-24	3	22	66	484	1452

1½

$N = 15$ $\quad \sum fx = 210$ $\quad \sum fx^2 = 3430$

$$\text{Variance} = \sigma^2 = \sum \frac{fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2$$ ½

$$= \frac{3430}{15} - \left(\frac{210}{15}\right)^2$$

$$= 228{\cdot}6 - 196$$ ½

$$= 32{\cdot}6$$ ½ 3

Assumed mean method :

Assumed mean $A = 12$

C.I.	f	x	$d = x - A$	fd	d^2	fd^2
0-4	1	2	– 10	– 10	100	100
5-9	2	7	– 5	– 10	25	50
10-14	5	12	0	0	0	0
15-19	4	17	5	20	25	100
20-24	3	22	10	30	100	300

1½

$N = 15$ $\quad \sum fd = 30$ $\quad \sum fd^2 = 550$

Qn. Nos.	Value Points		Marks allotted
	Variance = $\sigma^2 = \sum \frac{f d^2}{N} - \left(\frac{\Sigma f d}{N} \right)^2$	½	
	$= \frac{550}{15} - \left(\frac{30}{15} \right)^2$	½	
	$= 36{\cdot}6 - 4$		
	$= 32{\cdot}6$	½	3

Actual mean method :

C.I.	f	x	fx	$d = x - \overline{x}$	d^2	$f d^2$
0-4	1	2	2	– 12	144	144
5-9	2	7	14	– 7	49	98
10-14	5	12	60	– 2	4	20
15-19	4	17	68	3	9	36
20-24	3	22	66	8	64	192

1½

$N = 15$ $\Sigma fx = 210$ $\Sigma f d^2 = 490$

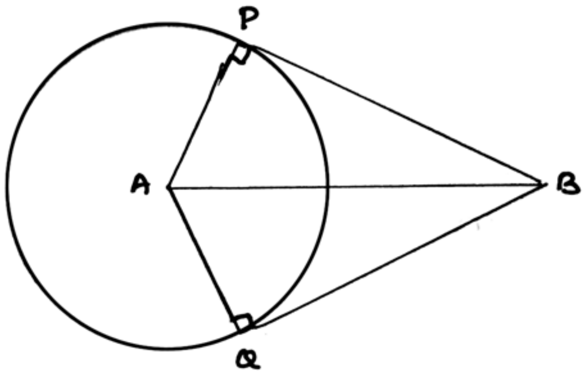
	Mean = $\overline{x} = \frac{\Sigma f x}{N}$		
	$= \frac{210}{15} = 14$	½	
	Variance = $\sigma^2 = \frac{\Sigma f d^2}{N}$	½	
	$= \frac{490}{15}$		
	$= 32{\cdot}6$	½	3

Qn. Nos.	Value Points		Marks allotted
43.	Solve ($2x+3$) ($3x-2$) + 2 = 0 by using formula.		
	OR		
	If one root of the equation $x^2+px+q=0$ is four times the other, prove that $4p^2-25q=0$.		
	Ans. :		
	$(2x+3)(3x-2)+2=0$		
	$2x(3x-2)+3(3x-2)+2=0$	½	
	$6x^2-4x+9x-6+2=0$		
	$6x^2+5x-4=0$	½	
	where $a=6$, $b=5$. $c=-4$		
	$\therefore \quad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	½	
	$=\dfrac{-5\pm\sqrt{5^2-4\times6\times(-4)}}{2\times6}$	½	
	$=\dfrac{-5\pm\sqrt{25+96}}{12}$		
	$=\dfrac{-5\pm\sqrt{121}}{12}$		
	$=\dfrac{-5\pm11}{12}$	½	
	$=\dfrac{-5+11}{12}$ or $\dfrac{-5-11}{12}$		
	$=\dfrac{6}{12}$ or $\dfrac{-16}{12}$		
	$x=\dfrac{1}{2}$ or $\dfrac{-4}{3}$.	½	3
	OR		

