

INTRODUCTION TO CIRCLES



KERALA SSLC MATHS

Chapter 2

CIRCLES വൃത്തങ്ങൾ



PART 1

Introduction to
circles

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GEOMETRY

Plane Geometry is about flat shapes like lines, circles and triangles ... shapes that can be drawn on a piece of paper

Solid Geometry is about three dimensional objects like cubes, prisms, cylinders and spheres.

Point: A point is a location in space. It is represented by a dot.

Line: A line is a collection of points that extend forever.

Line segment: A line segment is part of a line.

Ray: A ray is a collection of points that begin at one point (an endpoint) and extend forever on one direction.

Angle: Two rays with the same endpoint is an angle.

- Sum of angle on a straight line is 180° .
- Sum of angle of one whole turn or at a point is 360° .

Acute angle: An acute angle has a measure less than 90° .

Right angle: An angle whose measure is 90° is called right angle.

Obtuse angle: An angle whose measure is more than 90° is called obtuse angle.

Complementary angle: Two angles are called complementary angle if their sum is equal to 90° .

Supplementary angle: Two angles are called supplementary angle if their sum is equal to 180° .

Adjacent angle: when the angles have a common arm and a common vertex, they are called adjacent angle.

Vertex: The point where two rays meet is called a vertex.

Plane: A plane is a flat surface like a piece of paper.

Collinear points: Three or more points lying on the same line are called collinear points

Parallel lines When two lines never meet in space.

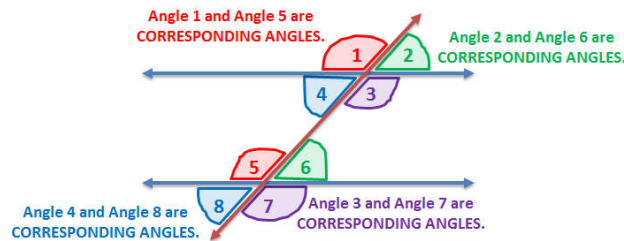
- ❖ Three or more parallel lines cut any two lines in the same ratio.
- ❖ In any triangle, a line drawn parallel to a side cuts the other two sides in the same ratio.
- ❖ In any triangle, the line draw parallel to one side, passing through the midpoint of another side, meets the third side also at its midpoint.
- ❖ The length of the line joining the midpoint of two sides of a triangle is half the length of the third side.
- ❖ In any triangle, all the perpendiculars from the vertices to the opposite side passes through a single point. It is called **orthocentre**.
- ❖ In any triangle, all the lines from the vertices to the midpoints of the opposite side passes through a single point. It is called **centroid**. That lines are called median line. Centroid divides the median line in the ratio 2:1.

Intersecting lines: When lines meet in space.

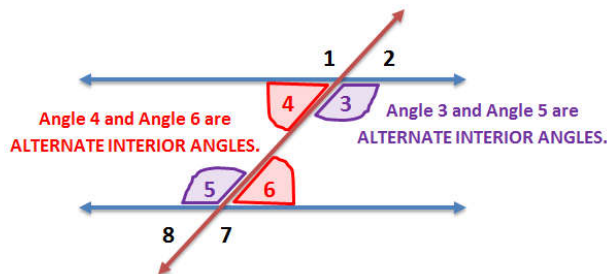
Perpendicular lines: If the angle between two lines is 90° , then those lines are called perpendicular lines.

Transversal: A straight line which cuts two or more given straight lines is called transversal.

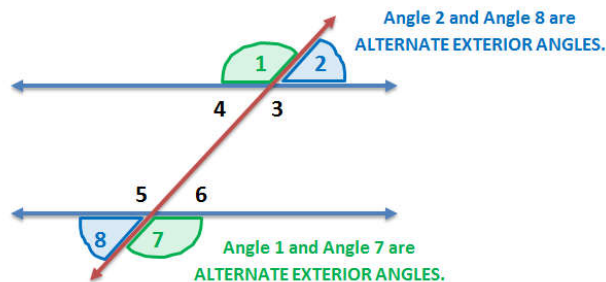
Corresponding Angles - a pair of angles at matching intersections of the parallel lines and the transversal



