I. **Mathematics 2020 (Key)** — Little Hearts School, Gangavathi, Radhika. Poling

1. B) equations have unique solution
2. C) 2
3. B) 1
4. D) $\frac{13}{12}$
5. A) 1:2
6. A) a tangent
7. D) $\frac{\theta}{360^\circ} \times 2\pi r$
8. C) $220 \, cm^3$

II

9. $\frac{23}{2^2 \times 5} = 1.15$ (terminating)
10. 3 zeroes
11. 2 (1+1)
12. $P(x,y) = \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$
13. Basic proportionality theorem: If a line is drawn parallel to one side of a triange to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
14.
$$\angle BOC + \angle BAC = 180^\circ$$
$$130^\circ + \angle BAC = 180^\circ$$
$$\angle BAC = 180^\circ - 130^\circ$$
$$\therefore \angle BAC = 50^\circ$$
15.
$$\frac{x+1}{2} \times \frac{1}{x}$$
$$x(x+1) = 2 \Rightarrow x^2 + x - 2 = 0$$

Little Hearts School
Gangavathi
By Radhika

16. $\pi r(r+1)$ Sq. units

III

17. $2x+y=11$ — ①
$x+y=8$ — ②
$\underline{-\quad -\quad -}$
$x=3$

substitute in eq ②

$x+y=8$
$3+y=8$
$y=8-3$
$y=5$
$\therefore x=3, y=5$

18. $5+8+11+\text{------}$ to 10 terms

$a=5,\ d=3$

$S_{10}=\frac{10}{2}[2(5)+(10-1)3]$

$=5[10+27]$

$=5[37]$

$\therefore S_{10}=185$

19. $2x-3y=8$ and $2(K-4)x-Ky=K+3$ are inconsistent

$\frac{a_1}{a_2}=\frac{b_1}{b_2}$

$a_1=2,\ b_1=-3,\ c_1=-8$
$a_2=2(K-4),\ b_2=-K,\ c_2=-(K+3)$

substitute

$\frac{2}{2(K-4)}=\frac{\cancel{-}3}{\cancel{-}K}$

$2K=6(K-4)$
$2K=6K-24$
$24=6K-2K$
$24=4K$
$\therefore K=6$

Little Hearts School, Gangavathi
By, Radhika

20. Given, $2x^2-5x+3=0$

Compare $ax^2+bx+c=0$

$a=2, b=-5, c=3$

Discriminant, $\Delta = b^2-4ac$

$= (-5)^2-4\cdot2\cdot3$

$= 25-24$

$\Delta = 1$

$\therefore \Delta > 0$

The roots are distinct real roots.

21. $P(x) = x^2-6x+K$

$a=1, b=-6, c=K$

let ~~α~~, β are the two roots

$\alpha=\alpha, \ \beta=2\alpha$

Sum

$\alpha+\beta = \alpha+2\alpha = \frac{-b}{a}$

$3\alpha = -(-6)$

$3\alpha = 6$

$\alpha = 2$

Product

$\alpha\beta = \frac{c}{a}$

$\alpha(2\alpha) = \frac{K}{1}$

$2\alpha^2 = K$

$2(2)^2 = K$

$K = 4\times2$

$\therefore K=8$

22. $(\overset{x_1}{-5},\overset{y_1}{7})$ & $(\overset{x_2}{-1},\overset{y_2}{3})$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(-1-(-5))^2+(3-7)^2}$

$=\sqrt{(-1+5)^2+(-4)^2}$

$=\sqrt{16+16}$

$=\sqrt{32} = \underline{4\sqrt{2}}$ units

(or)

Little Hearts School, Gangavati (4)