Qn. Nos.	Value Points	Marks allotted
	$x^2 + px + q = 0$ where $a = 1$, $b = p$, $c = q$	
	If m and n are the roots	
	then $m = 4n$ ½	
	$\therefore$ Sum of the roots $= m + n = \frac{-b}{a}$	
	$4n + n = \frac{-p}{1}$	
	$5n = -p$	
	$\therefore$ $n = \frac{-p}{5}$ ... (i) ½	
	Product of the roots $= mn = \frac{c}{a}$	
	$4n \times n = \frac{q}{1}$	
	$4n^2 = q$ ... (ii) ½	
	Substituting (i) in (ii)	
	Then $4\left(\frac{-p}{5}\right)^2 = q$ ½	
	$\frac{4p^2}{25} = q$	
	$4p^2 = 25q$ ½	
	$4p^2 - 25q = 0$ ½	3

Qn. Nos.	Value Points	Marks allotted
44.	Prove that "The tangents drawn from an external point to a circle are equal".	

Ans. :



½

Data : A is the centre of the circle.

B is an external point. BP and BQ are the tangents. ½

To prove : $BP = BQ$ ½

Construction : AP, AQ and AB are joined. ½

Proof : In ⊿ APB and ⊿ AQB,

$A\hat{P}B = A\hat{Q}B$ Radius drawn at the point of contact is perpendicular to the tangent. ½

hyp. AB = *hyp* AB Common side

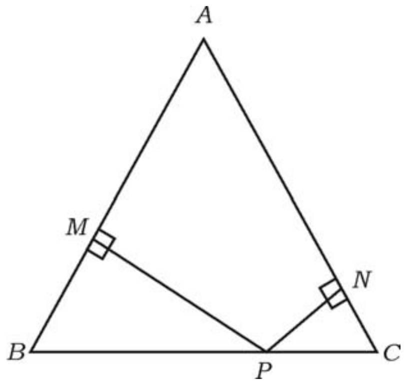
$AP = AQ$ Radii of the same circle.

$\therefore$ ⊿ $APB \cong$ ⊿ AQB RHS theorem.

$\therefore$ $BP = BQ$ CPCT. ½ **3**

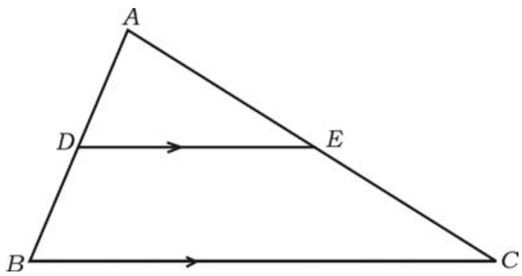
Qn. Nos.	Value Points	Marks allotted

45. In ΔABC, $AB = AC$. P is a point on BC such that $PN \perp AC$ and $PM \perp AB$ as shown in the figure. Prove that $\overline{MB}\,.\,\overline{CP} = \overline{NC}\,.\,\overline{BP}$.



OR

In ΔABC, $DE \parallel BC$. If $3DE = 2BC$ and the area of ΔABC is 81 cm^2, show that the area of ΔADE is 36 cm^2.



Ans. :

In ΔABC, $AB = AC$

$\therefore \quad \hat{B} = \hat{C}$ — angles opposite to equal sides — ½

In ⊿ BMP and ⊿ CNP

$B\hat{M}P = P\hat{N}C$ — right angles — ½

$M\hat{B}P = N\hat{C}P$ — equal angles — ½

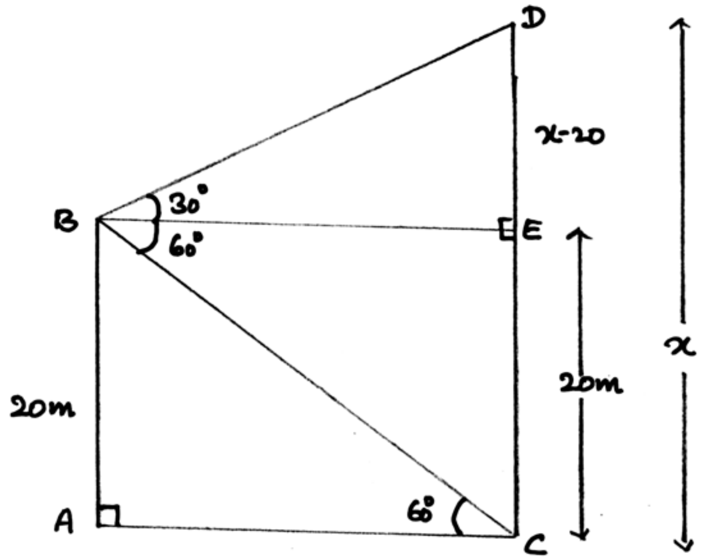
$\therefore$ ⊿ $MBP \sim$ ⊿ NCP — equiangular triangles — ½