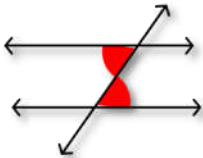
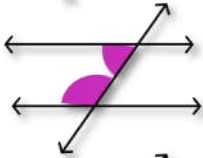
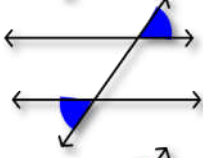
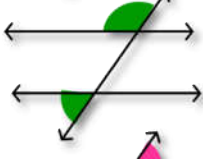
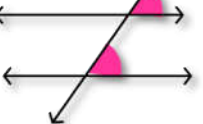
Alternate Interior Angles - a pair of angles in between the parallel lines that are on the opposite sides of the transversal



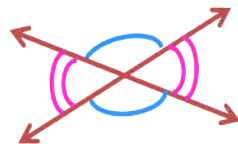
Alternate Exterior Angles - a pair of angles outside the parallel lines that are on the opposite sides of the transversal



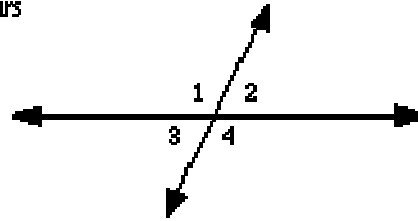
So, if the lines are parallel, what can you conclude about:

<u>Alternate Interior Angles</u>		Equal
<u>Co-Interior Angles</u>		Supplementary
<u>Alternate Exterior Angles</u>		Equal
<u>Co-Exterior Angles</u>		Supplementary
<u>Corresponding Angles</u>		Equal

Vertical Angles: The **opposite angles** formed by two intersecting lines. When two lines intersect, the resulting pairs of vertical angles are **equal**



Linear Pair of Angles: A pair of adjacent angles formed by intersecting lines. Angles 1 and 2 below are a linear pair. So are angles 2 and 4, angles 3 and 4, and angles 1 and 3. Linear pairs of angles are **supplementary**.

Linear Pairs
of Angles**MENSURATION**

Mensuration is the branch of geometry which deals with measurement of length, areas, volumes and so on

Perimeter : Perimeter is sum of all the sides. It is measured in cm, m etc.

Area : The area of any figure is the amount of surface enclosed within its boundary lines. This is measured in square unit like cm^2 , m^2 , etc.

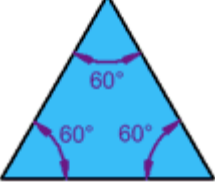
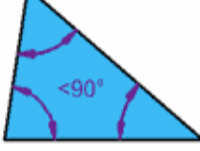
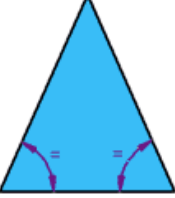
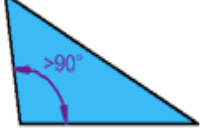
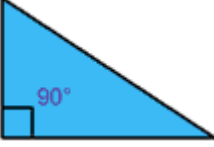

Volume : If an object is solid, then the space occupied by such an object is called its volume. This is measured in cubic units like cm^3 , m^3 , etc.

Basic conversions:

Length	Weight	Volume	Area
1 cm = 10 mm	1 c gm = 10 m gm	1 c. l = 10 m. l	1 m ² = 10,000 cm ²
1 m = 100 cm	1 gm = 1000 m gm	1 l = 1000 m. l	1 cm ² = 100 mm ²
1 km = 1000 m	1 kg = 1000 gm	1 k l = 1000 l	
1 m ³ = 1000 litre		1 m ³ = 1000000 cm ³	

TRIANGLES

- ❖ A triangle has three sides and three angles
- ❖ The three angles always add to 180°

	<p>Equilateral Triangle</p> <p>3 equal sides</p> <p>3 equal angles, always 60°</p> <p>Area = $\frac{\sqrt{3}a^2}{4}$</p>		<p>Acute Triangle</p> <p>All angles are less than 90°</p>
	<p>Isosceles Triangle</p> <p>Two equal sides</p> <p>Two equal angles</p>		<p>Obtuse Triangle</p> <p>Has an angle more than 90</p>
	<p>Right Triangle</p> <p>Has a right angle (90°)</p>		<p>Scalene Triangle</p> <p>No equal sides</p> <p>No equal angles</p>

ISOSCELES TRIANGLES :

- ❖ If two sides of a triangle are equal, then the angles opposite to these sides are also equal.
- ❖ If two angles of a triangle are equal, then the sides opposite to these angles are also equal.
- ❖ In an isosceles triangle, the perpendicular from the vertex joining the equal sides to the opposite side bisects this side and the angle at this vertex.

