Mathematics X Circles

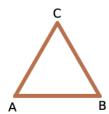
16

Concepts

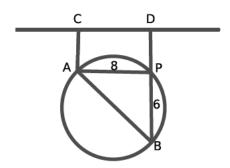
- a) Angle in the semicircle is 90°
- b) Angle outside the semicircle is less than 90°
- c) Angle inside the semicircle is greater than $90^{\circ}\,$

Worksheet16

1) ABC is a triangle in which AB = AC = BC

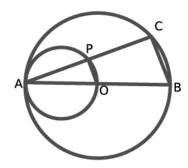


- a) What are the measure of its angles?
- b) What is the position of the vertex C based on the circle with diametre AB? Is it inside ,outside or on the circle.
- 2) In the figure AB is the diametre of the circle AC and PD are perpendicular to CD



a) What is the measure of angle APB?

- b) Suggest a suitable name to ACDP
- c) If $AP=8\mathrm{cm}$ and $BP=6\mathrm{cm}$ then what is the radius of this circle?
- 3) O is the centre of the circle with diametre $AB. {\it Another}$ circle is drawn with AO as the diametre



- a) What are the measure of $\angle APO$, $\angle ACB$
- b) Outer circle has radius $5 {\rm cm}$ and $BC=8 {\rm cm}$. What is the length OP?
- c) Is AP = PC? Why? [d)] What is the length of AC?
- 4) Draw a circle of radius $3\,\mathrm{cm}$ and construct a rectangle with vertices on the circle. One side of the rectangle should be $4\,\mathrm{cm}$. What is the length of other side?(Write the measurement)
- 5) Sides of triangle ABC are $AB=5\mathrm{cm}$, $AC=12\mathrm{cm}$, $BC=13\mathrm{cm}$
 - a) What kind of triangle is this?
 - b) What is the position of A based on the circle with diametre BC?
 - c) What is the position of ${\cal C}$ based on the circle with diametre AB?
 - d) What is the position of B based on the circle with diametre AB?

Mathematics X Circles

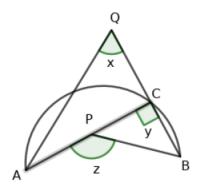
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Concepts

- a) Angle in the semicircle is $90^{\circ}\,$
- b) Angle outside the semicircle is less than $90^{\circ}\,$
- c) Angle inside the semicircle is greater than $90^{\circ}\,$

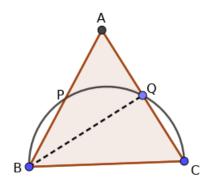
Worksheet17

1) In the figure AB is the diametre of a semicircle.Three angles x,y,z are marked outside, on the semicircle and inside the semicircle.



- a) What is the value of y?
- b) If x,y,z are in an arithmetic sequence,then what is x+z?
- c) If the common difference of the sequence is 50 then find \boldsymbol{x} and \boldsymbol{z}
- 2) a) Draw a circle of radius 3cm. Construct a square with vertices are on the circle.
 - b) What is the length of its side?
 - c) Calculate the area of the square.
- 3) In triangle ABC, AB=AC.A circle is drawn with one of these sides as diametre.Prove that the circle biscts the side BC

- 4) The sides of a triangle are $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$.
 - a) What kind of triangle is this?
 - b) What is the position of the vertex opposite to the side $\sqrt{5}$ based on a circle with this side as the diametre?
 - c) What is the position of other two vertices based on this circle?
- 5) ABC is an equilateral triangle.A semicircle is drawn with diametre BC. Semicircle intersect the sides at P and Q.



- a) What is the measure of angle BQC?(Draw angle in the figure)
- b) What are the angles of triangle ABQ and triangle BQC
- c) Prove that the semicircle bisects the side $AB\ \mbox{and}\ AC$

Mathematics X Circles

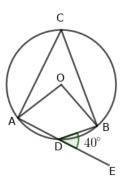
18

Concepts

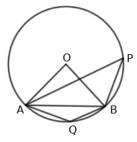
- a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement
- b) Angle formed by the arc in the complement is half the angle at the centre
- c) Sum of the angles at the centre and in the complement is $180^{\circ}\,$
- d) Angles on an arc are equal

Worksheet18

1) In the figure $BDE=40^{\circ}$

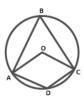


- a) What is the measure of angle ADB?
- b) Wha is the measure of angle ACB?
- c) What is the measure of angle AOB?
- 2) Triangle OAB is an equilateral triangle

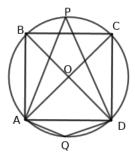


- a) What is the measure of angle AOB?
- b) What is the measure of angle APB?
- c) What is the measure of angle AQB?

