

MATHEMATICS

MODEL KEY ANSWER – ENGLISH MEDIUM

27-Jun-20

T.SHIVAKUMAR

10TH STANDARD ANNUAL EXAM -2020

Subject: Mathematics

Subject code: 81E

Time : 3 hours

KEY ANSWER

Max.marks: 80

Choose the correct answer given below ----- 1x8=8

1. Equations have unique solution
2. 2
3. 1
4. $\frac{13}{12}$
5. 1:2
6. tangent
7. $\frac{\theta}{360} \times \pi r^2$
8. 220 cm²

Answer the following questions ----- 1x8=8

9. Here, q =20, which is in the form $2^n \times 5^m = 2^2 \times 5^1$ So, the rational number $\frac{23}{20}$ is a terminating decimal expansion.
10. 3
11. 1
12. $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
13. **Basic Proportionality Theorem** states that "If a line is drawn **parallel** to one side of a triangle to intersect the other two sides in distinct points, the other two sides are **divided in the same ratio**".
14. 50⁰
15. $x^2+x-2=0$
16. $\pi r l + \pi r^2$

Solve the following questions ----- 2x8=16

17. $2x+y=11$ & $x+y=8$ by elimination method

$$2x+y=11$$

$$\underline{2x+2y=16}$$

$$Y=5 \quad \& \quad x=3$$

18. Here $a=5$, $d=3$ & $n=10$ we have to find S_{10} , the formula is $S_n = \frac{n}{2} (2a + (n-1)d)$

$$S_{10} = \frac{10}{2} (2(5) + (10-1)3)$$

$$= 5(10+27)$$

$$S_{10} = 185$$

19. If the pair of linear equations are inconsistent then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$a_1=2, a_2=2(k-4), b_1=-3, b_2=-k$$

$$2x(-k) = -3(2k-8)$$

$$-2k = -6k + 24$$

$$4k = 24$$

$$k = 6$$

20. The equation is $2x^2 - 5x + 3 = 0$, $a=2$, $b=-5$ & $c=3$

Nature of the roots will be depend on $b^2 - 4ac$

$$b^2 - 4ac = 25 - 4 \times 2 \times 3$$

$$= 1 > 0$$

If $b^2 - 4ac > 0$, then it will be two distinct real roots.

21. We know that sum of roots in a quadratic equation is given by

$$p+q = -b/a \text{ If } p, q \text{ are roots}$$

The quadratic equation is $x^2 - 6x + k$

One root is twice the other. So,

The roots are $p, 2p$

Comparing coefficients,

$$a=1, b=6, c=k$$

$$\text{So, } -b/a = p+2p$$

$$6/1 = 3p$$

$$6 = 3p$$

$$p = 2$$

So the roots are 2 and $2(2)=4$

Also we know that product of zeros $= c/a$

$$\text{That is, } pq = c/a$$

$$2 \times (4) = k/1$$

$$k = 8$$

OR

Given $p(x) = x^3 - 2x^2 + 3x + 4$ and $g(x) = x^2 - 3x + 1$

By division algorithm, $g(x) = \frac{p(x) - r(x)}{q(x)}$, So, $r(x)$ must be subtracted that result is exactly

divisible by $x^2 - 3x + 1$.

$$\begin{array}{r}
 \quad | \quad X+1 \\
 \hline
 x^2-3x+1 \quad | \quad x^3-2x^2+3x+4 \\
 \quad | \quad x^3-3x^2+1x \\
 \hline
 \quad | \quad x^2+2x+4 \\
 \quad | \quad x^2-3x+1 \\
 \hline
 \quad | \quad 5x+5 \\
 \quad | \quad 5(x+1)
 \end{array}$$

22. The distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(4)^2 + (-4)^2}$$

$$d = \sqrt{16 + 16}$$

$$d = \sqrt{32} = \sqrt{2 \times 16}$$

$$d = 4\sqrt{2} \text{ units}$$

OR

Given points are $(x_1, y_1) = (1, 6)$

$(x_2, y_2) = (4, 3)$ and ratio $m:n=1:2$

$$\begin{aligned}
 \text{The coordinates are } x, y \text{ is } & \left(\frac{mx_1 + nx_2}{m+n}, \frac{my_1 + ny_2}{m+n} \right) \\
 & \left(\frac{1x_1 + 2x_2}{3}, \frac{1y_1 + 2y_2}{3} \right) \\
 & \left(\frac{9}{3}, \frac{12}{3} \right)
 \end{aligned}$$

$(3, 4)$

23. The three points are $A(1,1)$, $B(3,2)$ and $C(5,3)$. We know that,

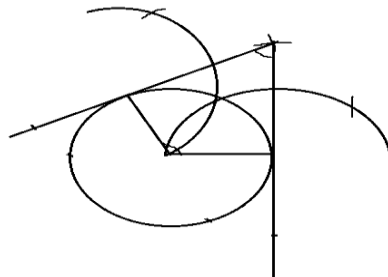
$$\text{Thus, area of triangle } A = \frac{1}{2}(1(2-3) + 3(3-1) + 5(1-2))$$

$$A = 0$$

Since, if area of triangle is 0 square units, then its vertices will be collinear

Hence, $A(1,1)$, $B(3,2)$ and $C(5,3)$ are not the vertices of triangle

24.



