Today's Maths Class - X - 15 (28 / 07 /2020)

<u>WORK SHEET</u>

Q. *Consider an arithmetic sequence 5 , 8 , 11 ....*

 a) *What is its common difference ?*

 b) *What is its 20th term ?*

 c) *Find the sum of first 20 terms of this sequence   ?*

<u>*Answer*</u> :

 a) $d = 8 - 5 = 3$

 b) $x_{20} = f + 19 \times d = 5 + 19 \times 3 = 5 + 57 = 62$

 c) $S_{20} = \dfrac{20}{2} \, (x_1 + x_{20}) = \dfrac{20}{2} \, (5 + 62) = 10 \times 67 = 670$

Q. *Consider an arithmetic sequence 6 , 10 , 14 ....*

 a) *What is its common difference ?*

 b) *What is its algebraic form ?*

 c) *Find the position of 122 in this sequence   ?*

 d) *Calculate the sum   6 + 10 + 14 + ... + 122*

<u>Answer .</u>

a) $d = 10 - 6 = 4$

b) $x_n = dn + f - d = 4 \times n + 6 - 4 = 4n + 2$

c) Take $x_n = 122$

$\qquad 4n + 2 = 122$

$\qquad 4n = 122 - 2 = 120$ ===> $n = \dfrac{120}{4} = 30$

d) $6 + 4 + 10 + ... + 122 = \dfrac{n}{2}(x_1 + x_n) = \dfrac{30}{2}(6 + 122)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 15 \times 128 = 1920$

<u>Solve the following questions</u>

1). *Consider an arithmetic sequence 6 , 11, 16 ....*

 a) *What is its common difference ?*

 b) *What is its 21ˢᵗ term ?*

 c) *Find the sum of first 21 terms of this sequence   ?*

2). *Consider an arithmetic sequence 7 , 13 , 19   ....*

 a) *What is its common difference ?*

 b) *What is its algebraic form ?*

 c) *Find the position of 91 in this sequence   ?*

 d) *Calculate the sum   7 + 13 + 19 + ... + 91*

3). *Consider the sequence of three digit numbers which leave a remainder 1 on divisible by 3 .*

 a) *What is its common difference ?*

 b) *Which is the smallest number in this sequence ?*

 c) *How many three digit numbers are there ,which leave a remainder 1 on divisible by 3?*

 d) *What is the sum of such numbers ?*

4). **Look at the number pattern given below**

1

2 3

4 5 6

7 8 9 10

....

....

a) **Write the next two more line of this pattern ?**

b) **How many numbers are there in the 10th line?**

c) **What is the last number in the 9th line ?**

d) **What is the first number in the 10th line ?**

e) **What is the last number in the 10th line ?**

f) **What is the sum of the numbers in the 10th line ?**

5). **The sum of 10th and 11th terms of an arithmetic sequence is 65 .**

a) **What is the sum of its first and 20th terms ?**

b) **What is the sum of first 20 terms of this sequence ?**

c) **If the 4th term of this sequence is 13 , what is its 17th term ?**

d) **What is the common difference of this sequence ?**

e) **What is the algebraic form of this sequence ?**

What did we learn in the last class ?

We learned how to find the algebraic form of the sum of an arithmetic sequence if its

algebraic form is given .

If the algebraic form of an arithmetic sequence is , $x_n = a\,n + b$

The algebraic form of its sum is , $S_n = a\,\dfrac{n(n+1)}{2} + b\,n$

Let's solve the following problem using this idea .

If the algebraic form of an arithmetic sequence is 3 n + 2 , find the sum of first 10 terms ?

Answer

$$x_n = 3\,n + 2$$

$$S_n = 3\,\frac{n(n+1)}{2} + 2\,n$$

$$S_{10} = 3 \times \frac{10 \times 11}{2} + 2 \times 10$$

$$= 3 \times 55 + 20$$

$$= 165 + 20$$

$$= 185$$

Another way of finding the sum

Is there any other way to find the sum of a particular number of terms of an arithmetic

sequence other than the algebraic form method ? Let's try .

