

## SUMS AND TERMS-NOTE-2

### PREVIOUS KNOWLEDGE

- **ARITHMETIC SEQUENCE:** A sequence got by starting a fixed Number and adding or subtracting a fixed number repeatedly.
- **COMMON DIFFERENCE (d):** The constant difference got by subtracting from any term the just previous term is called the common difference of an arithmetic Sequence.
- $x_1, x_2, x_3, x_4, x_5, x_6, \dots$  Are the terms of an arithmetic sequence and suffix denote position
- For any arithmetic sequence the sum of three consecutive terms is three times the middle term.
- In any arithmetic sequence the sum of the five consecutive terms is five times the middle term
- In any arithmetic sequence the sum of the odd number of consecutive terms is the product of number of terms and middle term.

$$\text{Sum} = \text{no of terms} \times \text{middle term}$$

E.g.: The sum of first 5 term of an arithmetic sequence is 40. write the sequence?

$$3^{\text{th}} \text{ term} = \text{middle term} = \frac{\text{sum}}{\text{no of terms}} = \frac{40}{5} = 8$$

$$\text{If } d=1 \quad 6, 7, 8, 9, 10$$

$$\text{If } d=2 \quad 4, 6, 8, 10, 12$$

### SUMS AND TERMS

- Sum of four consecutive Natural numbers

$$1 + 2 + 3 + 4$$

$$\text{Here } 1 + 4 = 5$$

$$2 + 3 = 5$$

$$\text{That is } 1 + 2 + 3 + 4 = 2 \times 5 = 10$$

	Sum of position	Sum of terms
1 <sup>st</sup> term+4 <sup>th</sup> term	1+4=5	1+4=5
2 <sup>nd</sup> term+3 <sup>rd</sup> term	2+3=5	2+3=5

- Sum of six consecutive even numbers

$$2 + 4 + 6 + 8 + 10 + 12$$

$$\text{Here } 2 + 12 = 14$$

$$4 + 10 = 14$$

$$6 + 8 = 14$$

$$\text{That is } 2 + 4 + 6 + 8 + 10 + 12 = 3 \times 14 = 42$$

	Sum of position	Sum of terms
1 <sup>st</sup> term+6 <sup>th</sup> term	1+6=7	2+12=14
2 <sup>nd</sup> term+5 <sup>th</sup> term	2+5=7	4+10=14
3 <sup>rd</sup> term+4 <sup>th</sup> term	3+4=7	6+8=14

- Sum of six consecutive terms of any arithmetic sequence.

Suppose  $x_n = an + b$  is algebra of any arithmetic sequence

$$x_1 = a + b$$

$$x_2 = 2a + b$$

$$x_3 = 3a + b$$

$$x_4 = 4a + b$$

$$x_5 = 5a + b$$

$$x_6 = 6a + b$$

Pair	Sum of position	Sum of terms
1 <sup>st</sup> term+6 <sup>th</sup> term $x_1 + x_6$	1+6=7	$(a + b) + (6a + b) = 7a + 2b$
2 <sup>nd</sup> term+5 <sup>th</sup> term $x_2 + x_5$	2+5=7	$(2a + b) + (5a + b) = 7a + 2b$
3 <sup>rd</sup> term+4 <sup>th</sup> term $x_3 + x_4$	3+4=7	$(3a + b) + (4a + b) = 7a + 2b$

That is  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 3 \times (7a + 2b)$

From this we can see that the sum of positions of two pairs of terms is 7 and the sum of pairs of the terms always  $7a + 2b$ . And also we can see that Sum of six consecutive terms is the product of no of pairs and sum of a pair.

In general

*In an arithmetic sequence, if the sum of positions of two pairs of terms are equal, then the sum of the pairs of the terms also equal.*

E.g.: consider an arithmetic sequence

5, 8, 11, 14, 17, 20.....

Pair	Sum of position	Sum of terms
1 <sup>st</sup> term+6 <sup>th</sup> term $x_1 + x_6$	1+6=7	5+20=25
2 <sup>nd</sup> term+5 <sup>th</sup> term $x_2 + x_5$	2+5=7	8+17=25
3 <sup>rd</sup> term+4 <sup>th</sup> term $x_3 + x_4$	3+4=7	11+14=25

$$5+8+11+14+17+20 = 3 \times 25 = 75$$

- Sum of even number of consecutive terms of any arithmetic sequence.

*Sum of even number of consecutive terms of any arithmetic sequence is the product of no of pairs and sum of a pair*

$$\text{Sum} = \text{no.of.pairs} \times \text{sum of a pair}$$

## MORE QUESTIONS TO PRACTICE

1. The first term of an arithmetic sequence is 5 and sum of first six terms is 105.
  - i. Split six terms into three pairs of equal sums
  - ii. What is its 6<sup>th</sup> term?
  - iii. What is the common difference?(use term difference/positional difference)
  - iv. Write the arithmetic sequence.

2.

Prove that for any four consecutive terms of an arithmetic sequence, the sum of the two terms on the two ends and the sum of the two terms in the middle are the same.

Suppose  $x_n = an + b$  is algebra of any arithmetic sequence

$$X_1 = \dots\dots\dots$$

$$X_2 = \dots\dots\dots$$

$$X_3 = \dots\dots\dots$$

$$X_4 = \dots\dots\dots$$

$$X_1 + X_4 = \dots\dots\dots$$

$$X_2 + X_3 = \dots\dots\dots$$

3.

Write four arithmetic sequences with 100 as the sum of the first four terms.

[Click here and watch the video class for better understand](#)

4.

Write the first three terms of each of the arithmetic sequences described below:

i.

First term 30; the sum of the first six terms is 300

ii.

First term 30; the sum of the first four terms is 300

Refer answer of question number 1