Solve the given problems ----- 3x9=27

25. Let us assume that $\sqrt{5}$ is a rational number. $\sqrt{5} = \frac{p}{q} \rightarrow (1)$
 $p, q \in \mathbb{Z}, q \neq 0, \text{HCF of } (p, q) = 1$

Equation 1, Squaring both side

We get $5 = \frac{p^2}{q^2}$

p^2, q^2 are not co-prime numbers. This is contradictory to our assumption that p and q are co-prime.

\Rightarrow Our assumption that $\sqrt{5}$ is a rational number is wrong.

$\therefore \sqrt{5}$ is an irrational number.

OR

HCF of 24 & 40 is

24)40(1	40=24x1+16
24	
16)24(1	24=16x1+8
16	
8)16(2	16=8x2+0
16	

HCF of 24 & 40 is 8

LCM of HCF (24, 40) & 20 is

$4 \overline{) 8, 20}$
 $2 \ 5 \Rightarrow 4 \times 2 \times 5 = 40$

26. Distance =12km speed of A be xkm/hr and speed of B is (x+2)km/hr

The time taken by A is $t_1 = \frac{12}{x}$

The time taken by B is $t_2 = \frac{12}{x+2}$

By question $t_2 = t_1 - \frac{1}{2}$

$\frac{12}{x+2} = \frac{12}{x} - \frac{1}{2}$

Solving the equation we get

$x^2+2x-48=0$ this is quadratic equation solve this by using any method

we get $x=6$ or $x=-8$

therefore time taken by A is $t_1 = \frac{12}{6} = 2$ hours

OR

27. $X = p \tan \theta + q \sec \theta$ & $y = p \sec \theta + q \tan \theta$

To prove $x^2 - y^2$

$$\begin{aligned} & (p \tan \theta + q \sec \theta)^2 - (p \sec \theta + q \tan \theta)^2 \\ &= (p^2 \tan^2 \theta + q^2 \sec^2 \theta + 2 p \tan \theta + q \sec \theta) - (p^2 \sec^2 \theta + q^2 \tan^2 \theta + 2 p \sec \theta + q \tan \theta) \\ &= p^2 (\tan^2 \theta - \sec^2 \theta) + q^2 (\sec^2 \theta - \tan^2 \theta) \\ &= p^2 (-1) + q^2 (1) \\ &= -p^2 + q^2 \end{aligned}$$

OR

$$\begin{aligned} & \frac{\cot 2(90-\theta) + \operatorname{cosec} 2\theta}{\tan 2\theta - 1 + \sec 2\theta - \operatorname{cosec} 2\theta} = \frac{1}{\sin 2\theta - \cos 2\theta} \\ & \frac{\tan 2\theta + \operatorname{cosec} 2\theta}{\tan 2\theta - 1 + \sec 2\theta - \operatorname{cosec} 2\theta} \\ & \frac{\sin 2\theta / \cos 2\theta + 1 / \sin 2\theta}{\sin 2\theta - \cos 2\theta / \cos 2\theta + \sin 2\theta - \cos 2\theta / \sin 2\theta} \\ & \frac{\sin 2\theta + \cos 2\theta}{\sin 2\theta - \cos 2\theta} = \frac{1}{\sin 2\theta - \cos 2\theta} \end{aligned}$$

Hence proof

28.