EQUAL (CONGRUENT) TRIANGLES

❖ When Sides are equal :

If three sides of a triangle are equal to the three sides of another triangle, then these triangles are congruent.

❖ Two sides and an Angle :

If two sides of a triangle and their included angle are equal to two sides of another triangle and their included angle, then these triangles are congruent.

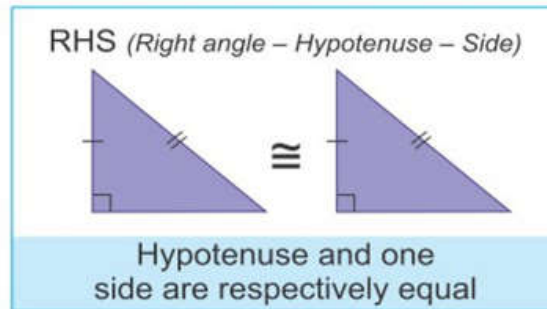
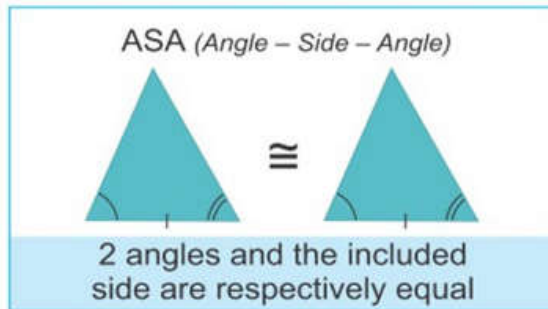
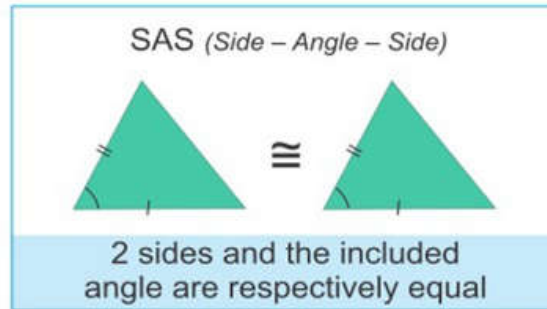
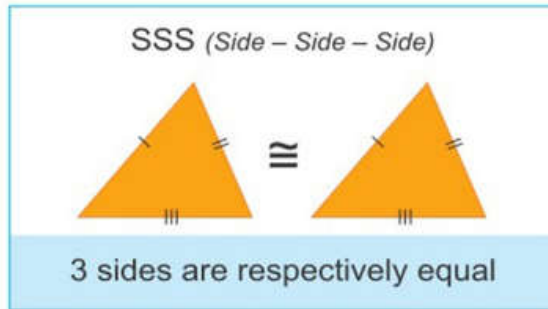
❖ One Side and Two Angles :

If one side and the two angles on it of a triangle are equal to one side and the two angles on it of another triangle, then these triangles are congruent.

❖ Right Angled triangles :

If the hypotenuse and one other side of a right angled triangle are equal to the hypotenuse and one other side of another right angled triangle, then these two triangles are congruent.

✓ **If two triangles are congruent, then the sides and angles of one are equal to the sides and angles of the other. Angles opposite to equal sides are equal and sides opposite to equal angles are equal.**

