2020–21 Academic year Works

Mathematics X
Circles
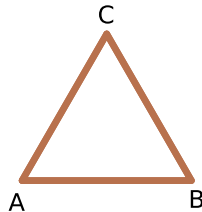
16

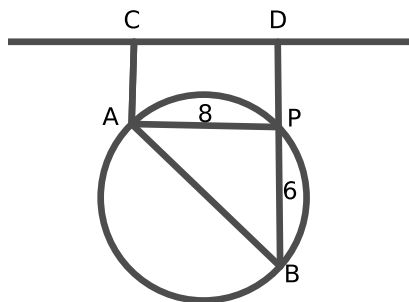Worksheet16

1) ABC is a triangle in which $AB = AC = BC$

a) What are the measure of its angles?

b) What is the position of the vertex C based on the circle with diametre AB? Is it inside ,outside or on the circle.

Answers

a) $60°$

b) Outside the circle

2) In the figure AB is the diametre of the circle. AC and PD are perpendicular to CD

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a) What is the measure of angle APB?

b) Suggest a suitable name to $ACDP$

c) If $AP = 8$cm and $BP = 6$cm then what is the radius of this circle?

Answers

a) $\angle APB = 90°$:Angle in the semicircle

b) $\angle APD = 180 - 90 = 90°$, Angles of $ACDP$ are $90°$ $ACDP$ is a square.

c) APB is a right trangle. $AB = \sqrt{8^2 + 6^2} = 10$. Radius $= 5$cm

3) O is the centre of the circle with diametre AB.Another circle is drawn with AO as the diametre
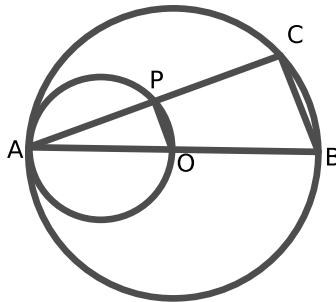
a) What are the measure of $\angle APO, \angle ACB$

b) Outer circle has radius 5cm and $BC = 8$cm . What is the length OP?

c) Is $AP = PC$? Why?

d) What is the length of AC?

Answers

a) $90°$. Reason $\angle APO, \angle ACB$ are the angles in the semicircle

b) Triangle APO,triangle ACB are similar. $\frac{AO}{AB} = \frac{OP}{BP}$
$\frac{5}{10} = \frac{OP}{8}, OP = 4$cm

c) AC is the chord of big circle. OP is perpendicular from centre to this chord. OP bisect AC.Therefore $AP = PC$

d) $AP = \sqrt{5^2 - 4^2} = 3$cm.$AC = 6$cm

4) Draw a circle of radius 3cm and construct a rectangle with vertices on the circle. One side of the rectangle should be 4cm . What is the lengh of other side?(Write the measurement)

Answers

a) Draw circle. Diametre $AB = 6$cm .Draw an of radius 4cm , centre at A which cut the circle at C. Join AC and BC

b) Draw an of radius 4cm , centre at B which cut the circle at D in the other side of the diametre at D. Join BD and AD

c) Angle ACB, angle ADB are $90°$ each. $ACBD$ is a rectangle

d) Measure the other side. $BC = \sqrt{6^2 4^2} = \sqrt{20} = 2\sqrt{5}$cm

5) Sides of triangle ABC are $AB = 5$cm, $AC = 12$cm ,$BC = 13$cm

2

a) What kind of triangle is this ?

b) What is the position of A based on the circle with diametre BC?

c) What is the position of C based on the circle with diametre AB?

d) What is the position of B based on the circle with diametre AB?

Answers

a) $5^2 + 12^2 = 13^2$
This is a right triangle

b) In triangleABC angle$A = 90°$ .A is on the circle

c) One angle is $90°$ and other two angles are less than $90°$. $\angle C < 90°$,therefore C is outside the circle

d) $\angle B < 90°$ .B is outside the circle.

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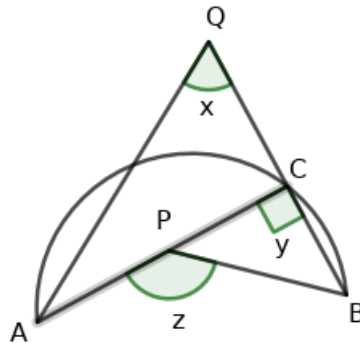# 2020–21 Academic year Works

Mathematics X
Circles

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Concepts

a) Angle in the semicircle is $90°$

b) Angle outside the semicircle is less than $90°$

c) Angle inside the semicircle is greater than $90°$

Worksheet17

1) In the figure AB is the diametre of a semicircle.Three angles x, y, z are marked outside, on the semicircle and inside the semicircle.

a) What is the value of y?

b) If x, y, z are in an arithmetic sequence,then what is $x + z$?

c) If the common difference of the sequence is 50 then find x and z

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Answers

a) $y = 90°$

b) $x + z = 2 \times 90 = 180°$ (Refer the propery of arithmetic sequence)

c) $d = 50 \therefore x = 90 - 50 = 40°$, $z = 90 - 50 = 140°$

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2)   a) Draw a circle of radius 3 cm. Construct a square with vertices are on the circle.

b) What is the length of its side?

c) Calculate the area of the square.

Answers

a) Draw a circle of radius 3 cm and diametre AB.

b) Draw another diametre CD perpendicular to AB. Join the end points of the diametre . $ACBD$ will be a square

c) Side of the square $AC = \sqrt{3^2 + 3^2} \overset{1}{=} 3\sqrt{2}$ cm.
Area $= 3\sqrt{2} \times 3\sqrt{2} = 18$ sq.cm

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3) In triangle ABC, $AB = AC$. A circle is drawn with one of these sides as diametre. Prove that the circle biscts the side BC

> **Answers**
>
> a) Circle intersect BC at P. $\angle BPA = 90°$.
>
> b) Triangle APB and triangle APC are equal triangles. $BP = CP$

4) The sides of a triangle are $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$.

    a) What kind of triangle is this?

    b) What is the position of the vertex opposite to the side $\sqrt{5}$ based on a circle with this side as the diametre?

    c) What is the position of other two vertices based on this circle?

> **Answers**
>
> a) $\sqrt{2}^2 + \sqrt{3}^2 = \sqrt{5}^2$. This is a right triagle.
>
> b) Angle opposite to the side of length $\sqrt{5}$ cm is $90°$. The vertex opposite to this side is on the circle.
>
> c) Other two angles are less than $90°$. Vertex is outside the circle

5) ABC is an equilateral triangle. A semicircle is drawn with diametre AB. Semicircle intersect the sides at P and Q.
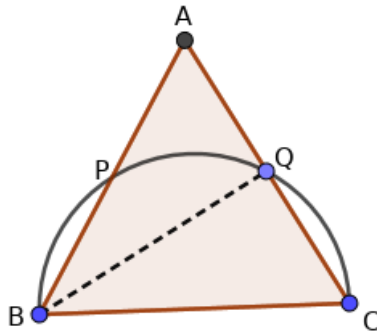
    a) What is the measure of angle BQC?(Draw angle in the figure)

    b) What are the angles of triangle ABQ and triangle BQC

    c) Prove that the semicircle bisects the side AB and AC

> **Answers**
>
> a) Draw a rough figure. $\angle BQC = 90°$.(Angle in the semicircle)
>
> b) $\angle A = 60°, \angle ABQ = 30°, \angle AQB = 90°, \angle QBC = 30°, \angle C = 60°$
>
> c) BQ is the altitude of the equilateral trangle. Therefore $AQ = CQ$

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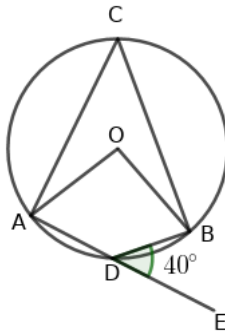**Concepts**

a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement

b) Angle formed by the arc in the complement is half the angle at the centre

c) Sum of the angles at the centre and in the complement is $180°$

d) Angles on an arc are equal
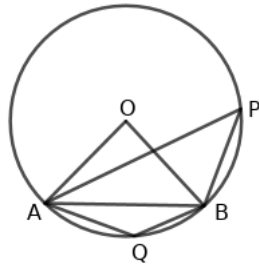

Worksheet18

1) In the figure $\angle BDE = 40°$

a) What is the measure of angle ADB?

b) Wha is the measure of angle ACB?

c) What is the measure of angle AOB?