$\therefore \quad \boxed{\dfrac{MB}{NC} = \dfrac{BP}{CP}} = \dfrac{MP}{NP}$ — AA - criteria — ½

$\therefore \quad MB\,.\,CP = BP\,.\,NC.$ — ½ — 3

OR

Qn. Nos.	Value Points		Marks allotted
	Given $3DE = 2BC$		
	$\therefore \quad \dfrac{DE}{BC} = \dfrac{2}{3}$	½	
	In $\Delta\ ADE$ and $\Delta\ ABC$,		
	$A\hat{D}E = A\hat{B}C$ Corresponding angles	½	
	$D\hat{A}E = B\hat{A}C$ Common angle	½	
	$\therefore \quad \Delta\ ADE \sim \Delta\ ABC$ Equiangular triangles	½	
	$\therefore \quad \dfrac{\text{Area of } \Delta\ ADE}{\text{Area of } \Delta\ ABC} = \dfrac{DE^2}{BC^2}$	½	
	$\dfrac{\text{Area of } \Delta\ ADE}{81} = \dfrac{2^2}{3^2}$		
	$\therefore \quad \text{Area of } \Delta\ ADE = \dfrac{4 \times \cancel{81}^{9}}{\cancel{9}}$		
	$= 36 \text{ cm}^2$.	½	3
46.	Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$.		
	OR		
	From the top of a building 20 m high, the angle of elevation of the top of a vertical pole is 30° and the angle of depression of the foot of the same pole is 60°. Find the height of the pole.		
	Ans. :		
	$= \left(1 + \dfrac{\cos A}{\sin A} - \dfrac{1}{\sin A}\right)\left(1 + \dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A}\right)$	½	
	$= \left(\dfrac{\sin A + \cos A - 1}{\sin A}\right)\left(\dfrac{\cos A + \sin A + 1}{\cos A}\right)$	½	
	$= \dfrac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A}$	½	
	$= \dfrac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A}$	½	

Qn. Nos.	Value Points		Marks allotted
	but $\sin^2 A + \cos^2 A = 1$		
	$= \dfrac{\cancel{1} + 2\sin A\cos A - \cancel{1}}{\sin A\cos A}$	½	
	$= \dfrac{2\cancel{\sin A}\ \cancel{\cos A}}{\cancel{\sin A}\ \cancel{\cos A}}$		
	$= 2$	½	3
	OR		
		½	
	In ◺ BED, $D\hat{B}E = 30°$		
	$\therefore \quad \tan 30° = \dfrac{DE}{BE}$	½	
	$\dfrac{1}{\sqrt{3}} = \dfrac{x-20}{BE}$		
	$\therefore \quad BE = \sqrt{3}\,(x-20)$	½	
	In ◺ ABC, $A\hat{C}B = 60°$		
	$\therefore \quad \tan 60° = \dfrac{AB}{AC}$	½	
	$\sqrt{3} = \dfrac{20}{\sqrt{3}\,(x-20)}$	½	

Qn. Nos.	Value Points		Marks allotted
	$3(x-20) = 20$		
	$3x - 60 = 20$		
	$\therefore \quad 3x = 80$		
	$x = \frac{80}{3} = 26{\cdot}6$ m.		
	Height of the pole = 26·6 m (approximate).	½	3
V. 47.	Solve the equation $x^2 + x - 6 = 0$ graphically.		