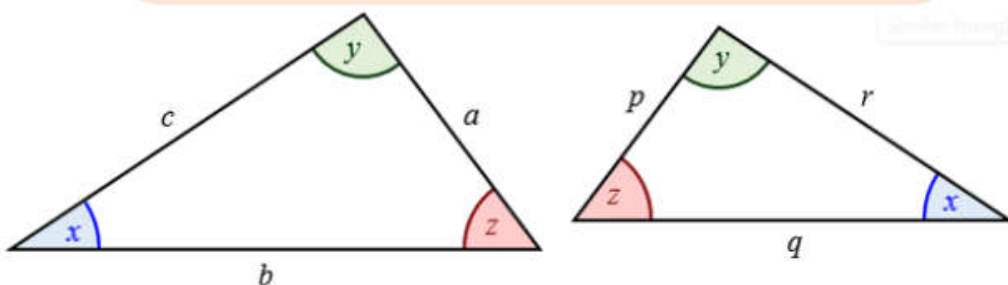


SIMILAR TRIANGLES

If three angles of a triangle are equal to three angles of another triangle, then the triangles are similar. In similar triangles, sides are proportional.

- Same shape, but not necessarily the same size.
- Corresponding angles are equal.
- Corresponding sides are in the same ratio.

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$$



To test for similar triangles:

- **AA** – If 2 corresponding angles are equal.
- **SSS** – If 3 corresponding sides are in the same ratio.
- **SAS** – Ratio of 2 pairs of corresponding sides are equal and their included angles are equal.

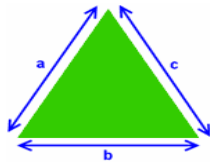
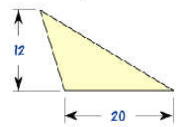
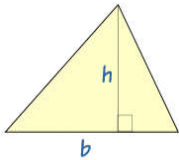
Perimeter of a triangle:

- ❖ The perimeter is the distance around the edge of the triangle: just add up the three sides.

Area

The area is half of the base times height.

$$\text{Area} = \frac{1}{2} bh$$

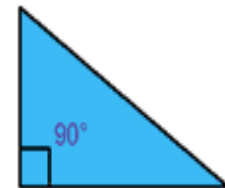
**Area of a Triangle from Sides (Heron's Formula)**

Step 1: Calculate "s" (half of the triangles perimeter) using:

$$S = \frac{a+b+c}{2}$$

Step 2: Then calculate the Area using:

$$\sqrt{s(s-a)(s-b)(s-c)}$$

**Pythagoras' Theorem**

$$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Altitude}^2$$

$$\text{Base}^2 = \text{Hypotenuse}^2 - \text{Altitude}^2$$

$$\text{Altitude}^2 = \text{Hypotenuse}^2 - \text{Base}^2$$

PYTHAGOREAN TRIPLETS:

- ✓ "Pythagorean triples" are integer solutions to the Pythagorean Theorem, $a^2 + b^2 = c^2$

- ✓ Common Pythagorean triplets:

3, 4, 5

6, 8, 10

5, 12, 13 etc

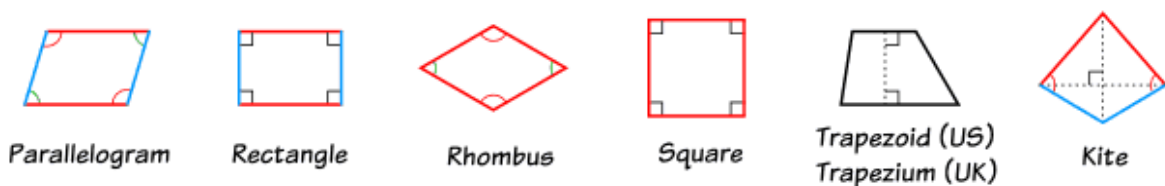
- ✓ For any natural number $n > 1$ $2n, n^2 - 1, n^2 + 1$ form a Pythagorean triplet.
- ✓ When m and n are any two positive integers ($m < n$)
 $n^2 - m^2, 2nm, n^2 + m^2$ form a Pythagorean Triplet

QUADRILATERALS

Any four-sided shape is a Quadrilateral.

- Four sides (edges)
- Four vertices (corners)
- The interior angles add up to **360 degrees**

Types of Quadrilateral:

**RECTANGLE**

- ✓ A rectangle is a four-sided shape where every angle is a right angle (90°).
- ✓ Opposite sides are parallel and of equal length.
- ✓ Diagonals are equal.
- ✓ Diagonals bisect each other.