Answers

a) $\angle ADB = 180 - 40 = 140°$

b) $\angle ACB = 180 - 140 = 40°$

c) $\angle AOB = 2 \times 40 = 80°$

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2) Triangle OAB is an equilateral triangle

    a) What is the measure of angle AOB?

    b) What is the measure of angle APB?

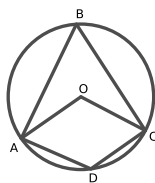   c) What is the measure of angle AQB?

Answers

    a) $\angle AOB = 60°$

    b) $\angle APB = \frac{1}{2} \times 60° = 30°$

    c) $\angle AQB = 180 - 30 = 150°$

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3) Draw a circle of radius 3cm.Construct the angles $30°$ and $150°$ with vertices on the circle using compasses and scale only.

Answers

   ⋆ Draw a circle of radius 3cm.Mark the center of the circle as O

   ⋆ Mark a point A on the circle.Draw the radius OA.

   ⋆ With A as the centre and OA as radius , draw an arc which cut the circle at B. Join OB, $\angle AOB = 60°$

   ⋆ Mark a point P on the complement of the arc AB,which makes $60°$ at the center. $\angle APB = \frac{1}{2} \times 60 = 30°$

   ⋆ Mark a point Q on the arc AB. $\angle AQB = 180 - 30 = 150°$

4) In the figure $\angle ABC, \angle AOC, \angle ADC$ are in an arithmetic sequence

    a) What is the relation between angle ABC and angle AOC

    b) What is the relation between angle ABC and ADC

    c) Find the measure of these angles
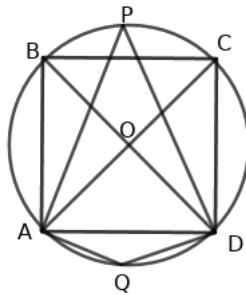
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a) $\angle AOC = 2 \times \angle ABC$

b) $\angle ABC + \angle ADC = 180°$

c) Let $\angle ABC = x, \angle AOC = y, \angle ADC = z$
x, y, z are in an arithmetic sequence.Therefore $2y = x + z$
From the relations noted above , $y = 2x, x + z = 180$

$$2y = x + z$$
$$2 \times 2x = x + z = 180$$
$$4x = 180, x = 45$$

$x = 45, y = 90, z = 135$.
$\angle ABC = 45°, \angle AOC = 90°, \angle ADC = 135°$

5) $ABCD$ is a square .The diagonals AC and BD intersect at O.

a) What is the measure of angle AOD?

b) What is the measure of angle APD?

c) What is the measure of angle AQD

a) Diagonals of a square are perpendicular to eachother.$\angle AOD = 90°$

b) $\angle APD = 45°$

c) $\angle AQD = 180 - 45 = 135°$

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Mathematics X
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$$\boxed{20}$$

**Concepts**

a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement

b) Angle formed by the arc in the complement is half the angle at the centre

c) Sum of the angle of an arc and its complement is $180°$

d) Angles on an arc are equal

$$\boxed{\text{Worksheet20}}$$

1) A, B, C, D are four points on a circle.
The chords AC, BD intersect at E. If $\angle BEC = 130°, \angle ECD = 20°$ then
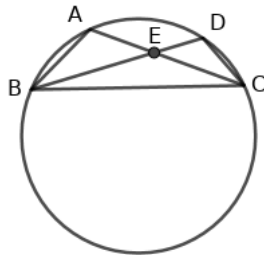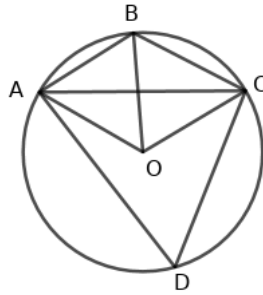

a) What is the measure of $\angle CED$?

b) What is the measure of $\angle CDE$?

c) What is the measure of $\angle BAC$?

Answers

a) $\angle CED = 180 - \angle BEC$
   $\angle CED = 180 - 130 = 50°$

b) $\angle CDE = 180 - (50 + 20) = 180 - 70 = 110°$

c) $\angle BAC = \angle BDC$
   $\angle BAC = 110°$

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2) O is the centre of the circle.If $\angle ACB = 20°, \angle CAB = 30°$ then
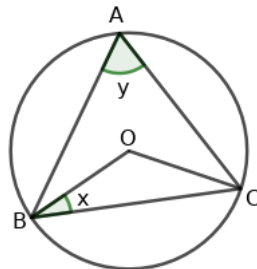

a) What is the measure of $\angle AOB$?

b) What is the measure of $\angle COB$?

c) What is the measure of $\angle AOC$?

d) What is the measure of $\angle ADC$?

e) What is the measure of $\angle ABC$?

Answers

a) Since $\angle ACB = 20°$,$\angle AOB = 2 \times \angle ACB = 40°$

b) Since $\angle CAB = 30°$,$\angle COB = 2 \times \angle ACB = 60°$

c) $\angle AOC = 40 + 60 = 100°$

d) $\angle ADC = \frac{1}{2} \times AOC = 50°$

e) $\angle ABC = 150 - 50 = 130°$

3) O is the centre of the circumcircle of triangle ABC.
If $\angle BAC = y, \angle OBC = x$ then

a) What is the measure of $\angle BCO$?

b) What is the measure of $\angle BOC$?

c) Prove that $x + y = 90°$

Answers

a) Since $OB = OC$ opposite angles of these sides in triangle OBC are equal.
$\angle BCO = x$.

b) $\angle BOC = 180 - 2x$

c) We know that $\angle BOC = 2 \times \angle BAC$
$180 - 2x = 2y, 2x + 2y = 180, x + y = 90°$

4) In triangle $ABC, \angle A = 70°, \angle B = 80°$. The vertices of the triangle are on the circumcircle of the triangle .Radius of the circumcircle is 3cm.Construct the triangle.

Answers

* ⋆ Draw a circle of radius 3 , mark its centre as$O$ and a point A on the circle. Draw the radius OA
* ⋆ Mark a point B on the circle such that $\angle AOB = 2 \times 70 = 140°$
* ⋆ Mark a point C on the circle such that $\angle BOC = 2 \times 80 = 160°$
* ⋆ Draw triangle ABC .