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We know that the if the algebraic form of an arithmetic sequence is , $x_n = a \times n + b$

the algebraic form of its sum is $\quad S_n = a\,\dfrac{n(n+1)}{2} + b\,n$

$$S_n = a\,\frac{n(n+1)}{2} + b\,n$$

$$= \frac{n}{2}\,a\,(n+1) + \frac{n}{2} \times 2\,b$$

$$= \frac{n}{2}\,(a\,n + a) + \frac{n}{2} \times 2\,b$$

$$= \frac{n}{2}\,(a\,n + a + 2\,b)$$

$$= \frac{n}{2}\,(a\,n + a + \underline{b + b})$$

$$= \frac{n}{2}\,[(a + b) + (a\,n + b)]$$

$$= \frac{n}{2}\,(x_1 + x_n)$$

Sum of first n terms of an arithmetic sequence is the product of half of n and the sum of first

and n^{th} terms .

$$S_n = \frac{n}{2}\,(x_1 + x_n)$$

.

Let's solve more problems .

1) Consider an arithmetic sequence 100 , 95 , 90 , ... Find the sum of terms from 20th

to 40th (including both the terms)

Answer

$$S_n = \frac{n}{2}\,(x_1 + x_n)$$

f = 100 , d = 95 – 100 = - 5

$$x_{20} = f + 19d = 100 + 19 \times (-5) = 100 - 95 = 5$$

$$x_{40} = x_{20} + 20d = 5 + 20 \times (-5) = 5 - 100 = -95$$

n = 40 − 20 + 1 = 21

$$Sum = \frac{21}{2}(x_{20} + x_{40})$$

$$= \frac{21}{2}[5 + (-95)]$$

$$= \frac{21}{2} \times -90$$

$$= -945$$

NB : *We can find 40 th term from the first term also .*

$$x_{40} = f + 39d = 100 + 39 \times (-5) = 100 - 195 = -95$$

2) *Find the product* $3^2 \times 3^7 \times 3^{12} \times \dots\dots\dots \times 3^{147}$

Answer

$$3^2 \times 3^7 \times 3^{12} \times \dots \times 3^{147} = 3^{(2+7+12+\dots+147)}$$

2 , 7 , 12 , ... *is an arithmetic sequence .*

$$f = 2, \quad d = 7 - 2 = 5, \quad x_n = 147$$

$$x_n - x_1 = (n - 1)d$$

$$x_n - x_1 = 147 - 2 = 145$$

$$(n - 1)d = 145 \implies (n - 1)5 = 145$$

$$n - 1 = \frac{145}{5} = 29$$

$$n = 29 + 1 = 30$$

$$2 + 7 + 12 + \ldots\ldots\ldots + 147 = \frac{n}{2}(x_1 + x_n)$$

$$= \frac{30}{2}(2 + 147)$$

$$= 15 \times 149 = 2235$$

$$3^2 \times 3^7 \times 3^{12} \times \ldots\ldots \times 3^{147} = 3^{2235}$$

NB :   *We can find the number of terms in the following way also .*

Algebraic form of the sequence , $x_n = d \times n + f - d$

$$= 5 \times n + 2 - 5 = 5n - 3$$

$$5n - 3 = 147$$

$$5n = 147 + 3$$

$$5n = 150$$

$$n = \frac{150}{5} = 30$$

More activities (Text book page 35)

(1)   Find the sum of the first 25 terms of each of the arithmetic sequences below.

i)   11, 22, 33, ...   ii) 12, 23, 34, ...   iii) 21, 32, 43, ...
iv) 19, 28, 37, ...   v) 1, 6, 11, ...

(2)   What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14, ...?

(3)   Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 6, 10, 14, ... and 15, 19, 23, ...

(4)   Find the sum of all three digit numbers, which are multiples of 9.

WORK SHEET

Q. *The sum of first n terms of an arithmetic sequence is* $3\,n^2 + 4\,n$.

 a) *What is its first term* ?

 b) *What is its common difference* ?

 c) *What is its algebraic form* ?