- 3) Draw a circle of radus $3 {\rm cm}$.Construct the angles 30° and 150° with vertices on the circle using compasses and scale only.
- 4) In the figure $\angle ABC, \angle AOC, \angle ADC$ are in an arithmetic sequence



- a) What is the relation between angle ABC and angle AOC
- b) What is the relation between angle $ABC\ \mathrm{and}\ ADC$
- c) Find the measure of these angles
- 5) ABCD is a square .The diagonals AC and BD intersect at O.



- a) What is the measure of angle AOD?
- b) What is the measure of angle $APD\mbox{?}$
- c) What is the measure of angle $AQD\,$

Mathematics X Circles

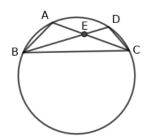
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Concepts

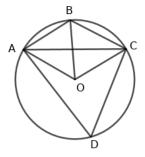
- a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement
- b) Angle formed by the arc in the complement is half the angle at the centre
- c) Sum of the angle of an arc and its complement is $180^{\circ}\,$
- d) Angles on an arc are equal

Worksheet20

1) A,B,C,D are four points on a circle. The chords AC,BD intersect at E.If $\angle BEC=130^\circ$, $\angle ECD=20^\circ$ then

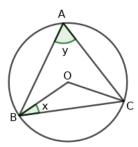


- a) What is the measure of $\angle CED$?
- b) What is the measure of $\angle CDE$?
- c) What is the measure of $\angle BAC$?
- 2) O is the centre of the circle.lf $\angle ACB=20^{\circ}, \angle CAB=30^{\circ}$ then

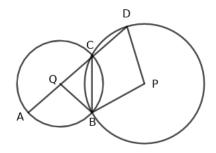


- a) What is the measure of $\angle AOB$?
- b) What is the measure of $\angle COB$?

- c) What is the measure of $\angle AOC$?
- d) What is the measure of $\angle ADC$?
- e) What is the measure of $\angle ABC$?
- 3) O is the centre of the circumcircle of triangle ABC. If $\angle BAC=y, \angle OBC=x$ then



- a) What is the measure of $\angle BCO$?
- b) What is the measure of $\angle BOC$?
- c) Prove that $x+y=90^{\circ}\,$
- 4) Draw a circle of radius 3cm. Construct triangle ABC in which $\angle A=70^{\circ}, \angle B=80^{\circ}$ with all its vertices on the circle.
- 5) P and Q are the centre of the circles shown in the figure. Circles intersect at B and C.If $\angle AQB=130^\circ$ then



- a) What is the measure of $\angle ACB$?
- b) What is the measure of $\angle BCD$?
- c) What is the measure of $\angle BPD$

Mathematics X Circles

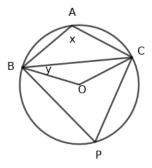
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Concepts

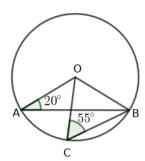
- a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement
- b) Angle formed by the arc in the complement is half the angle at the centre
- c) Sum of the angle of an arc and its complement is $180^{\circ}\,$
- d) Angles on an arc are equal

Worksheet21

1) In the figure $\angle BAC = x, \angle CBO = y, O$

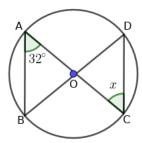


- a) What is the measure of $\angle BCO$?
- b) What is the measure of $\angle BOC$?
- c) What is the measure of $\angle BPC$?
- d) Prove that $x y = 90^{\circ}$?
- 2) In the figure O is the centre of the circle .If $\angle BCO = 55^{\circ}$, $\angle BAO = 20^{\circ}$ then



- a) What is the measure of $\angle OBC$?
- b) What is the measure of $\angle BOC$?

- c) What is the measure of $\angle AOC$?
- d) What is the measure of $\angle ABC$?
- 3) In the figure O is the centre of the circle.lf $\angle BAC=32^{\circ} {\rm then}$



- a) Find the angles of triangle ${\cal O}{\cal A}{\cal B}$
- b) What is the measure of $\angle DOC$?
- c) Find \boldsymbol{x}
- 4) This is the picture of a clock face. 1, 8, 5 are joined to make a triangle. Find the angles of this triangle.