5) P and Q are the centre of the circles shown in the figure. Circles intersect at B and C.If $\angle AQB = 130°$ then
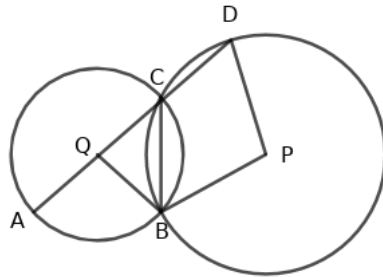
a) What is the measure of $\angle ACB$?

b) What is the measure of $\angle BCD$?

c) What is the measure of $\angle BPD$

Answers

a) $\angle ACB = \frac{1}{2} \times 130 = 65°$

b) $\angle BCD = 180 - 65 = 115°$

c) The central angle of the complement of the arcBCD is $2 \times 115 = 230°$.Therefore $\angle BPD = 360 - 230 = 130°$

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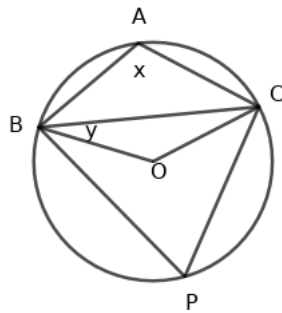
Mathematics X
Circles

$$\boxed{21}$$

Concepts

a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement

b) Angle formed by the arc in the complement is half the angle at the centre

c) Sum of the angle of an arc and its complement is $180°$

d) Angles on an arc are equal

$$\boxed{\text{Worksheet21}}$$

1) In the figure $\angle BAC = x, \angle CBO = y, O$ is the centre of the circle.
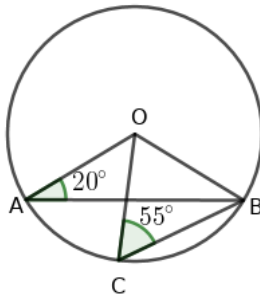

a) What is the measure of $\angle BCO$?

b) What is the measure of $\angle BOC$?

c) What is the measure of $\angle BPC$?

d) Prove that $x - y = 90°$?

Answers

a) $\angle BCO = y$
   (In triangle $BCO, OB = OC$. Angles opposite to equal sides are equal.)

b) $\angle BOC = 180 - 2y$

c) $\angle BPC = \frac{1}{2} \times \angle BOC = \frac{1}{2} \times (180 - 2y) = 90 - y$

d) $\angle BAC + \angle BPC = 180, x + 90 - y = 180, x - y = 180 - 90 = 90°$

1

2) In the figure O is the centre of the circle .If $\angle BCO = 55°, \angle BAO = 20°$ then
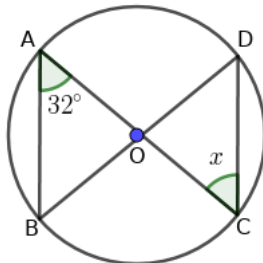

a) What is the measure of $\angle OBC$?

b) What is the measure of $\angle BOC$?

c) What is the measure of $\angle AOC$?

d) What is the measure of $\angle ABC$?

Answers

a) $\angle OBC = 55°$

   (In triangle $OBC, OB = OC$. Angles opposite to equal sides are equal.)

b) $\angle BOC = 180 - (55 + 55) = 180 - 110 = 70°$

c) In triangle $AOB, \angle B = 20°, \angle ACB = 180 - 40 = 140°$

   $\angle AOC = 140 - 70 = 70°$

d) $\angle ABC = \frac{1}{2} \times 70 = 35°$

3) In the figure O is the centre of the circle.If $\angle BAC = 32°$ then
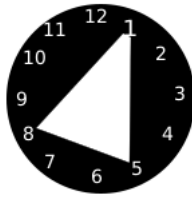

a) Find the angles of triangle OAB

b) What is the measure of $\angle DOC$?

c) Find x

Answers

a) In triangle $OAB, OA = OB$. Therefore $\angle B = 32°, \angle AOB = 180 - 64 = 116°$

b) $\angle DOC = 116°$

   (Opposite angles are equal)

c) In triangle $OCD, \angle D = x$

   $x + x + 116 = 180, 2x = 64, x = 32$

2

4) This is the picture of a clock face. $1, 8, 5$ are joined to make a triangle. Find the angles of this triangle.

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Answers

★ The central angle of the arc in between two numbers (say $1, 2$) is $\frac{1}{12} \times 360 = 30°$.
The central angle of the arc between 1 and 5 is $4 \times 30 = 120°$.
The angle of the triangle with vertex at 8 is $\frac{1}{2} \times 120 = 60°$

★ The central angle of the arc between 8 and 5 is $3 \times 30 = 90°$.
The angle of the triangle with vertex at 1 is $\frac{1}{2} \times 90 = 45°$

★ The central angle of the arc between 8 and 1 is $5 \times 30 = 150°$.
The angle of the triangle with vertex at 5 is $\frac{1}{2} \times 150 = 75°$

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5) Angles of a triangle are in the ratio $1 : 2 : 3$. Vertices of this triangle are on a circle of radius 3cm. Construct the triangle

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Answers

Draw a circle of radius 3cm
Divide the angle around the centre in the ratio $1 : 2 : 3$.
The resulting angles are $60°, 120°, 180°$ by drawing radii. Join the ends of the radii on the circle.

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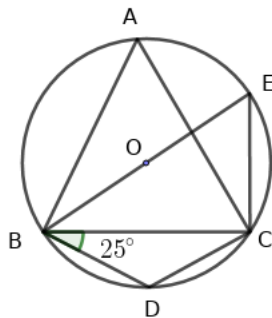
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Worksheet23

1 In the figure $BD = CD$,$\angle DBC = 25°$

a) What is the measure of $\angle BDC$?

b) What is the measure of $\angle BAC$?

c) What is the measure of $\angle EBC$?

Answers

a) In triangle BDC, $BD = CD$. Angle opposite to these sides are equal .$\angle BCD = 25°$
$\angle BDC = 180 - (25 + 25) = 130°$

b) $\angle BAC = 180 - 130 = 50°$

c) $\angle BEC = \angle BAC = 180 - 130 = 50°$, $\angle EBC = 180 - (90+50) = 180 - 140 = 40°$

2) Two circles intersect at B and E as in the figure. The points $A - B - C$ are along a line. Also the points $D - E - F$ are also on a line.
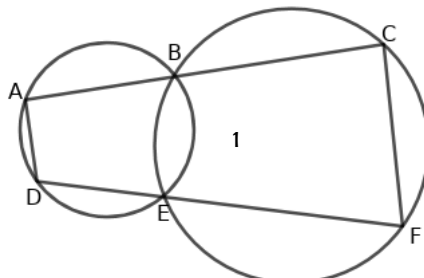
a) Prove that AD is parallel to CF

b) If $AC = DF$ suggest a suitable name to the quadrilateral $ADFC$

c) Prove that $ADFC$ is a cyclic quadrilateral.

Answers
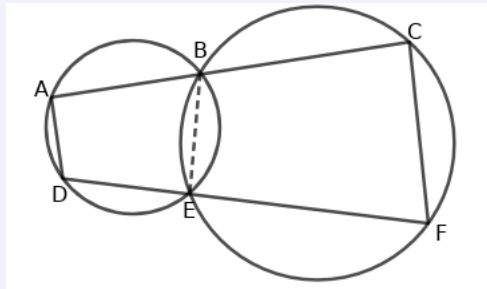
a) Draw BE . $ABED$ is a cyclic quadrilateral.
   If $\angle DAB = x$ then $\angle BED = 180 - x$, $\angle BEF = 180 - (180 - x) = x$.
   $BEFC$ is cyclic . $\angle C = 180 - x$.
   In quadrilateral $ADFC$, $\angle A + \angle C = x + 180 - x = 180°$
   Co interior angle sum is $180°$. AD is parallel to CF

b) $ADFC$ is a trapezium .Since $AC = DF$ is an isosceles trapezium .

c) Angles at the ends of parallel sides of an isosceles trapezium are equal .Since $\angle A = \angle D$
   and $\angle A + \angle C = 180°$ then $\angle D + \angle C = 180°$.
   $ADFC$ is a cyclic quadrilateral.