Answer

a) $f = 3 + 4 = 7$

b) $d = 2 \times 3 = 6$

c) $x_n = dn + f - d = 6 \times n + 7 - 6 = 6\,n + 1$

Q. *Consider the arithmetic sequence* 7 , 11 , 15 ....

 a) *What is its common difference* ?

 b) *What is algebraic form of the sequence* ?

 c) *What is its sum of first n terms* ?

Answer

a) $d = 11 - 7 = 4$

b) $x_n = dn + f - d = 4 \times n + 7 - 4 = 4\,n + 3$

c) $S_n = 2\,n^2 + 5\,n$ $\left(p = \dfrac{4}{2}\ ,\ q = 7 - 2 = 5\right)$

Let's solve the following questions

1.) *The sum of first n terms of an arithmetic sequence is* $2n^2 + 5n$.

 a) *What is its first term* ?

 b) *What is its common difference* ?

c) *What is its algebraic form* ?

2.) *The sum of first n terms of an arithmetic sequence is* $n^2 + 4n$.

 a) *What is its first term* ?

 b) *What is its common difference* ?

c) *What is the algebraic form of the sequence* ?

3.) *Consider the arithmetic sequence* 8 , 12 , 16 ....

 a) *What is its common difference* ?

 b) *What is its algebraic form* ?

c) *What is its sum of first n terms* ?

4.) *Fifth term of an arithmetic sequence is* 23 *and its tenth term is* 43 .

 a) *What is its common difference* ?

 b) *What is its first term* ?

 b) *What is its algebraic form* ?

c) *What is its sum of first n terms* ?

5.) *The algebraic form of an arithmetic sequence is* $8n + 1$

 a) *What is its common difference* ?

 b) *What is its first term* ?

 c) *What is its sum of first n terms* ?

Let's discuss some problems related to arithmetic sequence .

Prove that the sum of any number of terms in the arithmetic sequence 16 , 24 , 32, ...

starting from the first term added to 9 gives a perfect square ?

Answer .

$$d = 24 - 16 = 8$$

$$x_n = d\,n + f - d = 8\,n + 16 - 8 = 8\,n + 8$$

$$S_n = p\,n^2 + q\,n \qquad\qquad (\quad p = \frac{d}{2} = \frac{8}{2} = 4$$

$$\qquad = 4\,n^2 + 12\,n \qquad\qquad p + q = f = 16 \quad)$$

$$S_n + 9 = (4\,n^2 + 12\,n) + 9$$

$$\qquad = (2\,n)^2 + 2 \times (2\,n) \times 3) + 3^2$$

$$\qquad = (2\,n + 3)^2$$

$S_n + 9$ *is a perfect square .*

Find the difference between the sums of the first 50 terms of the arithmetic sequences

5, 8 , 11 , ... and 9 , 12 , 15 , ... ?

Answer .

Difference between the sums =

$$9 + 12 + 15 + ... + x_{50} -$$
$$(5 + 8 + 11 + ... + y_{50})$$

$$=$$

$$9 + 12 + 15 + 18 + 21 + \ldots\ldots\ldots\ldots + x_{50} -$$

$$5 + 8 + 11 + 14 + 17 + \ldots\ldots\ldots + y_{50}$$

$$\overline{}$$

$$4 + 4 + 4 + 4 + 4 + \ldots\ldots\ldots + 4$$

$$= 50 \times 4 = 200$$

Find the difference between the sums of the first 50 terms of the arithmetic sequences

5, 8 , 11 , ... and 6 , 10 , 14 , ... ?

Answer .

Difference between the sums =

$$6 + 10 + 14 + \ldots\ldots\ldots + x_{50} -$$

$$(5 + 8 + 11 + \ldots\ldots\ldots + y_{50})$$

$$=$$

$$6 + 10 + 14 + 18 + 22 + \ldots\ldots\ldots + x_{50} -$$

$$5 + 8 + 11 + 14 + 17 + \ldots\ldots\ldots + y_{50}$$

$$\overline{}$$

$$1 + 2 + 3 + 4 + 5 + \ldots\ldots\ldots + \ldots\ldots$$

$$= \text{ Sum of first 50 natural numbers .}$$

$$= \frac{50 \times 51}{2}$$

$$= 1275$$

Find the difference between the sums of the first 50 terms of the arithmetic sequences

5, 8 , 11 , ... and   6 , 11 , 16 , ... ?