5) Angles of a triangle are in the ratio 1:2:3. Vertices of this triangle are on a circle of radius $3 \, \text{cm}$. Construct the triangle

Mathematics X Circles

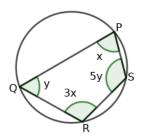
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Concepts

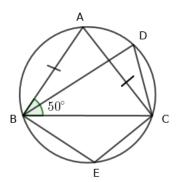
- a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.
- b) The sum of the opposite angles of a cyclic quadrilateral is 180° .
- c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is 180° it will be a cyclic quadrilateral.
- d) Square, rectangle and isosceles trapezium are $\mbox{\rm cyclic}$.

Worksheet22

1) In the figure PQRS is a cyclic quadrilateral. $\angle P=x, \angle Q=y, \angle R=3x, \angle S=5y$.

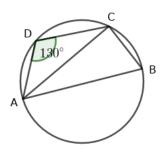


- a) Find x and y
- b) Find the angles of the quadrilateral.
- 2) ABC is an isosceles triangle with $AB = AC, \angle ABC = 50^{\circ}$.



- a) Name two cyclic quadrilaterals in this picture.
- b) What is the measure of angle D?
- c) What is the measure of $\angle BEC$?

3) ABCD is a cyclic quadrilateral. AB is the diametre of the circle , AD=CD and $\angle ADC=130^{\circ}$.



- a) What is the measure of $\angle ACB$?
- b) What is the measure of $\angle ABC$?
- c) Find $\angle DCB$.
- d) What is the measure of $\angle BAD$?
- 4) Prove that any cyclic parallelogram is a rectangle.
- 5) In triangle ABC, AB=AC.P and Q are the mid points of the side AB and AC.
 - a) Draw a rough diagram and join the points ${\cal P}$ and ${\cal Q}.$
 - b) Prove that BPQC is a cyclic quadrilateral.
 - c) If $\angle A$ in triangle ABC is 20° , find the angles of the trapezium BPQC

Mathematics X Circles

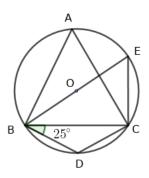
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Concepts

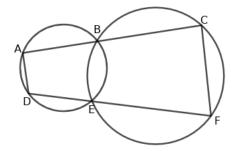
- a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.
- b) The sum of the opposite angles of a cyclic quadrilateral is 180° .
- c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is 180° it will be a cyclic quadrilateral.
- d) Square, rectangle and isosceles trapezium are $\mbox{\rm cyclic}$.

Worksheet23

1 In the figure $BD=CD \angle DBC=25^\circ$

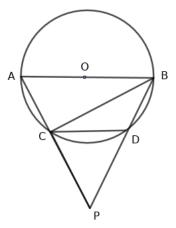


- a) What is the measure of $\angle BDC$?
- b) What is the measure of $\angle BAC$?
- c) What is the measure of $\angle EBC$?
- 2) Two circles intersect at B and E as in the figure. The points A-B-C are along a line. Also the points D-E-F are also on a line.

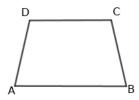


a) Prove that AD is parallel to CF

- b) If AC=DF suggest a suitable name to the quadrilateral ADFC
- c) Prove that ADFC is a cyclic quadrilateral.
- 3) AB is the diametre of the circle.CD is a chord of length equal to radius of the circle.



- a) What is the measure of $\angle COD$?
- b) What is the measure of $\angle CBD$?
- c) What is the measure of $\angle DCP$?
- d) Find the measure of $\angle CPD$
- 4) In the figure ABCD is a quadrilateral in which AB is parallel to CD and AD=BC



Prove that ABCD is a cyclic quadrilateral.

- 5) The angles of the quadrilateral ABCD are in the ratio 1:2:3:4 in an order.
 - a) If the smallest angle is x, what are the other angles?
 - b) Find the measure of all the angles of $ABCD\,$
 - c) Is ABCD a cyclic quadrilateral.
 - d) How should the ratio numbers interchange to make this cyclic?

Mathematics X Circles

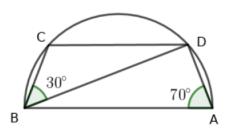
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Concepts

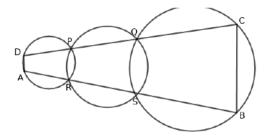
- a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.
- b) The sum of the opposite angles of a cyclic quadrilateral is 180° .
- c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is 180° it will be a cyclic quadrilateral.
- d) Square, rectangle and isosceles trapezium are cyclic.