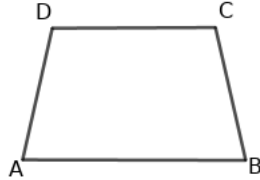**3)** AB is the diametre of the circle. CD is a chord of length equal to radius of the circle.
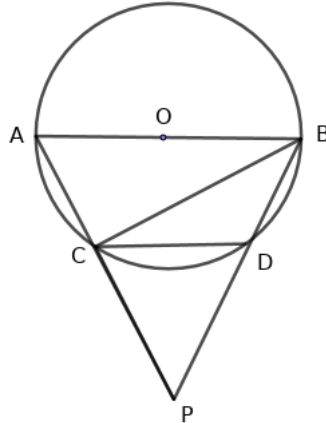
a) What is the measure of $\angle COD$?

b) What is the measure of $\angle CBD$?

c) What is the measure of $\angle DCP$?

d) Find the measure of $\angle CPD$

2

4) In the figure $ABCD$ is a quadrilateral in which AB is parallel to CD and $AD = BC$

Prove that $ABCD$ is a cyclic quadrilateral.

5) The angles of the quadrilateral $ABCD$ are in the ratio $1 : 2 : 3 : 4$ in an order.

a) If the smallest angle is x,what are the other angles?

b) Find the measure of all the angles of $ABCD$

c) Is $ABCD$ a cyclic quadrilateral.

d) How should the ratio numbers interchange to make this cyclic?

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Concepts

a) An arc of a circle can make three type of angles. Angle on the arc, angle at the centre and angle in the complement

b) Angle formed by the arc in the complement is half the angle at the centre

c) Sum of the angle of an arc and its complement is $180°$

d) Angles on an arc are equal

Worksheet19

1) $OABC$ is a parallelogram.Three vertices are on a circle and one at the centre.P is a point on the circle
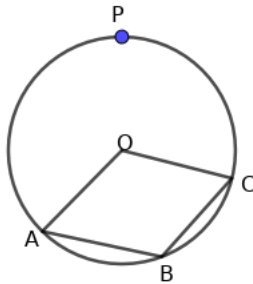
a) Draw AP and CP, mark the angle$APC = x$. What is $\angle AOC$
b) What is angle ABC?
c) Find x
d) Find the angles of the parallelogram

Answers

a) $\angle AOC = 2x$

b) $\angle ABC = 2x$ Opposite angles of a parallelogram are equal

c) $\angle APC + \angle ABC = 180°$,$x + 2x = 180, 3x = 180, x = 60$

d) Angles are $120°, 60°, 120°, 60°$

2) In the figure O is the centre of the circle,$\angle BAO = 20°, \angle BCO = 10°$
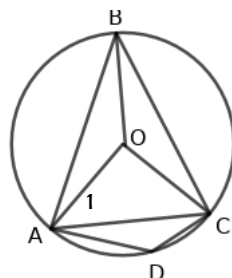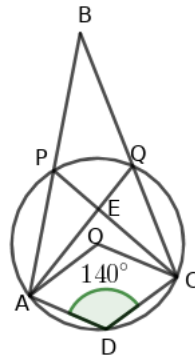
a) What is the measure of angle ABC?

b) What is the measure of angle AOC?

c) What is the measure of angle ADC?

d) Find the angles of triangle AOC

e) If the diametre of the circle is 10cm then find the length of the chord AB

Answers

a) In triangle OAB, $OA = OB$. Angles opposite to the equal sides are equal. Similarly in the case of triangle OBC also.
$\angle ABC = 20 + 10 = 30°$

b) $\angle AOC = 2 \times 30 = 60°$

c) $\angle ADC = 180 - 30 = 150°$

d) Triangle AOC, $OA = OC$, $\angle OAC = \angle OCA = \frac{180-60}{2} = 60° \triangle OAC$ is an equilateral triangle. Angles are $60°$ each.

e) $OA = AC = OC = 5$cm, radius 5cm.

---

3) In the figure O is the centre of the circle. If angle $ADC = 140°$, angle $AEC = 60°$ then

a) What is the measeure of $\angle APC$ and $\angle AQC$

b) What is the measure of angle AOC?

c) Fnd the angles of the quadrlateral $PEQB$

Answers

a) $\angle APC = 180 - 140 = 40°$, $\angle AQC = 40°$

b) $\angle AOC = 2 \times 40 = 80°$

c) In the quadrilateral $\angle AEQ = \angle AEC = 60°$, $\angle EPB = 180 - 40 = 140°$, $\angle EQB = 140°$
$\angle PBQ = 360 - (140 + 140 + 60) = 20$. Angles are $140°, 60°, 140°, 20°$

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4) In the figure O is the centre of the circle, $\angle OAC = 45°$ then

2
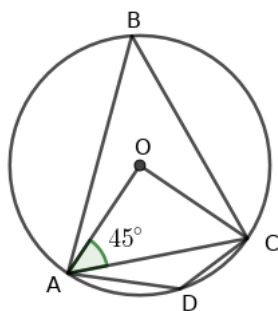
a) What kind of triangle is OAC?

b) What is the measure of angle ABC?

c) What is the measure of angle ADC?

d) If the radius of the circle is 6cm then what is the length of the chord AC.

Answers

a) $OA = OC, \angle OAC = \angle OCA = 45°, \angle AOC = 90°$. $\triangle OAC$ is an isosceles right triangle

b) $\angle ABC = \frac{1}{2} AOC = 45°$

c) $\angle ADC = 180 - 45 = 135°$

d) $AC = \sqrt{6^2 + 6^2} = 6\sqrt{2}$cm

---

5) Draw a circle of radius 3cm , construct an equilateral triangle with vertices on the circle. What is the length of the side?

Answers

★ Draw a circle with centre O and radius 3cm. Mark a point A on the circle and radius OA.

★ Divide the angle around the centre $120°$ each and mark the points B, C on the circle

★ Draw triangle ABC .

★ Length of side $= 3\sqrt{3}$cm

2020–21 Academic year Works

Mathematics X
Circles

22

Concepts

a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.

b) The sum of the opposite angles of a cyclic quadrilateral is $180°$.

c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is $180°$ it will be a cyclic quadrilateral.

d) Square, rectangle and isosceles trapezium are cyclic .

Worksheet22

1) In the figure $PQRS$ is a cyclic quadrilateral. $\angle P = x, \angle Q = y, \angle R = 3x, \angle S = 5y$.
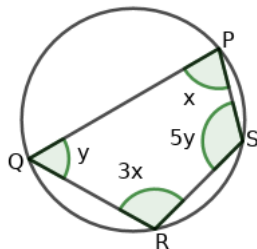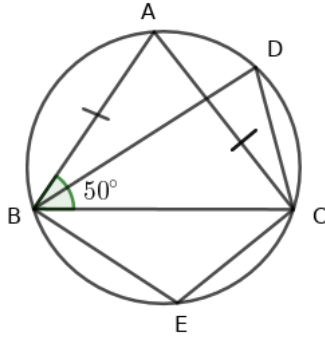

a) Find x and y

b) Find the angles of the quadrilateral.

Answers

a) $\angle P + \angle R = 180°$
$x + 3x = 180, 4x = 180, x = 45$
$\angle Q + \angle S = 180°, y + 5y = 180, 6y = 180, y = 30$

b) $\angle P = 45°, \angle R = 3 \times 45 = 135°$ $\angle Q = 30°, \angle S = 5 \times 30 = 150°$

1

2) ABC is an isosceles triangle with $AB = AC, \angle ABC = 50°$.