By, Radhika

22. Given (1,6) & (4,3) in 1:2

$x_1\ y_1 \quad x_2\ y_2 \quad m_1\ m_2$

$$P(x,y) = \left[\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$$

$$= \left[\frac{1(4)+2(1)}{1+2}, \frac{1(3)+2(6)}{1+2}\right]$$

$$= \left[\frac{4+2}{3}, \frac{3+12}{3}\right]$$

$$= \left[\frac{6}{3}, \frac{15}{3}\right]$$

$\therefore P(x,y) = [2,5]$

23. A(1,1), B(3,2) and C(5,3)

$x_1\ y_1 \quad x_2\ y_2 \quad x_3\ y_3$

Area of $\Delta ABC = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2}[1(2-3)+3(3-1)+5(1-2)]$$

$$= \frac{1}{2}[1(-1)+3(2)+5(-1)]$$

$$= \frac{1}{2}[-1+6-5]$$

$$= \frac{1}{2}[6-6]$$

$$= \frac{0}{2}$$

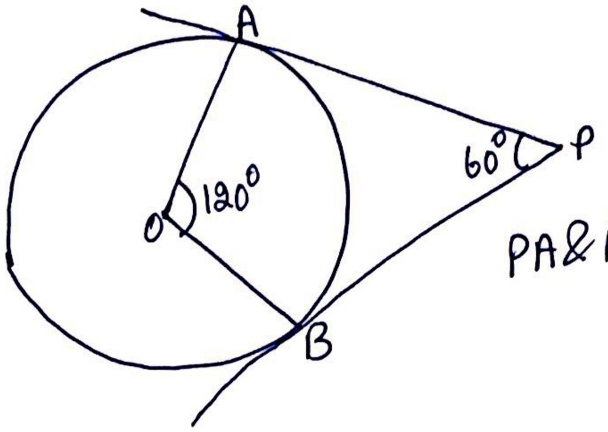
$\therefore$ Area of $\Delta ABC = 0$

$\therefore$ The points are collinear.

$\therefore$ These vertices cannot form a triangle.

Little Hearts School, Gangavathi
By, Radhika

24.



PA & PB are tangents

IV.

25. $\sqrt{5} = \frac{a}{b}$

a and b are coprime.

$b\sqrt{5} = a$

[Shortcut written]

S.O.B.S

$5b^2 = a^2$

5 divides a^2, 5 divides a

$a = 5c$

$5b^2 = (5c)^2$

$\cancel{5}b^2 = \overset{5}{\cancel{25}}c^2$

$b^2 = 5c^2$

5 divides b^2, 5 divides b

(or)

H.C.F 24 & 40

$$
\begin{array}{l}
24)\,40\,(1 \\
\quad\ \ 24 \\
\hline
\quad 16)\,24\,(1 \\
\qquad\ \ 16 \\
\hline
\qquad 8)\,16\,(2 \\
\qquad\quad\ 16 \\
\hline
\qquad\quad\ \ \boxed{0}
\end{array}
$$

H.C.F (24,40) = 8

L.CM of 8 and 20 is 40.

26.

Little Hearts School, Gangavathi ⓒ
By, Radhika

A ———— 12 km

B ————

2 km/h

$\frac{1}{2}\frac{\cancel{30}}{\cancel{60}}$

$$\frac{12}{x} - \frac{12}{x+2} = \frac{1}{2}$$

$$\frac{12(x+2) - 12x}{x(x+2)} = \frac{1}{2}$$

$$\frac{\cancel{12x} + 24 - \cancel{12x}}{x^2+2x} = \frac{1}{2}$$

$$\frac{24}{x^2+2x} = \frac{1}{2}$$

$$x^2+2x-48=0$$

$$x^2+8x-6x-48=0$$

$$x(x+8)-6(x+8)=0$$

$$x=-8,\ x=6$$

$$A,\ T = \frac{D}{S} = \frac{\cancel{12}^{2}}{\cancel{6}} = \underline{2\ hr.}$$

$$B,\ T = \frac{D}{S} = \frac{12}{x+2} = \frac{12}{6+2} = \frac{12}{8} = 1\frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \underline{1\tfrac{1}{2}\ hr}$$

27. Given, $x = p\tan\theta + q\sec\theta$, $y = p\sec\theta + q\tan\theta$

Consider, L.H.S

$$x^2 - y^2$$

$$(p\tan\theta + q\sec\theta)^2 - (p\sec\theta + q\tan\theta)^2$$

$$(p\tan\theta)^2 + (q\sec\theta)^2 + 2pq\tan\theta\sec\theta - \left((p\sec\theta)^2 + (q\tan\theta)^2 + 2pq\sec\theta\tan\theta\right)$$

$$p^2\tan^2\theta + q^2\sec^2\theta + 2pq\cancel{\tan\theta\sec\theta} - p^2\sec^2\theta - q^2\tan^2\theta - \cancel{2pq\sec\theta\tan\theta}$$

$$p^2(\tan^2\theta - \sec^2\theta) + q^2(\sec^2\theta - \tan^2\theta)$$

Little Hearts School, Gangavathi :
By - Radhika

$$p^2(-1) + q^2(1)$$
$$-p^2 + q^2$$
$$= q^2 - p^2$$
$$= R.H.S \qquad \therefore x^2 - y^2 = q^2 - p^2$$