Ans. :

$$x^2 + x - 6 = 0$$

$$\therefore \quad y = x^2 + x - 6$$

x	0	1	2	3	– 1	– 2	– 3	– 4
y	– 6	– 4	0	6	– 6	– 4	0	6

Table —	2	
Drawing parabola —	1	
Identifying roots —	1	4

Alternate method :

$x^2 + x - 6 = 0 \qquad \therefore \quad y = x^2 , \; y = 6 - x$

$$y = x^2$$

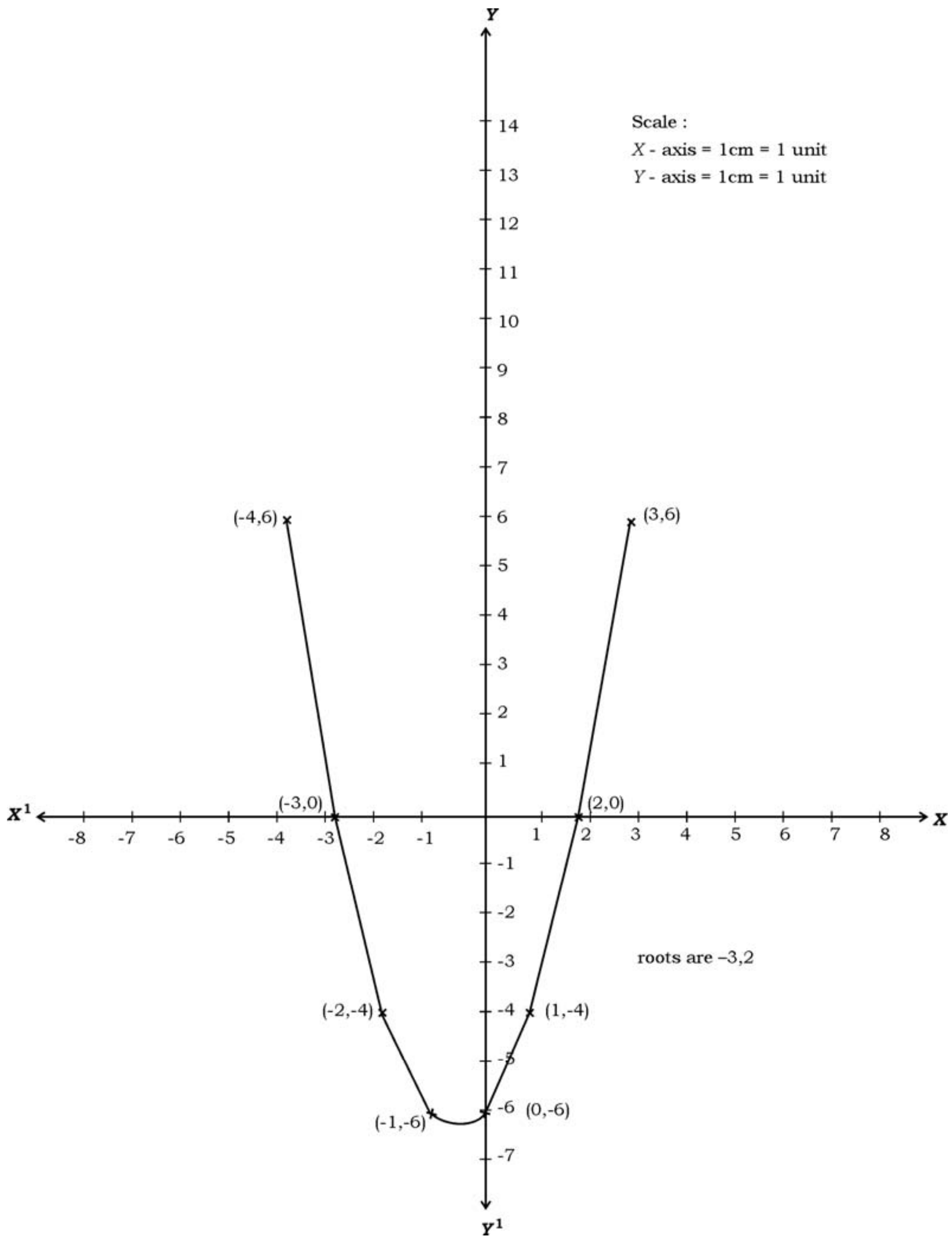