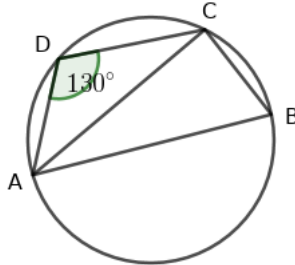a) Name two cyclic quadrilaterals in this picture.

b) What is the measure of angle D?

c) What is the measure of $\angle BEC$?

<div style="border:1px solid blue">

Answers

a) Quadrilateral $ABEC$ and quadrilateral $DBEC$ are cyclic.

b) $\angle ABC = \angle ACB = 50°$
$\therefore \angle A = 180 - 100 = 80°$
$\therefore \angle D = 80°$

c) $\angle BEC = 180 - 80 = 100°$

</div>

3) $ABCD$ is a cyclic quadrilateral. AB is the diametre of the circle , $AD = CD$ and $\angle ADC = 130°$.

a) What is the measure of $\angle ACB$?

b) What is the measure of $\angle ABC$?

c) Find $\angle DCB$.

d) What is the measure of $\angle BAD$?

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Answers

a) $\angle ACB = 90°$ (Angle in the semicircle)

b) $\angle ABC = 180 - 130 = 50°$

c) Since $CD = AD$,the angles opposite to the equal sides of triangle ADC are equal.
$\angle DCA = 25°, \angle DCB = 90 + 25 = 115°$

d) $\angle BAD = 180 - 115 = 65°$

</div>

2

4) Prove that any cyclic parallelogram is a rectangle.

- ★ $ABCD$ is a parallelogram .(Draw rough figure)

- ★ Opposite angles are equal. $\angle A = \angle C, \angle B = \angle D$

- ★ Sum of the opposite angles is $180°$

- ★ $\angle A + \angle C = 180°, \angle A = \angle C \therefore \angle A = 90°, \angle C = 90°$

- ★ $\angle B + \angle D = 180°, \angle B = \angle D \therefore \angle B = 90°, \angle D = 90°$

- ★ $ABCD$ is a square

5) In triangle ABC, $AB = AC$. P and Q are the mid points of the side AB and AC.

   a) Draw a rough diagram and join the points P and Q.

   b) Prove that $BPQC$ is a cyclic quadrilateral.

   c) If $\angle A$ in triangle ABC is $20°$, find the angles of the trapezium $BPQC$

   a) Figure
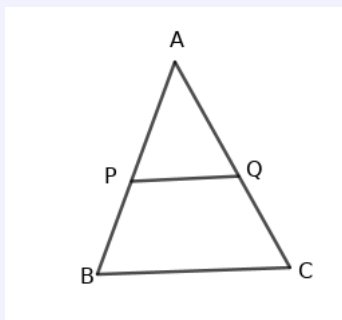
   b) Since $AB = AC$, $\angle B = \angle C$.

Line joining the mid points of two sides of a triangle is parallel to the third side. PQ is parallel to BC.

In $PBCQ$, $\angle B + \angle P = 180°$ (co interior angles)

Since $\angle C = \angle B \angle C + \angle P = 180°$

$PQCB$ is a cyclic quadrilateral

   c) $\angle A = 20°$

$\angle B = \angle C = \frac{180-20}{2} = 80°$

$\angle B + \angle P = 180°, \angle P = 100°, \angle Q = 100°$

Angles are $\angle P = 100°, \angle Q = 100°, \angle B = 80°, \angle C = 80°$

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2020–21 Academic year Works

Mathematics X
Circles

23

Concepts

a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.

b) The sum of the opposite angles of a cyclic quadrilateral is $180°$.

c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is $180°$ it will be a cyclic quadrilateral.

d) Square, rectangle and isosceles trapezium are cyclic .

Worksheet23

1) C, D are two points in a semicircle of diametre AB.
If $\angle BAD = 70°, \angle DBC = 30°$ then
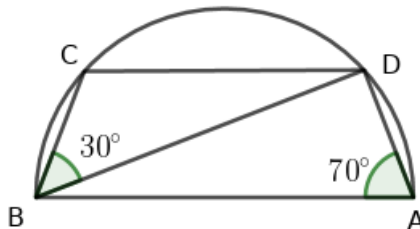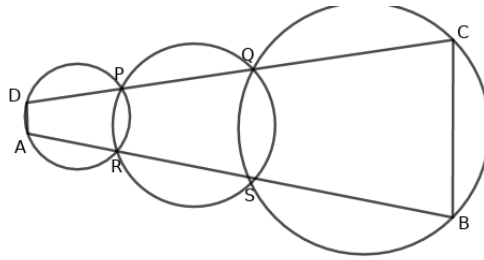

a) What is the measure of $\angle BCD$?

b) What is the measure of $\angle CDB$?

c) What is the measure of $\angle ADC$?

d) What is the measure of $\angle ABD$?

Answers

a) $\angle BCD = 180 - 70 = 110°$

b) $\angle CDB = 180 - (30 + 110) = 180 - 140 = 40°$

c) $\angle ADC = \angle ADB + \angle BDC = 90 + 40 = 130°$

d) $\angle ABD = 180 - (90 + 70) = 180 - 160 = 20°$

1

2) In the figure we can see three intersecting circles .$D - P - Q - C$ are on a line .$A - R - S - B$ are also on a line


a) Prove that the quadrilateral $ABCD$ is cyclic

b) If $\angle CDA = \angle DAB$ then what type of quadrilateral is $ABCD$?

c) If $\angle CDA = \angle DAB = 40°$ then find other two angles of $ABCD$

Answers

a) Join PR and QS in the figure.
$DARP$ is a cyclic quadrilateral.
If $\angle ADP = x$,then $\angle ARP = 180 - x$,$\angle PRS = x$.
Since PRSQ is cyclic, $\angle PQS = 180 - x$,$\angle SQC = x$, $\angle SBC = 180 - x$.
In the quadrilateral $ABCD$,$\angle D + \angle B = x + 180 - x = 180°$
$ABCD$ is cyclic.

b) Quadrilateral $ABCD$ will be an isosceles trapezium.

c) $\angle ABC = 140°$, $\angle DCB = 140°$

3 In traingle ABC, P is a point on BC.
$AB = AP$,the line through A parallel to BC and the line through C parallel to AP intersect at D .Prove that $ABCD$ is cyclic
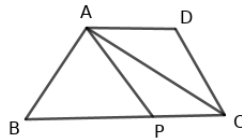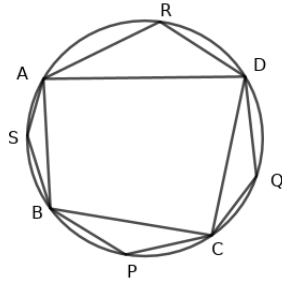

Answers

a) In traingle ABP,$AB = AP$. Angles opposite to the equal sides are equal.
$\angle ABP = x$ then $\angle APB = x$,$\angle APC = 180 - x$.
Since $APCD$ is a parallelogram $\angle D = 180 - x$.
$\angle B + \angle D = x + 180 - x = 180°$.
$ABCD$ is cyclic

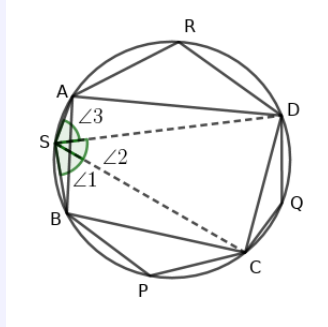4) The parallelogram which is not a rectangle is not cyclic. Justify this statement