(or)

27. Prove that $\dfrac{\cot^2(90^\circ-\theta)}{\tan^2\theta - 1} + \dfrac{\csc^2\theta}{\sec^2\theta - \csc^2\theta} = \dfrac{1}{\sin^2\theta - \cos^2\theta}$

consider L.H.S

$$\frac{\cot^2(90^\circ-\theta)}{\tan^2\theta - 1} + \frac{\csc^2\theta}{\sec^2\theta - \csc^2\theta}$$

$$= \frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\csc^2\theta}{\sec^2\theta - \csc^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - 1} + \frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}} + \frac{\frac{1}{\sin^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}}$$

$$= \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{\sin^2\theta - \cos^2\theta} + \frac{1}{\cancel{\sin^2\theta}} \times \frac{\cancel{\sin^2\theta}\cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$$

$= R.H.S$

Hence it is proved

Little Hearts School, Gangavathi
By- Radhika

28.

C.I	F	C.F
20-40	7	7
40-60	15	22
60-80	20	42
80-100	8	50

$$\text{Median} = l + \left[\frac{\frac{n}{2} - C_f}{f}\right] \times h$$

$l = 60,\ \frac{n}{2} = \frac{50}{2} = 25,\ C_f = 22,\ F = 20,\ h = 20$

$$\text{Median} = 60 + \left[\frac{25 - 22}{20}\right] \times 20$$

$$= 60 + \frac{3}{\cancel{20}} \times \cancel{20}$$

$$= 60 + 3$$

$\therefore$ Median $= 63$

(or)

<u>Mode</u>

	9	f_0
l 5 - 7	15	f_1
	9	f_2

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 5 + \left[\frac{15 - 9}{2(15) - 9 - 9}\right] \times 2$$

$$= 5 + \left[\frac{6}{30 - 18}\right] \times 2$$

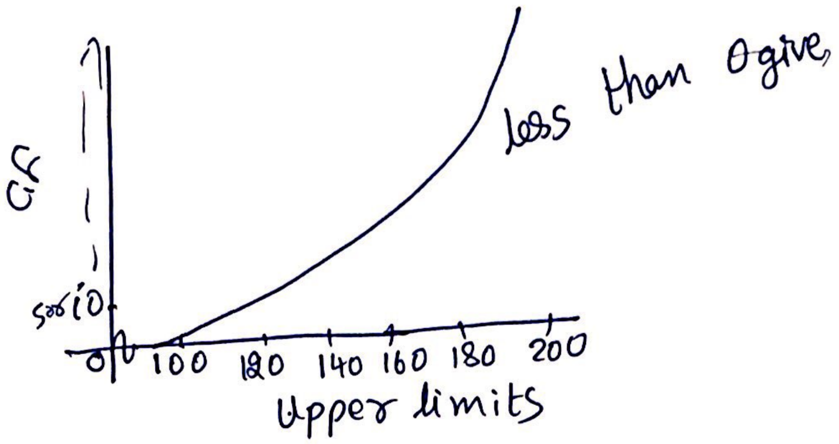
Little Hearts School, Gangavathi
By- Radhika

$= 5 + \left[\frac{\cancel{6}}{\cancel{12}_{6}}\right] \times \cancel{2}$

$= 5 + 1$

$\therefore$ Mode $= 6$

29.