x	0	1	2	3	– 1	– 2	– 3
y	0	1	4	9	1	4	9

$$y = 6 - x$$

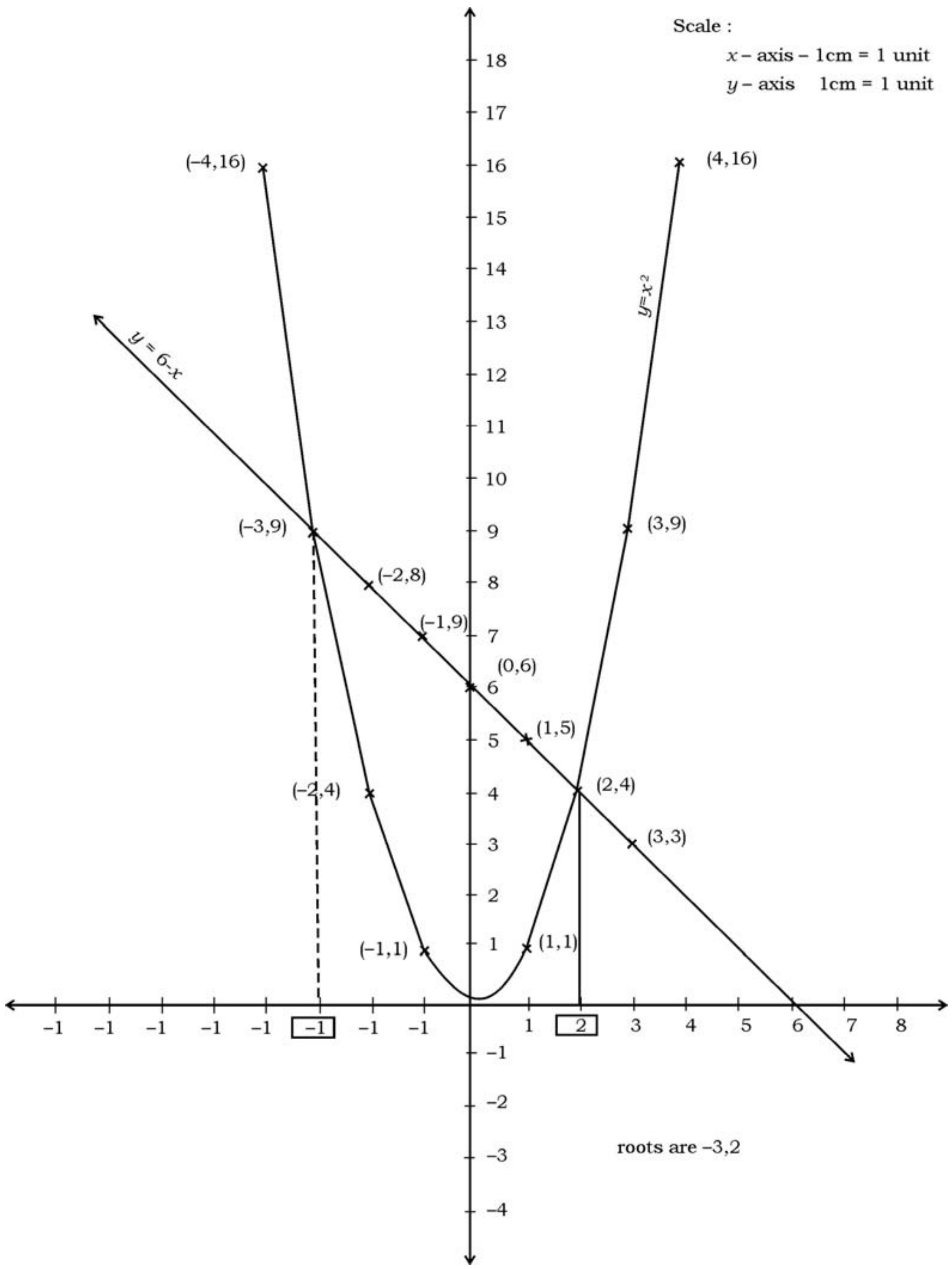
x	0	1	2	3	– 1	– 2	– 3
y	6	5	4	3	7	8	9

Table —	2	
Drawing parabola —	1	
Identifying roots —	1	4

Qn. Nos.	Value Points	Marks allotted



Qn. Nos.	Value Points	Marks allotted
	Alternate method :	