Answers

★ $ABCD$ is a parallelogram(Draw a parallelogram and name it as $ABCD$ in an order.
$\angle A = \angle C$

★ Since $ABCD$ is not a rectangle $\angle A \neq 90°$, $\angle C \neq 90°$

★ $\angle A + \angle C \neq 180°$.
$\therefore ABCD$ is not cyclic

2

5) A, B, C, D, P, Q, R, S are the points on a circle Find $\angle P + \angle Q + \angle R + \angle S$

Answers

a) Draw SC and SD.
   $\angle SBPC$ is cyclic. $\angle 1 + \angle P = 180°$

b) $SCQD$ is cyclic. $\therefore \angle 2 + \angle Q = 180°$

c) $SDRA$ is cyclic. $\therefore \angle 3 + \angle R = 180°$.

d) Adding,
   $\angle 1 + \angle 2 + \angle 3 + \angle P + \angle Q + \angle R = 3 \times 180$
   $\angle S + \angle P + \angle Q + \angle R = 540°$

2020–21 Academic year Works

Mathematics X
Circles

24

Concepts

a) If the vertices of a qudrilateral are on a circle we call it cyclic quadrilateral.

b) The sum of the opposite angles of a cyclic quadrilateral is $180°$.

c) The converse of the above statement is also true. If the sum of the opposite angles of a quadrilateral is $180°$ it will be a cyclic quadrilateral.

d) Square, rectangle and isosceles trapezium are cyclic .

Worksheet24

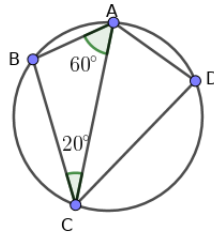**1)** In the figure O is the centre of the circle, $\angle DAB = 50°$
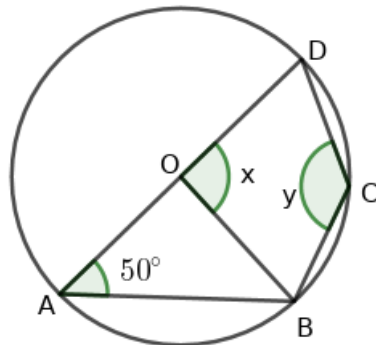
a) Find x

b) Find y

c) If $BC = CD$ then what is the measure of $\angle ADC$?

d) If $BC = CD$ then what is the measure of $\angle ABC$?

Answers

a) $x = 2 \times 50 = 100°$

b) $ABCD$ is a cyclic quadrilateral . $y + 50 = 180$, $y = 180 - 50 = 130$

c) Draw BD. In traingle $BDC, CD = CB$
Angles opposite to the equal sides are equal .
$\angle CDB = \angle DBC = \frac{180-130}{2} = 25°$
In triangle ODB, $OD = OB$. Angles opposite to the equal sides are equal. $\angle ODB = \angle OBD = \frac{180-100}{2} = 40°$
$\angle ADC = 25 + 40 = 65°$

d) $\angle ABD = 90°, \angle DBC = 25°$
$\angle ABC = 90 + 25 = 115°$

1

Prepared by Sri John PA

**2)** In the figure $\angle BAC = 60°, \angle BCA = 20°$

    a) Looking into the figure Riswan said: AC is the diametre of the circle .Can you agree with his opinion? Why?

    b) What is the measure $\angle ADC$

    c) If $\angle DAC : \angle DCA = 3 : 1$ then find these angles.

Answers

    a) $\angle B = 180 - (60 + 20) = 100° \neq 90°$
        AC is not the diametre

    b) $ABCD$ is cyclic .$\angle ADC = 180 - 100 = 80°$

    c) In triangle $ACD, \angle A + \angle C = 180 - 80 = 100°$.
        Divide 100 in the ratio $3 : 1, \angle A = 75°, \angle C = 25°$

---

3 In the figure $ABCDE$ is a regular pentagon.Prove that $ABCE$ is a cyclic quadrilateral.
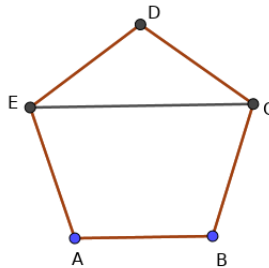
Answers

    ★ One angle of the regular pentagon is $= \frac{(5-2) \times 180}{5} = 108°$

    ★ In triangle $EDC, ED = CD, \angle DEC = \angle DCE = \frac{180-108}{2} = 36°$

    ★ $\angle ECB = 108 - 36 = 72°$. In the quadrilateral $ABCE, \angle A + \angle C = 108 + 72 = 180°$.
      $ABCE$ is cyclic.

---

4) Prove that the trapezium having diagonals equal is cyclic

2
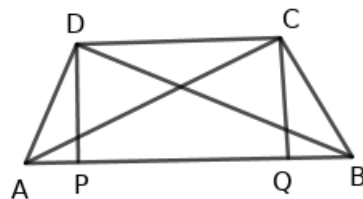
- ⋆ In the figure $ABCD$ is a trapezium having equal diagonals .AB is parallel to CD ,also $AC = BD$

- ⋆ Draw DP and , CQ perpendicular to AB.We can see traingles DPB and CQA.These are right angled triangles .Hypotenues are equal . Other two sides are equal $(DP = CQ)$.These triangles are equal . $\angle CAQ = \angle DBP$

- ⋆ $\angle CAQ = \angle DBP$ are the angles of triangle ACB and triangle BDA. Consider triangle ACB, and triangle BDA, AB is the common side , $AC = BD$, including angles are equal .These triangles are equal

- ⋆ Base angles of trapezium $ABCD$ are equal . This is an isosceles trapezium . Since $\angle A = \angle B, \angle A + \angle D = 180°, \angle A + \angle C = 180°$. $ABCD$ is cyclic

5) $ABCD$ is a cyclic quadrilateral. If $\angle A - \angle C = 60°$ then find the measure of $\angle C$.What is the measure of $\angle A$?

- ⋆ $\angle A - \angle C = 60°, \angle A + \angle C = 180°$.

- ⋆ Adding these equations ,$2 \times \angle A = 240, \angle A = 120°, \angle C = 180 - 120 = 60°$

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Prepared by Sri John PA

Mathematics X
Circles

25

Concepts

a) Two chords of a circle AB and CD intersect at the point P inside the circle .It can be proved that $PA \times PB = PC \times PD$

b) This relation can be used to construct a rectangle having equal area of another rectangle.

c) If the chords intersect outside the circle ,the same relation holds. $PA \times PB = PC \times PD$

Worksheet 25

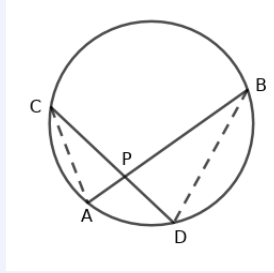1) In the figure two chords AB and CD intersect inside a circle at P.