Worksheet23

1) C, D are two points in a semicircle of diametre AB. If $\angle BAD=70^{\circ}, \angle DBC=30^{\circ}$ then



- a) What is the measure of $\angle BCD$?
- b) What is the measure of $\angle CDB$?
- c) What is the measure of $\angle ADC$?
- d) What is the measure of $\angle ABD$?
- 2) In the figure we can see three intersecting circles .D-P-Q-C are on a line .A-R-S-B are also on a line



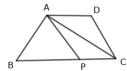
a) Prove that the quadrilateral ABCD is cyclic

b) If $\angle CDA = \angle DAB$ then what type of quadrilateral is ABCD?

c) If $\angle CDA = \angle DAB = 40^{\circ} \mathrm{then}$ find other two angles of ABCD

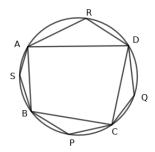
3 In traingle ABC, P is a point on BC.

AB=AP,the line through A parallel to BC and the line through C parallel to AP intersect at D .Prove that ABCD is cyclic



4) The parallelogram which is not a rectangle is not cyclic. Justify this statement

5) ABCD is a cyclic quadrialteral. Find $\angle P + \angle Q + \angle R + \angle S$



Mathematics X Circles

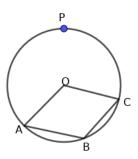
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Concepts

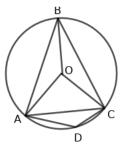
- a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement
- b) Angle formed by the arc in the complement is half the angle at the centre
- c) Sum of the angle of an arc and its complement is 180°
- d) Angles on an arc are equal

Worksheet19

1) OABC is a parallelogram.Three vertices are on a circle and one at the centre.P is a point on the circle

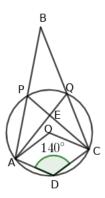


- a) Draw AP and CP, mark the angle APC = x. What is $\angle AOC$
- b) What is angle ABC?
- c) Find \boldsymbol{x}
- d) Find the angles of the parallelogram
- 2) In the figure O is the centre of the circle, $\angle BAO = 20^{\circ}, \angle BCO = 10^{\circ}$

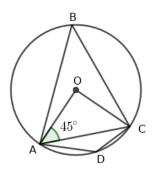


- a) What is the measure of angle ABC?
- b) What is the measure of angle AOC?

- c) What is the measure of angle ADC?
- d) Find the angles of triangle AOC
- e) If the diametre of the circle is $10\mathrm{cm}$ then find the length of the chord AB
- 3) In the figure O is the centre of the circle.lf angle $ADC=140^{\circ}$, angle $AEC=60^{\circ}$ then



- a) What is the measure of $\angle APC$ and $\angle AQC$
- b) What is the measure of angle AOC?
- c) Fnd the angles of the quadrlateral \!PEQB $\,$
- 4) In the figure O is the centre of the circle, $\angle AOC = 45^{\circ}$ then



- a) What kind of triangle is OAC?
- b) What is the measure of angle ABC?
- c) What is the measure of angle $ADC\ensuremath{\mathbf{?}}$
- d) If the radius of the circle is $6\mathrm{cm}$ then what is the length of the chord AC.
- 5) Draw a circle of radius $3 \, \text{cm}$, construct an equilateral triangle with vertices on the circle. What is the length of the side?

Mathematics X Circles

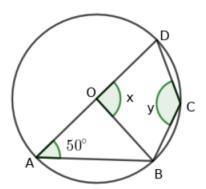
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Concepts

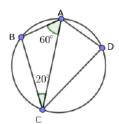
- a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.
- b) The sum of the opposite angles of a cyclic quadrilateral is 180° .
- c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is 180° it will be a cyclic quadrilateral.
- d) Square, rectangle and isosceles trapezium are cyclic .

Worksheet24

1) In the figure O is the centre of the circle, $\angle DAB = 50^{\circ}$



- a) Find x
- b) Find y
- c) If BC = CDthen what is the measure of $\angle ADC$?
- d) If BC = CDthen what is the measure of $\angle ABC$?
- 2) In the figure $\angle BAC = 60^{\circ}$, $\angle BCA = 20^{\circ}$

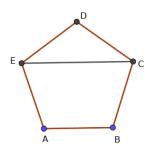


a) Looking into the figure Riswan said: AC is the diametre of the circle .Can you agree with his opinion? Why?

b) What is the measure $\angle ADC$

c) If $\angle DAC: \angle DCA=3:1$ then find these angles.