30. Total no of balls $= 3 + 5 + 8 = 16$

$n(S) = 16$

$r = 3$
$w = 5$
$B = 8$

a) a red ball

$P(R) = \frac{n(A)}{n(S)}$

$= \frac{3}{16}$

b) not a white ball

$P(\overline{w}) = P(r) + P(B)$

$= \frac{3}{16} + \frac{8}{16}$

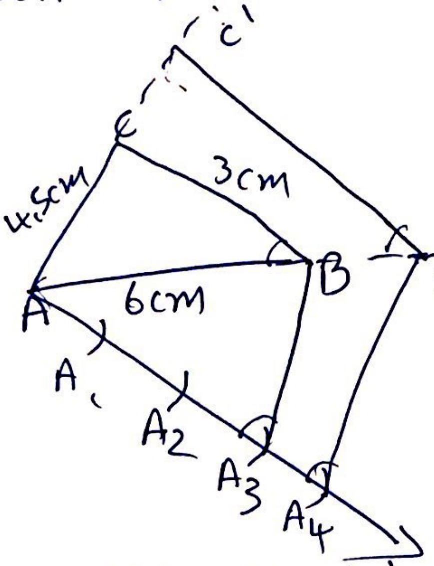
$= \frac{3+8}{16}$

$\therefore P(\overline{w}) = \frac{11}{16}$

Little Hearts School, Gangavathi
By - Radhika

31. Text book theorem

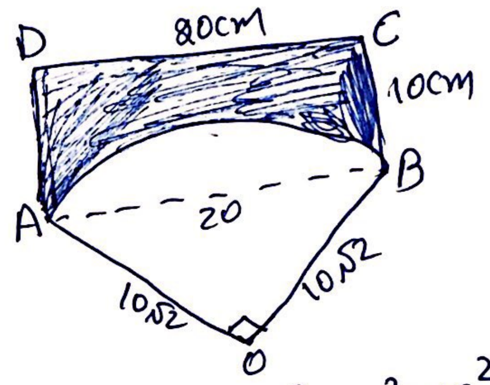
32.



(Rough sketch)

$\Delta ABC \sim \Delta AB'C'$

33)



Area of rectangle ABCD $= 20 \times 10$
$= 200 cm^2$

ΔAOB, $AB^2 = AO^2 + OB^2$
$400 = 200 + 200$
$400 = 400$
$\therefore \angle O = 90^\circ$

Area of Segment,

$= $ Area of quadrant − Area of Δ^{le}

$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times b \times h$

$= \frac{1}{4} \times \frac{22}{7} \times (10\sqrt{2})^2 - \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2}$

$= \frac{1}{\cancel{4}_2} \times \frac{\cancel{22}^{11}}{7} \times \cancel{200}^{100} - \frac{1}{\cancel{2}}\cancel{200}^{100}$

$= \frac{1100}{7} - 100$

$= \frac{1100 - 700}{7}$

Area of segment $= \frac{400}{7} cm^2$

Area of shaded region = Area of rectangle - Area of seg.

$$= 200 - \frac{400}{7}$$

$$= \frac{1400-400}{7}$$

$$= \frac{1000}{7}$$

Area of shaded region $= 142.86 cm^2$

(or)

33. Given $r = 21cm$

angle $= 120^\circ$

Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{\cancel{120^\circ}}{\cancel{360^\circ}_3} \times \frac{22}{\cancel{7}} \times \cancel{21}^3 \times 21$$

$$= 22 \times 21$$

$$= 462 cm^2$$

Total length of wire $= \overset{(r)}{21} + \overset{(r)}{21} +$ length of arc

$$= 42 + \frac{120}{360^\circ} \times 2\pi r$$

$$= 42 + \frac{\cancel{120}}{\cancel{360}_3} \times 2 \times \frac{22}{\cancel{7}} \times \cancel{21}^3$$

$$= 42 + 44$$

length of wire $= 86 cm$

Little Hearts School (12)
Gangavathi.
By - Radhika

34. Graph

Intersects at $(2,5)$

35. Let the 5 terms be

$$\overset{a_1}{a-2d},\ \overset{a_2}{a-d},\ \overset{a_3}{a},\ \overset{a_4}{a+d},\ \overset{a_5}{a+2d}$$

Sum $a-\cancel{2d}+a-\cancel{d}+a+a+\cancel{d}+a+\cancel{2d}=55$

$5a=55$

$a=11$

Given, $a_4=S_2+5$

$a_4=a_1+a_2+5$

$\cancel{a+3d=a+a+d+}$

$a+d=a-2d+a-d+5$

$\cancel{11}+d=\cancel{11}-2d+11-d+5$ $(a=11$

$\therefore d+3d=16$

$4d=16$

$d=4$

$\therefore$ 5 terms are $a-2d,\ a-d,\ a,\ a+d,\ a+2d$

$11-2(4),\ 11-4,\ 11,\ 11+4,\ 11+2(4)$

$11-8,\ 7,\ 11,\ 15,\ 11+8$

$\therefore$ $\underline{3, 7, 11, 15, 19}$ are the 5 terms of an A.P.