.
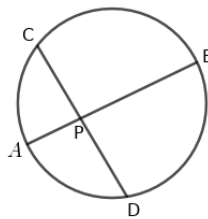
a) Join AC and BD. Establish the similarity of triangle PAC and PBD

b) What are the equal angles of these triangles

c) Prove that $PA \times PB = PC \times PD$

1

a) $\angle ACD = \angle ABD, \angle CAB = \angle CDB$(Angle in the same arc are equal)

In the triangles ACP and DBP
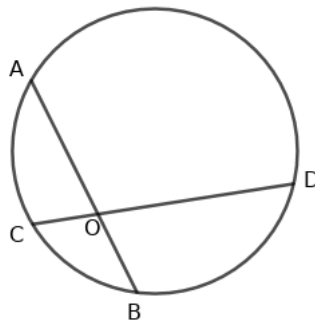
$\angle ACP = \angle DBP, \angle CAP = \angle PDB$

These triangles are similar.

b) $\angle C = \angle B, \angle A = \angle D$

c) Sides opposite to the equal angles are proportional.$\frac{PA}{PD} = \frac{PC}{PB}$
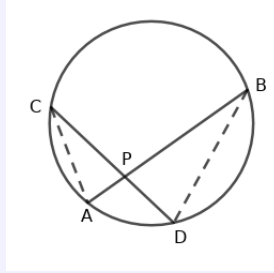
$PA \times PB = PC \times PD$

2) In the figure the chord AB has length 8cm and $OA = 5$cm.

a) What is the length of OB?

b) If$OC = 2.5$cm, what is the length of OD?

a) $OB = 8 - 5 = 3$cm

b) $OA \times OB = OC \times OD$
$5 \times 3 = 2.5 \times OD, OD = \frac{15}{2.5} = 6$cm

3) In the figure $AB = 5$cm, $BD = 4$cm, $CD = 9$cm.

.
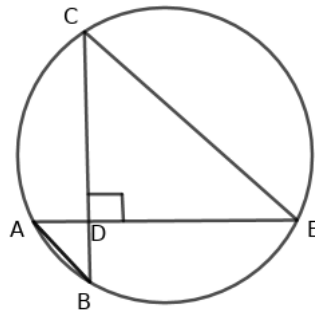
    a) What is the length of AD?

    b) Calculate the length of DE?

    c) Is CE the diameter of the circle? why?

    d) Find the length of CE

    a) $AD = \sqrt{5^2 - 4^2} = 3$cm

    b) $DA \times DE = CD \times DB$
        $3 \times DE = 9 \times 4$
        $DE = 12$cm

    c) CE is not the diametre . If it is a diametre ,$\angle D = 90°$ has the vertex on the circle

    d) $CE = \sqrt{12^2 + 9^2} = 15$cm

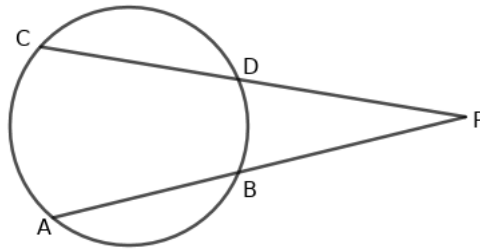**4)** If AB and CD are two chords of a circle which when produced meet at a point P . If $PA = PC$ show that $AB = CD$.

Answers

★ $PA \times PB = PC \times PD$
Cancel PA and PC on both sides.
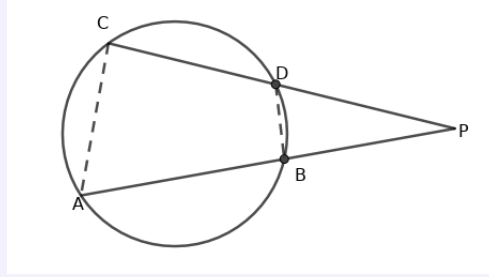$PB = PD, PA - AB = PC - CD$
Again cancel PA and PC on both sides . $-AB = -CD$,
$AB = CD$

5) In the figure AB and CD are two chords of a circle which when produced meet at a point P

a) Draw AC and BD , complete the quadrilateral $ABDC$

b) Establish the similarity of the triangles PAC and PDB

c) Establish the relation $PA \times PB = PC \times PD$

4

- $ABDC$ is a cyclic quadrilateral
  If $\angle C = x, \angle B = 180 - x$ and $\angle PBD = x$.
  $\angle ACP = \angle DBP$

- If $\angle A = y, \angle BDC = 180 - y$ and $\angle BDP = y$.
  $\angle CAP = \angle BDP$

- Triangles PCA and PBD are similar. Sides opposite to the equal angles are proportional.
  $\frac{PA}{PD} = \frac{PC}{PB}$
  $PA \times PB = PC \times PD$

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2020–21 Academic year Works

Mathematics X
Circles

25

Concepts

a) Two chords of a circle AB and CD intersect at the point P inside the circle .It can be proved that $PA \times PB = PC \times PD$

b) This relation can be used to construct a rectangle having equal area of another rectangle.

c) If the chords intersect outside the circle ,the same relation holds. $PA \times PB = PC \times PD$

Worksheet 25

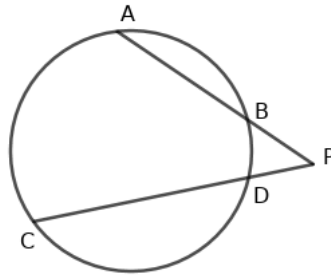1) The chords AB and CD intersect at O .This point divide each chord into two segments
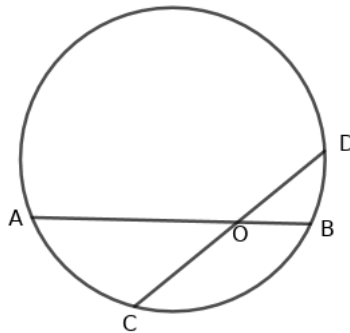
a) What is the relation between these segments?

b) If $CD = 10$cm and $OD = 4$cm then what is the length OC?

c) If $OA = 8$cm , $OC = 6$cm and $OD = 4$cm then what is the length OB?

Answers

a) $OA \times OB = OC \times OD$

b) $OC = CD - OD = 10 - 4 = 6$ cm

c) $8 \times OB = 6 \times 4, OB = 3$cm

1

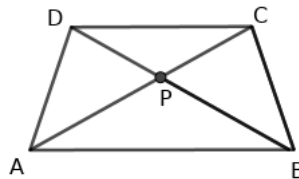**2)** The chords AB and CD intersect at P outside the circle.

    a) What is the relation between PA, PB, PC and PD?

    b) If $AB = 5$cm , $PB = 3$cm , $PD = 2$cm then what is the length CD?

Answers

    a) $PA \times PB = PC \times PD$

    b) $(5+3) \times 3 = (PD + CD) \times PD$
       $(5+3) \times 3 = (2 + CD) \times 2$
       $2 + CD = 12, CD = 10$cm

---

3) In the trapezium $ABCD$, $AD = BC$ and AB is parallel to CD . The diagonals AC and BD intersect at P.

    a) What is the relation between $\angle ADB$ and $\angle ACB$? How can we realize this relation?

    b) If $\angle DAC = 30°$ then what is the measure of $\angle DBC$?

    c) What is the relation between the segments made by P on the diagonals?

Answers

    a) $ABCD$ is an isosceles trapezium.Therefore it is a cyclic trapezium.
      A circle passes through A, B, C, D.In this circle the arc AB makes two angles $\angle ADB, \angle ACD$ in its complement .These are equal
      $\angle ADB = \angle ACD$

    b) The arc CD makes two angles $\angle DAC, \angle DBC$ in its complement .These are equal
      Since $\angle DAC = 30°$,$\angle DBC = 30°$

    c) AC ಿ BD are the diagonals .These are the chords of the circle passing through the vertices . These chods meet at P .
      $PA \times PC = PB \times PD$

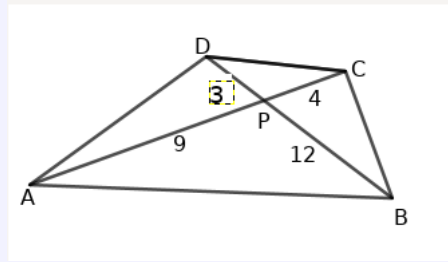**4)** In the quadrlateral $ABCD$, the diagonals AC and BD intersect at P. If $PA = 9$cm $PB = 12$cm, $PC = 4$cm and $PD = 3$cm then

    a) Draw a rough diagram and mark the mesaurements

    b) Is this a cyclic quadrilateral ? How can we realize this ?