3 In the figure ABCDE is a regular pentagon.Prove that ABCE is a cyclic quadrilateral.



- 4) Prove that the trapezium having diagonals equal is cyclic
- 5) ABCD is a cyclic quadrilateral. If $\angle A \angle C = 60^\circ$ then find the measure of $\angle C$. What is the measure of $\angle A$?

Mathematics X Circles

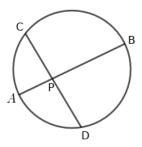
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Concepts

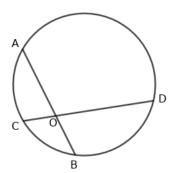
- a) Two chords of a circle AB and CD intersect at the point P inside the circle .It can be proved that $PA \times PB = PC \times PD$
- b) This relation can be used to construct a rectangle having equal area of another rectangle.
- c) If the chords intersect outside the circle ,the same relation holds. $PA \times PB = PC \times PD$

Worksheet 25

1) In the figure two chords AB and CD intersect inside a circle at P.

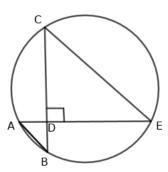


- a) Join AC and BD. Establish the similarity of triangle PAC and PBD
- b) What are the equal angles of these triangles
- c) Prove that $PA \times PB = PC \times PD$
- 2) In the figure the chord AB has length $8\mathrm{cm}$ and $OA=5\mathrm{cm}$.



- a) What is the length of OB?
- b) If $OC=2.5\mathrm{cm}$, what is the length of OD?

3) In the figure $AB=5\mathrm{cm}$, $BD=4\mathrm{cm}$, $CD=9\mathrm{cm}$.



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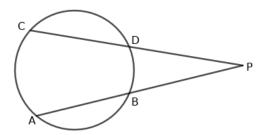
a) What is the length of $AD\mbox{\bf ?}$

b) Calculate the length of DE?

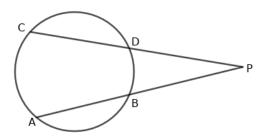
c) Is CE the diameter of the circle? why?

d) Find the length of ${\cal DE}$

4) If AB and CD are two chords of a circle which when produced meet at a point P . If PA=PC show that AB=CD.



5) In the figure AB and CD are two chords of a circle which when produced meet at a point P



a) $\operatorname{Draw} AC$ and BD , complete the quadrilateral ABDC

b) Establish the similarity of the triangles PAC and PDB

c) Establish the relation $PA \times PB = PC \times PD$

Mathematics X Circles

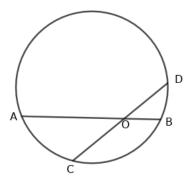
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Concepts

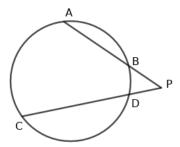
- a) Two chords of a circle AB and CD intersect at the point P inside the circle .lt can be proved that $PA \times PB = PC \times PD$
- b) This relation can be used to construct a rectangle having equal area of another rectangle.
- c) If the chords intersect outside the circle ,the same relation holds. $PA \times PB = PC \times PD$

Worksheet 25

1) The chords AB and CD intersect at O .This point divide each chord into two segments

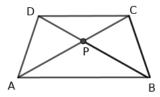


- a) What is the relation between these segments?
- b) If $CD=10\mathrm{cm}$ and $OD=4\mathrm{cm}$ then what is the length OC?
- c) If $OA=8\mathrm{cm}$, $OC=6\mathrm{cm}$ and $OD=4\mathrm{cm}$ then what is the length OB?
- 2) The chords AB and CD intersect at P inside the circle.



- a) What is the relation between PA, PB, PC and PD?
- b) If $AB=5\mathrm{cm}$, $PB=3\mathrm{cm}$, $PD=2\mathrm{cm}$ then what is the length CD?

3) In the trapezium ABCD, AD=BC and AB is parallel to CD . The diagonals AC and BD intersect at P.

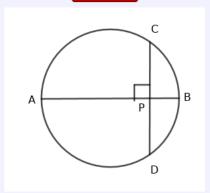


- a) What is the relation between $\angle ADB$ and $\angle ACB$? How can we realize this relation?
- b) If $\angle DAC = 30^{\circ}$ then what is the measure of $\angle DBC$?
- c) What is the relation between the segments made by $\!P$ on the diagonals?
- 4) In the quadrlateral ABCD , the diagonals AC and BD intersect at P. If $PA=9{\rm cm}$ $PB=12{\rm cm}$, $PC=4{\rm cm}$ and $PD=3{\rm cm}$ then
 - a) Draw a rough diagram and mark the mesaurements
 - b) Is this a cyclic quadrilateral? How can we realize this?
 - c) If $\angle A=40^\circ$ and $\angle B=70^\circ$ find other two angles of the quadrilateral
- 5) Draw a rectangle of sides 4 cm and 6 cm. Construct another rectangle with area equal to the area of the first rectangle and one side 7 cm in length.

