Qn. 26 **(Orukkam - 2017)**

The 5th term of an arithmetic sequence is 40 and 10th term is 20. Find 15th term. How many term of his sequence makes the sum 0.

Ans $x_5 = 40, x_1 + 4d = 40$ ..... (1)

$x_{10} = 20, x_1 + 9d = 20$ ..... (2)

(2) + (1) $5d = -20$

$d = -4$

$x_1 + 4 \times -4 = 40$

$x_1 = 40 + 16$

$= 56$

$x_{15} = x_1 + 14d$

$= 56 + 14 \times -4$

$= 56 + -56 = 0$

$a = 56, d = -4$

$$\frac{n}{2}[2 \times 56 + (n-1) \times -4) = 0$$

$$\frac{n}{2}[112 - 4n + 4] = 0$$

$$\frac{n}{2}[116 - 4n] = 0$$

$116 - 4n = 0$

$4n = 116$

$n = 29$

29 terms of this sequence makes the sum 0

Qn. 27 **(Orukkam - 2017)**

Prove that 5, 8, 11, contains no perfect squares.

Ans Sequence = 5, 8, 11, ...

$x_n = 3n + 2$

Each term in this sequence will give remainder 2 when divided by 4.

When a perfect square is divided by 3 remainder is 1. But in the sequence 5, 8, 11, ... When terms are divided by 3 remainder is 2. So a perfect square will not be a term in this sequence.

1. The first term of an arithmetic sequence is 10 and the third term is 24. What is its common difference. (2)
2. The 3^{rd} term of an arithmetic sequence is 9 and 10^{th} term is 23.
   i) Find the common difference
   ii) Write its 20^{th} term?
   iii) Write the algebraic expression of the sequence. (3)
3. In the arithmetic sequence 3, 7, 11 is 101 is a term? What about 103? (3)
4. Sum of 'n' terms of an arithmetic sequence is $4n^2+3n$
   i) Find first term.
   ii) Find the common difference.
   iii) Write down the sequence. (3)
5. In an arithmetic sequence, the ratio of the first term to the second term is 2:3. What is the ratio of the third term to fifth term? (3)
6. Write down the arithmetic sequence with common difference 8.
   Find the 25^{th} term of the sequence. Write down the algebraic expression for the sequence. (3)
7. The sum of three consecutive terms of an arithmetic sequence is 24 and product is 440. Find the three consecutive terms? (3)

ANSWERS

1. $x_1 = 10$

   Let the common difference be 'd'.

   Then third term = $x_1 + 2d$

   $x_1 + 2d = 24$

   $2d = 24 - 10$

   $= 14$

   $d = 7$

2. i) $f + 2d = 9$ .....(1)

   $f + 9d = 23$ ..... (2)

   (2) -(1) $\rightarrow$ $7d = 14$

   $d = 2$

   $f = 9 - 2d$

   $= 9 - 4 = 5$

   ii) 20^{th} term $= f + 19d$

   $= 5 + 38 = 43$

   iii) $x_n = dn + (f - d)$

   $= 2n + 3$

3. From any term of this sequence if we subtract 3 then we get the multiple of 4.

   101 - 3 = 98. 98 is not a multiple of 4. Therefore the number 101 is not a term of this sequence. 103 - 3=100, 100 is a term of this sequence.

4. Sum of n term of the arithmetic sequence is $4n^2+3n$

i) $x_1 = 4+3 = 7$

$x_1+x_2 = 6 \times 2^2 + 3 \times 2$

$= 16 + 6 = 22$

$x_2 = 22 - 7 = 15$

ii) Common difference = 15 - 7= 8

iii) The sequence = 7, 15, 23, ...

5. Let the first term be 'f'

Second term = f + d

That is $\frac{f}{f+d} = \frac{2}{3}$

$3f = 2f + 2d$

$f = 2d$

Third term = f+2d

= f+f = 2f

Fifth term = f+4d

= f+2×2d

= f+2f = 3f

Ratio = 2f : 3f

= 2 : 3

4. Sum of n term of the arithmetic sequence is $4n^2+3n$

i) $x_1 = 4+3 = 7$

$x_1+x_2 = 6 \times 2^2 + 3 \times 2$

$= 16 + 6 = 22$

$x_2 = 22 - 7 = 15$

ii) Common difference = 15 - 7= 8

iii) The sequence = 7, 15, 23, ...

5. Let the first term be 'f'

Second term = f + d

That is $\frac{f}{f+d} = \frac{2}{3}$

$3f = 2f + 2d$

$f = 2d$

Third term = f+2d

= f+f = 2f

Fifth term = f+4d

= f+2×2d

= f+2f = 3f

Ratio = 2f : 3f

= 2 : 3

6. Sequence is 2, 10, 18, ...

25^{th} term $= 2+24\times8$

$= 2+192 = 194$

Algebraic expression $= 8n - 6$

7. Let three consecutive terms are $(a-d)$, a, $a+d$.

$a - d + a + a + d = 24$

$3a = 24$

$a = 8$

Product = 440

$(a-d)(a+d)\,a = 440$

$a(a^2-d^2) = 440$

$$a^2-d^2 = \frac{440}{8}$$

$64 - d^2 = 55$

$-d^2 = 55-64$

$-d^2 = -9$

$d^2 = 9$

$d = \pm 3$

If $d = 3$

The terms are 5, 8, 11

if $d = -3$, the terms are 11, 8, 5