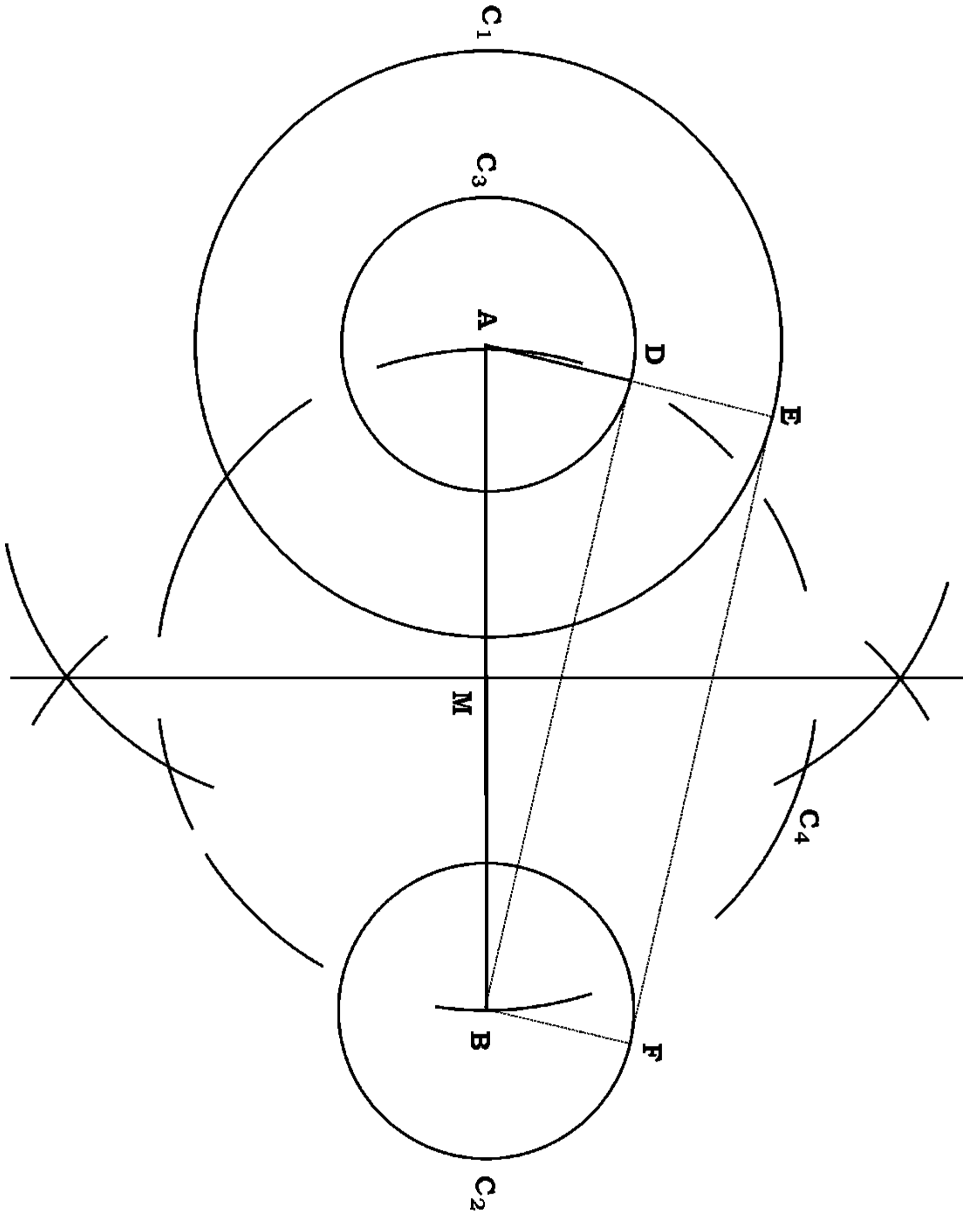


Qn. Nos.	Value Points	Marks allotted
48.	Construct a direct common tangent to two circles of radii 4 cm and 2 cm whose centres are 9 cm apart. Measure and write the length of the direct common tangent.	