Mathematics X Circles



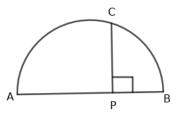
Concepts



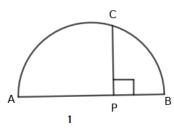
- a) In the case of the intersecting chords of a circle,if one chord AB is the diametre of the circle and other chord CD is perpendicular to the diametre ,then $PA \times PB = PC^2$
- b) This relation is used to construct a square with same area of a rectangle.lt can be used to draw the lines of irrational lengths.

Worksheet 26

1) AB is the diametre of a semicircle, P is a point on AB and PC is perpendicular to AB

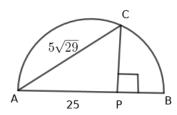


- a) Prove that $PA \times PB = PC^2$
- b) If $PA=9\mathrm{cm}$, PB=4 cm then what is the length PC?
- c) What is the area of the square with side PC?
- 2)]AB is the diametre of a semicircle, P is a point on AB and PC is perpendicular to AB

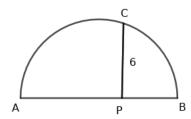


- a) If $PC=6\mathrm{cm}$,and $PB=3\mathrm{cm}$ then what is the length of PA
- b) What is the radius of the circle?
- c) What is the area of the square drawn with side PC?

3) In the figure AB is the diametre of the semicircle, PC is perpendicular to AB. $AC=5\sqrt{29}{\rm cm}$ and $PA=25{\rm cm}$.



- a) What is the length of PC?
- b) What is the lenght PB?
- c) What is the radius of the circle?
- 4) Draw a semicircle of suitable diametre .Construct a line of length $\sqrt{12} cm$ perpendicular to the diametre whose one end is on the diameter and other end is on the semicircle.Explain the principle of construction.
- 5) In the figure AB is the diametre of the circle and PC is perpendicular to the diametre. PA:PB=2:1 and PC=6cm.

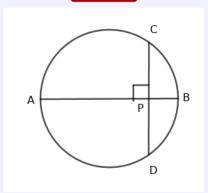


- a) Write the relation between PA, PB and PC?
- b) Find the lengths ${\cal P}{\cal A}$ and ${\cal P}{\cal B}$
- c) What is the radius of the circle?

Mathematics X Circles

26

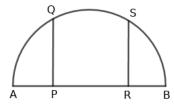
Concepts



- a) In the case of the intersecting chords of a circle,if one chord AB is the diametre of the circle and other chord CD is perpendicular to the diametre ,then $PA \times PB = PC^2$
- b) This relation is used to construct a square with same area of a rectangle.lt can be used to draw the lines of irrational lengths.

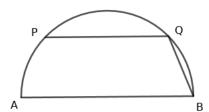
Worksheet 26

1) AB is the diametre of a semicircle.The lines PQ and RS are perpendicular to AB.If PQ=RS then



- a) What is the relation between the lengths PA, PB and PQ ?
- b) What is the relation between the lenghths AR,BR and RS
- c) Prove that PA = BR
- 2) a) Draw an equilateral triangle of altitude $3\ \mathrm{cm}$
 - b) What is the lenght of one side?
 - c) What is the radius of its incircle?
- 3) Draw a rectangle of sides 5cm and 3cm . Construct a square whose area is same as the area of the rectangle

- 4) a) Draw a semicircle of suitable diametre .Draw a line of length $\sqrt{12} {\rm cm}$ whose one end on AB and other end on the semicircle.
 - b) Draw a chord of length $\sqrt{48}$ cm by make the semicircle as the circle
- 5) AB is the diametre of a semicircle. $PQ=\sqrt{14} {\rm cm} \ RS=\sqrt{18} \ {\rm cm}$. These lines are perpendicular to the diametre . Find the length of AB?
- 6) AB is the diametre of a semicircle , PQ is parallel to the diametre If $AB=8{\rm cm}$, BQ=2 cm then find the langth PQ.



7) Draw an equilateral triangle of one side $\sqrt{18} {\rm cm}$