Answer .

Difference between the sums $=$

$6 \ + \ 11 \ + \ 16 \ + ... + \ x_{50} \ -$

$$(5 \ + \ 8 \ + \ 11 \ + ... + \ y_{50})$$

$=$

$6 \ + \ 11 \ + \ 16 \ + \ 21 \ + \ 26 \ + ... + \ x_{50} \ -$

$5 \ + \ 8 \ + \ 11 \ + \ 14 \ + \ 17 \ + ... + \ y_{50}$

_____

$1 \ + \ 3 \ + \ 5 \ + \ 7 \ + \ 9 \ + ... + \ ......$

$=$ *Sum of first 50 odd numbers* .

$$= 50^2$$

$$= 2500$$

Prove that the square of each term of the arithmetic sequence 4 , 7 , 10 , ... also belongs to this sequence ?

Answer .

$$d = 7 - 4 = 3$$

$$x_n = d \, n + f - d = 3 \, n + 4 - 3 = 3 \, n + 1$$

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$$x_n{}^2 = (3n+1)^2$$

$$= (3n)^2 + 2 \times 3n \times 1 + 1^2$$

$$= 9n^2 + 6n + 1$$

$$x_n{}^2 - 4 = (9n^2 + 6n + 1) - 4$$

$$= 9n^2 + 6n - 3$$

$$= 3 \times 3n^2 + 3 \times 2n - 3 \times 1$$

$$= 3(3n^2 + 2n - 1)$$

Since the term difference is exactly divisible by the common difference $x_n{}^2$, is also a term of this sequence . That is , square of each term of the sequence belongs to this sequence.

More activities (*Text book page 35*)

(2) What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14, ...?

(3) Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 6, 10, 14, ... and 15, 19, 23, ...

(4) Find the sum of all three digit numbers, which are multiples of 9.

(5) The expressions for the sum to n terms of some arithmetic sequences are given below. Find the expression for the nth term of each:

i) $n^2 + 2n$ ii) $2n^2 + n$ iii) $n^2 - 2n$

iv) $2n^2 - n$ v) $n^2 - n$

WORKSHEET

1) *Consider the arithmetic sequence 5 , 7 , 9 , ...*

 a) *What is its common difference ?*

 b) *What is its algebraic form ?*

 c) *Add 4 to the sum of first three terms .*

 d) *What is the sum of first n terms ?*

 e) *Prove that the sum of any number of terms in this sequence starting from the first term*

   added to 4 gives a perfect square ?

2) *Consider the arithmetic sequence 6 , 11 , 16 , ...*

 a) *What is its common difference ?*

 b) *Is 36 a term of this sequence ? Give reason .*

 c) *What is its algebraic form ?*

 d) *Prove that the square of each term of this sequence also belongs to it ?*

3) *The sum of first 13 terms and the sum of next 12 terms of an arithmetic sequence are*

   equal . If its common difference is 3 ,

 a) *How many times of the common difference will be the difference between 14th and first*

   terms of this sequence ?

 b) *What is the 13th term of this sequence ?*

 c) *What is the sum of first 25 terms of this sequence ?*

4) *The sum of first 7 terms of an arithmetic sequence is 119 and the sum of first 20 terms*

   is 860.

 a) *What is its fourth term ?*

 b) *What is its 17th term ?*

c) *What is the algebraic form of this sequence ?*

5) *The sum of 6th and 10th terms of an arithmetic sequence is 66 .*

 a) *What is the sum of first and 15th terms of this sequence ?*

 b) *What is its 8th term ?*

 c) *What is the sum of first15 terms of this sequence ?*

6) *Consider the arithmetic sequence 4 , 12 , 20 , ………*

 a) *What is the common difference of this sequence ?*

 b) *What is the sum of first 4 terms of this sequence ?*

 c) *Can the sum of any 25 terms of this sequence be 1090 ? Why ?*