Ans. :

$R = 4$ cm, $r = 2$ cm $\therefore$ $R - r = 4 - 2 = 2$ cm

$d = 9$ cm



Length of the tangent $EF = 8 \cdot 8$ cm

Qn. Nos.	Value Points	Marks allotted
	Drawing AB and marking mid-point — 1	
	Drawing C_1, C_2, C_3 — 1½	
	Joining DB, EF — 1	
	Measuring and writing the length of the tangent — ½	4
49.	Prove that "In a right angled triangle, square on the hypotenuse is equal to sum of the squares on the other two sides".	

Ans. :

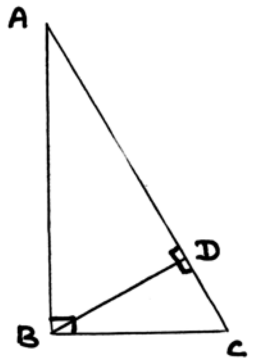


Figure — ½
Data — ½
To prove — ½
Construction — ½

Data : In ⊿ ABC, $A\hat{B}C = 90°$

To prove : $AC^2 = AB^2 + BC^2$

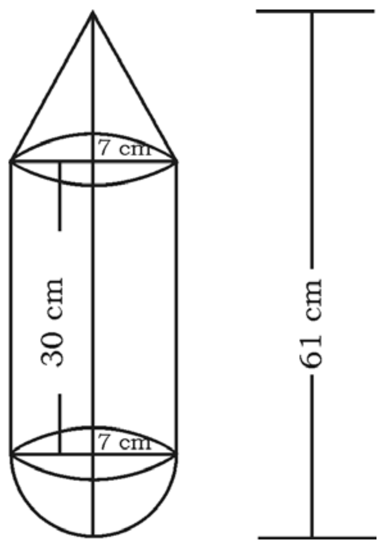
Construction : $BD \perp AC$ drawn.

Proof : Comparing ⊿ ABC and ⊿ ABD

Statement	*Reason*	
$A\hat{B}C = A\hat{D}B$	Right angles	
$B\hat{A}C = B\hat{A}D$	common angle	
∴ ⊿ BAC ~ ⊿ DAB	Equiangular triangles	½
∴ $\frac{BA}{DA} = \frac{AC}{AB}$	AA — criteria	

∴ $\boxed{AB^2 = AC \cdot AD}$ ... (i) ½

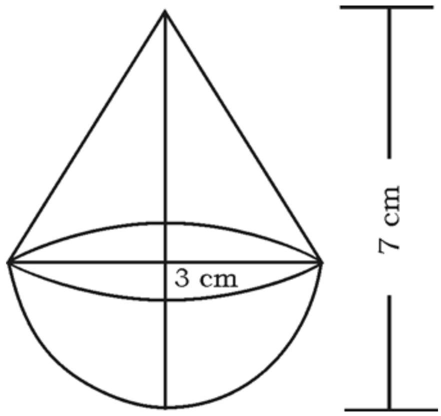
Qn. Nos.	Value Points		Marks allotted
	Comparing $\triangle ABC$ and $\triangle BDC$		
	$A\hat{B}C = B\hat{D}C$	Right angles	
	$A\hat{C}B = B\hat{C}D$	common angle	
	$\therefore \triangle BCA \sim \triangle DCB$	Equiangular triangles ½	
	$\therefore \frac{BC}{DC} = \frac{AC}{BC}$	AA — criteria	
	$\therefore \boxed{BC^2 = AC \cdot DC}$ ... (ii)		
	By adding (i) and (ii)		
	$AB^2 + BC^2 = AC \times AD + AC \times DC$		
	$= AC(AD + DC)$ $\because AD + DC = AC$		
	$= AC \times AC$		
	$\therefore \boxed{AB^2 + BC^2 = AC^2}$	½	4
50.	A solid is in the shape of a cylinder with a cone attached at one end and a hemisphere attached to the other end as shown in the figure. All of them are of the same radius 7 cm. If the total length of the solid is 61 cm and height of the cylinder is 30 cm, calculate the cost of painting the outer surface of the solid at the rate of Rs. 10 per 100 cm^2.		