C.I	f	fc
20-40	7	7
40-60	15	22
60-80	20	42
80-100	8	50

$N=50$

$N/2=25$, $fc=22$, $h=20$, $LRL=60$ and $f=20$

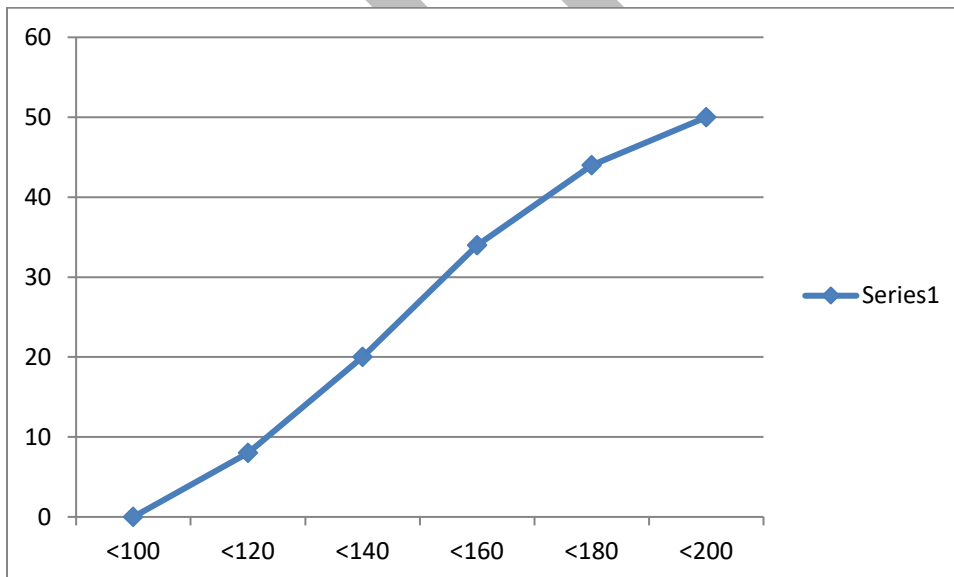
$$\begin{aligned} \text{Median} &= LRL + \frac{\frac{N}{2} - Fc}{f} \times h \\ &= 60 + \frac{25 - 22}{20} \times 20 \\ &= 63 \end{aligned}$$

C.I	f	
1-3	6	
3-5	9	f ₀
5-7	15	f ₁
7-9	9	f ₂
9-11	1	

$$\begin{aligned} \text{Mode} &= \text{LRL} + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 5 + \left(\frac{15 - 9}{30 - 9 - 9} \right) \times 2 \\ &= 5 + 12/12 \\ &= 6 \end{aligned}$$

29.

Daily income	no.of workers
<100	0
<120	8
<140	20
<160	34
<180	44
<200	50



$$\begin{aligned}
 30. N(s) &= n(R) + n(W) + n(B) \\
 &= 3 + 5 + 8 \\
 &= 16
 \end{aligned}$$

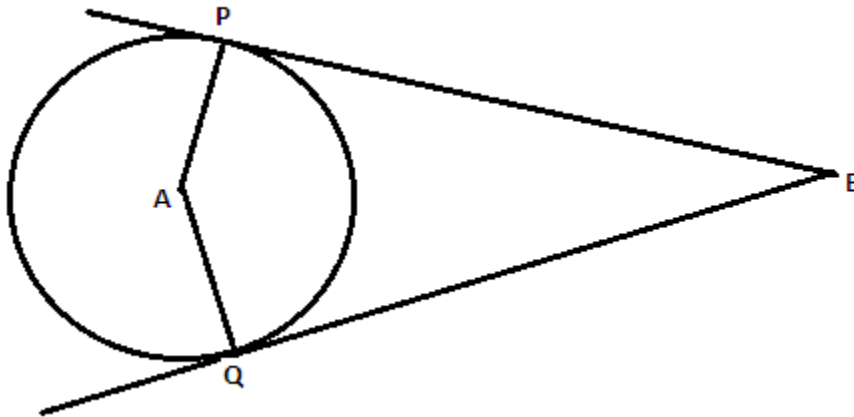
a) red ball, the probability is $p(a) = \frac{3}{16}$

b) not a white ball means, red+blue = 11
 the probability is $p(a) = \frac{11}{16}$

31.

Theorem: The tangents drawn from an external point to a circle

- (a) are equal
- (b) subtend equal angles at the centre
- (c) are equally inclined to the line joining the centre and the external point



Data : A is the centre of the circle. B is an external point.

BP and BQ are the tangents.

AP, AQ and AB are joined.

