SUMS-NOTE-1

- **Sum of natural numbers**
  - ✓ Sum of first 20 natural numbers

    $$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

    For that, take

    $$S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 +$$
    $$S = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$
    $$2S = 11+11+11+11+11+11+11+11+11+11$$
    $$2S = 10 \times 11$$
    $$S = \frac{10 \times 11}{2} = \frac{110}{2} = 55$$

  - ✓ From this sum of first 20 natural numbers

    $$1+2+3+ \ldots\ldots +20 = \frac{20 \times 21}{2} = \frac{420}{2} = 210$$

  - ✓ Sum of first 100 natural numbers

    $$1+2+3+ \ldots\ldots +100 = \frac{100 \times 101}{2} = \frac{10100}{2} = 5050$$

    From these we can make a general statement

    Sum of first n natural numbers is

    $$1+2+3+ \ldots\ldots + n = \frac{n \times (n+1)}{2}$$

- **Sum of even numbers**
  - ✓ Sum of first 10 even numbers

    $$2+4+6+8+10+12+14+16+18+20$$

    For that

    $$2+4+6+8+10+12+14+16+18+20 = 2\,(1+2+3+4+5+6+7+8+9+10)$$

    $$= 2 \left(\frac{10 \times 11}{2} \right)$$
    $$= 10 \times 11$$
    $$= 110$$

✓ Sum of first 20 even numbers

$$2+4+6+ \ldots\ldots.. +40 = 20 \times 21 = 20 \times 21 = 420$$

From this,

Sum of first n even numbers

$$2 + 4 + 6 + 8 + \ldots\ldots.. + 2n = n \times (n + 1)$$

n^{th} even number is 2n

E.g.: Find the sum 2+4+6+8+ ……….. 50.

Here the number of even number is $\frac{50}{2} = 25$

$$2+4+6+8+ \ldots\ldots.. 50 = 25 \times 26 = 650$$

➢ Sum of odd numbers

Sum of 2 odd numbers	$1 + 3 = 4 = 2^2$
Sum of 3 odd numbers	$1 + 3 + 5 = 9 = 3^2$
Sum of 4 odd numbers	$1 +3 + 5 + 7 = 16 = 4^2$

n^{th} odd number is $2n - 1$

Sum of 10 odd numbers $1 + 3 + \ldots.. + 19 = 10^2 = 100$

From these we can make a general statement

Sum of first n odd numbers is

$$1 + 3 + 5 + \ldots\ldots\ldots\ldots. + 2n - 1 = n^2$$

E.g.: Find the sum 1+3+5+ …………… + 29

29 is the $(\frac{29+1}{2})^{th}$ term. That is 15th term so

$$1+3+5+ \ldots\ldots\ldots + 29 = 15^2 = 225$$

MORE EXAMPLES

1. Find the sum of 41 + 42 + 43 + 44 + ………….. + 60

(40+1) + (40+2) + (40+3) + …………. + (40+20)

There are twenty 40s are adding there. That is

41 + 42 + 43 + 44 + ………….. + 60= (40+1) + (40+2) + (40+3) + …………. + (40+20)

$$=20 \times 40 + (1+2+3+ \ldots\ldots\ldots….. + 20)$$

$$=800 + (\frac{20 \times 21}{2})$$

$$=800 +210 =1010$$

2. Find the sum of $1\sqrt{3} + 3\sqrt{3} + 5\sqrt{3} + 7\sqrt{3} + \ldots\ldots + 19\sqrt{3}$

$$1\sqrt{3} + 3\sqrt{3} + 5\sqrt{3} + 7\sqrt{3} + \ldots\ldots + 19\sqrt{3} = \sqrt{3}\,(1 + 3 + 5 + \ldots + 19)$$
$$= \sqrt{3} \times 10^2 \quad \text{(sum of first 10 odd numbers)}$$
$$= 100\sqrt{3}$$

MORE QUESTIONS TO PRACTICE

1. Find the sum of first 10 multiple of 3 (Hint: refer sum of even number)
2. Find the sum of 4 + 8 + 12 + ... +40
3. What is the sum of first 25 odd numbers?
4. The sum of some odd number is 2500.
   a) How many odd numbers are added?
   b) Which is the last odd number?
5. Find the sum of
   a) 1+3+5+7+....................+15
   b) 2+6+10+14+.................+30
   c) 5+9+13+17+.................+33
6.

Calculate in head, the sums of the following arithmetic sequences.

i) $51 + 52 + 53 + \ldots + 70$

ii) $1\frac{1}{2} + 2\frac{1}{2} + \ldots + 12\frac{1}{2}$

iii) $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + \ldots + 12\frac{1}{2}$

[Click here to watch video classes of arithmetic sequence for the youtube channel "anwer classes"](#)