- | | |
|--|--|
| • <u>Area of a Rectangle</u> | = length \times breadth = lb |
| • <u>Perimeter of a Rectangle</u> | = $2(\text{length} + \text{breadth}) = 2(l + b)$ |
| • <u>Diagonals of a Rectangle</u> | = $\sqrt{\text{length}^2 + \text{breadth}^2}$ |

SQUARE

- A Square is a flat shape with 4 equal sides and every angle is a right angle
- Opposite sides are parallel (so it is a Parallelogram).
- A square also fits the definition of a rectangle (all angles are 90°), and a rhombus (all sides are equal length).
- Diagonals are equal and perpendicular bisectors.

✓ <u>Perimeter of a Square</u>	= $4a$
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✓ <u>Area of a Square</u>	= a^2 = $\frac{d^2}{2}$
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✓ <u>Diagonals of a Square</u>	= $\sqrt{2} a$
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PARALLELOGRAM

- A parallelogram has opposite sides parallel and equal in length. Also opposite angles are equal.
- Opposite angles add up to 180° , so they are supplementary angles.
- Squares, Rectangles and Rhombuses are all Parallelograms.
- Diagonals of a Parallelogram bisect each other.

<u>Area of a Parallelogram</u>	Area	= bh
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<u>Perimeter of a Parallelogram</u>
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Perimeter = 2 (base + side length)
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RHOMBUS

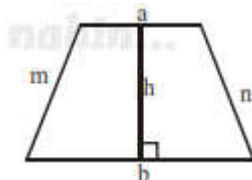
- A Rhombus is a four-sided shape where all sides have equal length.(it is a square)
- Also opposite sides are parallel *and* opposite angles are equal. (it is a Parallelogram).
- The **altitude** is the distance at right angles to two sides
- The diagonals "d₁" & "d₂" of a rhombus bisect each other at right angles.

✓ <u>Area of a Rhombus</u>	Area = bh = $\frac{(d_1 \times d_2)}{2}$
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✓ <u>Perimeter of a Rhombus</u>	The Perimeter is the distance around the edges. Perimeter = 4s
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TRAPEZIUM(TRAPEZOID)

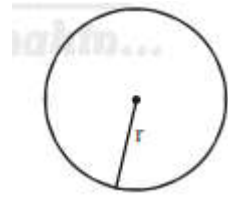
- A pair of parallel sides
- The parallel sides are the "bases"
- The other two sides are the "legs"
- The distance from one base to the other is called the "altitude"

**Area of a Trapezoid**

$$\text{Area} = \frac{a+b}{2} \times h$$

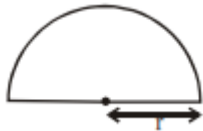
Perimeter of a Trapezoid

$$\text{Perimeter} = a+b+m+n$$

CIRCLE

$$\text{Perimeter (Circumference)} = 2\pi r = \pi d$$

$$\text{Area} = \pi r^2 \quad \left[= \frac{22}{7} = 3.14 \right]$$

SEMI CIRCLE

$$\text{Perimeter} = \pi r + 2r$$

$$\text{Area} = \frac{1}{2} \pi r^2$$

Chord: The line segment joining any two points of the circle is called a chord.

Radius: The distance between centre of the circle and any point on the circle is called its radius.

Diameter: A chord that passes through the centre of a circle is called its diameter. Diameter is the largest chord in a circle.

Arc: The part of a circle that is cut off by a chord is called an arc of the circle.

$$\text{Length of arc} = 2\pi r \times \frac{x}{360}$$

Sector: A sector is a figure enclosed by two radii and the area lying between them.

$$\text{Area of sector} = \pi r^2 \times \frac{x}{360}$$

Segment of a circle: A region bounded by an arc and a chord is called segment of a circle.

- ❖ The perpendicular bisector of any chord of a circle passes through its centre. (The perpendicular from the centre of a circle to a chord bisects the chord)
- ❖ Chords are of the same distance from the centre are of the same length.
- ❖ In any triangle, the perpendicular bisectors of all three sides intersect at a single point. It is called circumcentre.

POLYGONS

$$\text{Sum of angles} = (n - 2) \times 180^{\circ}$$

$$\text{Sum of exterior angles} = 360^{\circ}$$

$$\text{Number of diagonals} = \frac{n(n-3)}{2}$$