(or)

35. Given,

$a_6 = 2a_3 + 1$ — (1)

$a_4 + a_5 = 5a_2$ — (2)

Find $a_{10} = ?$

Consider eqn (1)

$a_6 = 2a_3 + 1$

$a + 5d = 2(a + 2d) + 1$

$a + 5d = 2a + 4d + 1$

$a - 2a = 4d - 5d + 1$

$-a = -d + 1$

$d - a = 1$ — (3)

(or) $a - d = -1$ — (3)

Consider eqn (2)

$a_4 + a_5 = 5a_2$

$a + 3d + a + 4d = 5(a + d)$

$2a + 7d = 5a + 5d$

$2a - 5a + 7d - 5d = 0$

$-3a + 2d = 0$

$-(3a - 2d) = 0$

$3a - 2d = 0$ — (4)

solve eqn (3) & eqn (4) → (3) × 3

$\cancel{3a} - 3d = -3$

$\cancel{3a} \underset{+}{-} 2d = 0$

$-d = -3$

$d = 3$

$d = 3$ substitute in eqn (3)

$a - d = -1$

$a - 3 = -1$

$a = 3 - 1$

$a = 2$

10th term

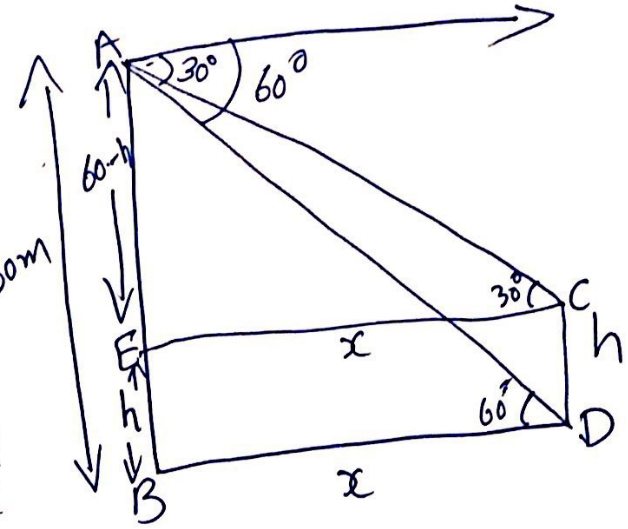
$a_{10} = a + 9d$

$= 2 + 9(3)$

$= 2 + 27$

$a_{10} = 29$

36.



Consider ~~ΔAB~~

ΔABD,

$\tan 60^\circ = \frac{AB}{BD}$

$\sqrt{3} = \frac{60}{BD} = \frac{60}{x}$

$x = \frac{60}{\sqrt{3}}$

$x = 20\sqrt{3}$ m

Consider, ΔAEC, $\tan 30^\circ = \frac{AE}{EC}$

$\frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\frac{1}{\cancel{\sqrt{3}}} = \frac{60-h}{20\cancel{\sqrt{3}}}$ (Substitute x)

$20 = 60 - h$

$h = 60 - 20$

$\therefore h = 40$ m

$\therefore$ Height of the pole $= 40$ m.

37. $h = 16$, $r_1 = 20$, $r_2 = 8$

$V = \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= \frac{1}{3} \times 3.14 \times 16 \left[(20)^2 + (8)^2 + 20 \times 8\right]$

$= \frac{50.24}{3}\left[400 + 64 + 160\right]$

$= \frac{50.24}{\cancel{3}}\left(\overset{208}{\cancel{624}}\right)$

$= 10449.92 \, cm^3$

$= \frac{10449.92}{1000} \, l$

$V = 10.44992 \, l$

Cost of 1 lt = 20₹

Cost of milk in container $= 10.4492 \times 20$

$= 208.99840$

$= ₹\, 209$

38. Theorem from text book.

Thank you

Mrs Radhika Polina
Little hearts School
Gangavathi