OR

Qn. Nos.	Value Points	Marks allotted

A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of right circular cone mounted on a hemisphere as shown in the figure. If radii of the cone and hemisphere are each equal to 3 cm and the height of the toy is 7 cm, calculate the number of such toys that can be formed.



Ans. :

Height of the cone = Total height of the solid – (height of the cylinder + radius of the hemisphere)

= 61 – (30 + 7)

= 61 – 37 = 24 cm. ½

But 7, 24, 25 are Pythagorian triplets

∴ Slant height of the cone = l = 25 cm. ½

TSA of the solid = LSA of the cone + LSA of the cylinder + LSA of the hemisphere ½

= $\pi rl + 2\pi rh + 2\pi r^2$ 1

= $\pi r (l + 2h + 2r)$

= $\frac{22}{7} \times 7$ (25 + 2 × 30 + 2 × 7) sq.cm. ½

= 22 × 99

= 2178 sq.cm. ½

Cost of painting at the rate of Rs. 10 per 100 cm^2 = $\frac{2178 \times 10}{100}$

= Rs. 217·8 ½ 4

Qn. Nos.	Value Points		Marks allotted
	Alternate method :		
	Height of the cone = h = 24 cm	½	
	Slant height of the cone = l = 25 cm.	½	
	∴ LSA of the cone = πrl		
	= $\pi \times 7 \times 25$ sq.cm		
	= $175\,\pi$ sq.cm.	½	
	LSA of the cylinder = $2\pi rh$		
	= $2\pi \times 7 \times 30$ sq.cm		
	= $420\,\pi$ sq.cm.	½	
	LSA of the hemisphere = $2\pi r^2$		
	= $2\pi \times 7^2$		
	= 98π sq.cm.	½	
	TSA of the solid = LSA of the cone + LSA of the cylinder + LSA of the hemisphere	½	
	= ($175\,\pi + 420\,\pi + 98\,\pi$) sq.cm.		
	= $\frac{22}{\not 7} \times \not{693}^{\,99}$		
	= 2178 sq.cm.	½	
	Cost of painting = $\frac{2178 \times 1\not 0}{10\not 0}$		
	= Rs. 217·8	½	4
	OR		

Qn. Nos.	Value Points		Marks allotted
	Volume of the metal cylinder $= \pi r^2 h$ cubic units	½	
	$\begin{pmatrix} r = 6 \text{ cm} \\ h = 15 \text{ cm} \end{pmatrix}$ $= \pi \times 36 \times 15$ c.c.	½	
	Volume of the toy = Volume of the cone + Volume of the hemisphere $\begin{pmatrix} r = 3 \text{ cm} \\ h = 7 - 3 = 4 \text{ cm} \end{pmatrix}$		
	$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$	½	
	$= \frac{\pi r^2}{3}(h + 2r)$	½	
	$= \frac{\pi \times 3^{\not 2}}{\not 3}(4 + 6)$		
	$= 3 \times 10 \times \pi$	½	
	Number of toys $= \frac{\text{Volume of the cylinder}}{\text{Volume of the toy}}$	½	
	$= \frac{\overset{18}{\not 36} \times \overset{3}{\not 15} \times \not\pi}{\not 3 \times \underset{2}{\not 10} \times \not\pi}$	½	
	$= 18$	½	4
	Alternate method :		
	Cylinder: $r_1 = 6$ cm, $h_1 = 15$ cm; *Cone*: $r_2 = 3$ cm, $h_2 = 4$ cm; *Hemisphere*: $r_2 = 3$ cm	½	
	Number of toys $= \frac{\text{Volume of the metal cylinder}}{\text{Volume of the toy}}$	½	

Qn. Nos.	Value Points		Marks allotted
	$= \dfrac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3}$	1½	
	$= \dfrac{\pi(6^2 \times 15)}{\frac{1}{3} \times \pi \times 3^2 (4+6)}$	1	
	$= \dfrac{36 \times 15}{3 \times 10}$		
	$= 18.$	½	4