To prove : (a) $BP = BQ$

(b) $\angle PAB = \angle QAB$

(c) $\angle PBA = \angle QBA$

Proof: Statement

Reason

In ΔAPB and ΔAQB

$AP = AQ$

radii of the same circle

$\angle APB = \angle AQB = 90^\circ$

Radius drawn at the point of contact

hyp $AB =$ hyp AB

Common side

$\therefore \Delta APB \cong \Delta AQB$

RHS Theorem

\therefore (a) $PB = QB$

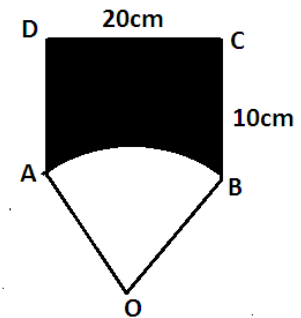
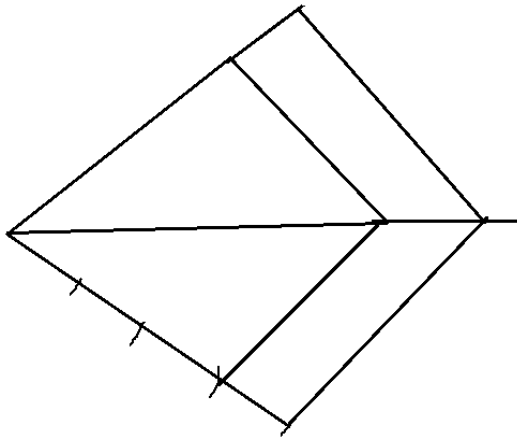
CPCT

(b) $\angle PAB = \angle QAB$

(c) $\angle PBA = \angle QBA$

QED

32.



33.

Area of rectangle is $=20 \times 10 = 200 \text{ cm}^2$.

Here $\angle AOB = 90^\circ$, OAEB is an sector of radius $OA = OB = 10\sqrt{2}$

Area of sector $= \frac{1}{2} r^2 \theta$.

$$= \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2} \times \frac{\pi}{2}$$

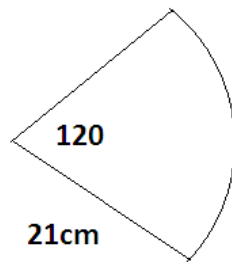
$$= 157.14 \text{ cm}^2.$$

Area of shaded region = area of rectangle – (area of sector - area of AOB)

$$= 200 - (157.14 - 100)$$

$$= 142.85 \text{ cm}^2.$$

OR



$$\begin{aligned} \text{Area of cloth used} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{120}{360} \times 22/7 (21)^2 \\ &= 462 \text{ cm}^2. \end{aligned}$$

$$\begin{aligned} \text{Length of the metallic wire is} & \frac{\theta}{360} \times 2\pi r \\ &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{120}{360} \times 2 \times 3.14 \times 21 \\ &= 44 \text{ cm.} \\ &= 44 + 21 + 21 = 86 \text{ cm} \end{aligned}$$

Solve ----- **4x4=16**

34.

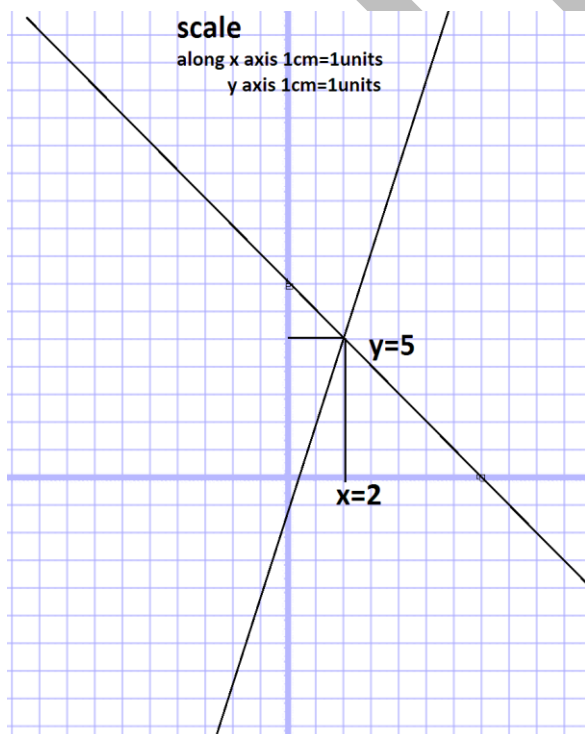
Graphical method The equations are $x+y=7$ & $3x-y=1$

$$x+y=7$$

x	0	7
y	7	0

$$3x-y=1$$

x	0	0.33
y	-1	0



35. Let the terms be $a-2d, a-d, a, a+d, a+2d$

According to first condition sum of its terms is 55

$$\text{So } a-2d+a-d+a+a+d+a+2d=55$$

$$5a=55$$

$$a=11$$

according to 2nd condition $a_4=a_1+a_2+5$

$$a+d=a-2d+a-d+5$$

$$4d=16$$

$$d=4$$

therefore the terms are $a-2d, a-d, a, a+d, a+2d$

$$3, 7, 11, 15 \text{ \& } 19$$

OR

According to first condition $a_6=2(a)_3+1$

$$a+5d=2(a+2d)+1$$

$$a-d=-1 \text{ -----} \rightarrow (1)$$

according to 2nd condition $a_4+a_5=5(a_2)$

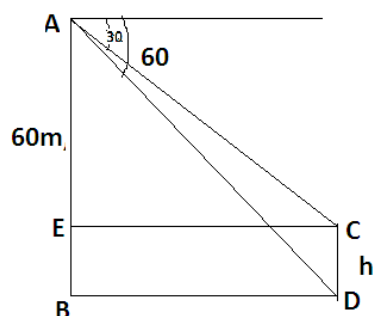
$$a+3d+a+4d=5a+5d$$

$$3a-2d=0 \text{ -----} \rightarrow (2)$$

Solve equation 1 and 2 we get

$$a=2 \text{ and } d=3$$

36.



In triangle ABD, $\angle D=60^\circ$.

$$\text{Tan } 60^\circ = \frac{AB}{BD} =$$

$$\sqrt{3} = \frac{60}{BD} \Rightarrow BD = \frac{60}{\sqrt{3}} = EC$$

Similarly in triangle AEC, $\tan 30^\circ = \frac{AE}{EC}$

$$\frac{1}{\sqrt{3}} = \frac{AE}{60/\sqrt{3}}$$

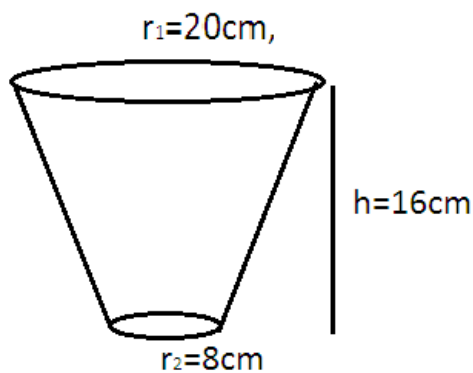
AE = 20m.

Therefore height of the pole is CD = AB - AE

$$= 60 - 20$$

$$= 40\text{m.}$$

37.



$$\begin{aligned} \text{Volume } V &= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 \cdot r_2) \\ &= \frac{1}{3} \times 3.14 \times 16 (400 + 64 + 160) \\ &= \frac{3.14 \times 16 \times 624}{3} \\ &= 10449.92 \text{ cm}^2. \end{aligned}$$

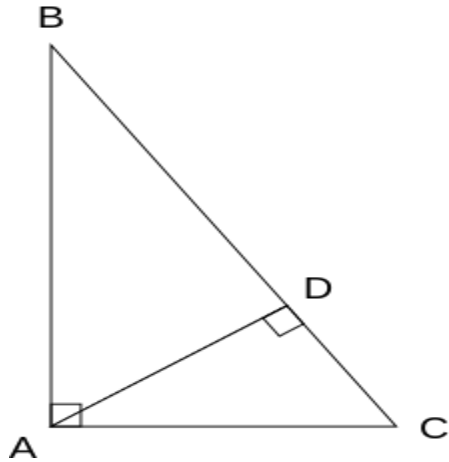
Convert this to the liter we get 10.44.

Therefore cost of the milk per liter is 10.44×20

Approximately Rs. 208.9 = Rs. 209

38. Pythagoras Theorem

In a right angles triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Data : In $\triangle ABC$, $\angle ABC = 90^\circ$

To Prove : $AB^2 + BC^2 = CA^2$

Construction : Draw $BD \perp AC$.

Proof: Statement	Reason
Compare $\triangle ABC$ and $\triangle ADB$, $\angle ABC = \angle ADB = 90^\circ$ $\angle BAD$ is common.	(\square) Data and construction
$\therefore \triangle ABC \sim \triangle ADB$	(\square) Equiangular triangles
$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$	(\square) AA similarity criteria
$\therefore AB^2 = AC \cdot AD$ (1)	
Compare $\triangle ABC$ and $\triangle BDC$, $\angle ABC = \angle BDC = 90^\circ$ $\angle ACB$ is common	(\square) Data and construction
$\therefore \triangle ABC \sim \triangle BDC$	(\square) Equiangular Triangles
$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC} \Rightarrow$ $BC^2 = AC \cdot DC$ (2)	(\square) AA similarity criteria
By adding (1) and (2) we get $AB^2 + BC^2 = (AC \cdot AD) + (AC \cdot DC)$ $AB^2 + BC^2 = AC (AD + DC)$ $AB^2 + BC^2 = AC \cdot AC = AC^2$	[\square] $AD + DC = AC$
$\therefore AB^2 + BC^2 = AC^2$	