2

c) If $\angle A = 40°$ and $\angle B = 70°$ find other two angles of the quadrilateral

Answers

a) Picture

b) $PA \times PC = 9 \times 4 = 36$
$PB \times PD = 12 \times 3 = 36$
$PA \times PC = PB \times PD$
There will a circle with the diagonals as the chords . $ABCD$ is a cyclic quadrilateral.

c) $\angle A = 40° \therefore \angle C = 180 - 40 = 140°$
$\angle B = 70° \therefore \angle D = 180 - 70 = 110°$

5) Draw a rectangle of sides 4cm and 6cm . Construct another rectangle with area equal to the area of the first rectangle and one side 7cm in length.

Answers

★ Draw a circle with the sides $AB = 6$ cm and $AD = 4$cm . Produce AB to E , 1 more such that $AE = 7$

★ Draw a arc with centre A and radius AE.This arc intresect DA produced at F.Produce BA such that $AD = AG$

★ Draw circumcircle of triangle BFG. This circle intersect AD at H.Draw a rectangle with sides AE and AH.Area of this rectangle will be equal to the area of the rectangle $ABCD$. Note :The first rectangle have area $AG \times AB.AE \times AH$ is the area of second rectangle . These areas are equal because $AB \times AG = AF \times AH$
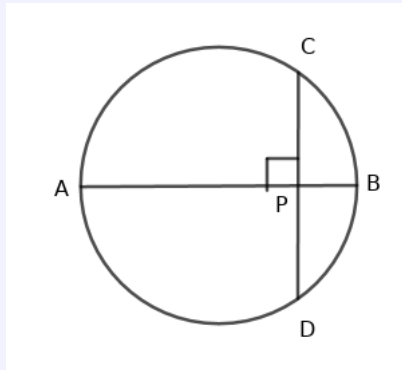
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# 2020-21 Academic year Works

Mathematics X

Circles

Concepts

a) In the case of the intersecting chords of a circle,if one chord AB is the diametre of the circle and other chord CD is perpendicular to the diametre ,then $PA \times PB = PC^2$

b) This relation is used to construct a square with same area of a rectangle.It can be used to draw the lines of irrational lengths.

Worksheet 26

1) AB is the diametre of a semicircle, P is a point on AB and PC is perpendicular to AB
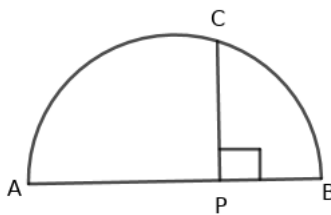
a) Prove that $PA \times PB = PC^2$

b) If $PA = 9$cm , $PB = 4$ cm then what is the length PC?

c) What is the area of the square with side PC?

Answers

a) Draw AC, BC.Consider triangle APC and triangle BPC.If $\angle PAC = x$ then $\angle PCA = 90 - x$, $\angle PCB = 90 - (90 - x) = x$, $\angle PBC = 90 - x$

These are similar triangles .Sides opposite to the equal angles are proportional . $\frac{PC}{PB} = \frac{PA}{PC}$
$PA \times PB = PC^2$.

b) $PC^2 = 9 \times 4 = 36$, $PC = 6$cm

c) Area $PC^2 = 36$sq.cm

1

2) $]AB$ is the diametre of a semicircle, P is a point on AB and PC is perpendicular to AB
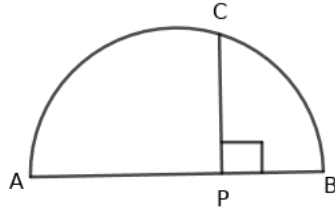
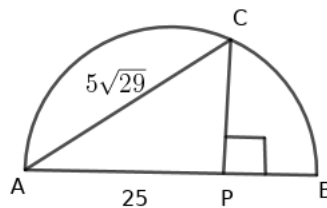   a) If $PC = 6$cm,and $PB = 3$cm then what is the length of PA

   b) What is the radius of the circle ?

   c) What is the area of the square drawn with side PC?

Answers

   a) $PA \times PB = PC^2$
      $PA \times 3 = 6^2, PA = 12$cm

   b) $AB = 12 + 3 = 15$cm ,Radius 7.5cm.

   c) Area $PC^2 = 36$sq.cm

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3) In the figure AB is the diametre of the semicircle, PC is perpendicular to AB. $AC = 5\sqrt{29}$cm and $PA = 25$cm.

   a) What is the length of PC?

   b) What is the lenght PB?

   c) What is the radius of the circle?

Answers

   a) $PC = \sqrt{(5\sqrt{29})^2 - 25^2} = 10$ cm

   b) $PA \times PB = PC^2$,$25 \times PB = 10^2, PC = 4$cm

   c) $AB = 25 + 4 = 29$, Radius= 14.5cm

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4) Draw a semicircle of suitable diametre .Construct a line of length $\sqrt{12}$cm perpendicular to the diametre whose one end is on the diameter and other end is on the semicircle.Explain the principle of construction.

a) Take two positive integers having the product 12.Draw a circle with the sum of these numbers as the diametre.
$12 = 6 \times 2$, diametre $= 6 + 2 = 8$cm.
Draw a line AB of lenght 8cm , and a semicircle with AB as the diametre.

b) Mark a point P at the diatance 6 cm from A. Draw a perpendicular to AB at P.This line cut the semicircle at C

c) $PC^2 = PA \times PB = 6 \times 2 = 12$
$PC = \sqrt{12}$സെന്റിമീറ്റർ

5) In the figure AB is the diametre of the circle and PC is perpendicular to the diametre. $PA : PB = 2 : 1$ and $PC = 6$cm.
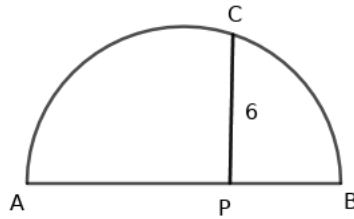
a) Write the relation between PA, PB and PC?

b) Find the lengths PA and PB

c) What is the radius of the circle?

a) $PA \times PB = PC^2$

b) If $PB = x$,$2x \times x = 6^2$,$2x^2 = 36, x^2 = 18, x = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$
$PA = 6\sqrt{2}, PB = 3\sqrt{2}$

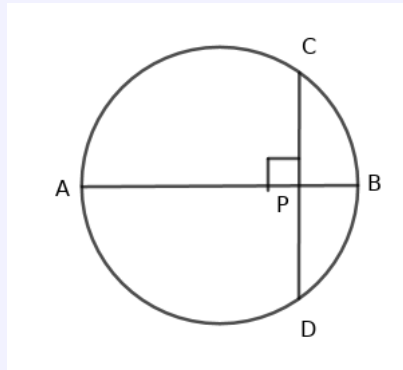c) $AB = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$
Radius $= \frac{9\sqrt{2}}{2}$ cm

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# 2020-21 Academic year Works

Mathematics X
Circles

$\boxed{26}$

a) In the case of the intersecting chords of a circle,if one chord AB is the diametre of the circle and other chord CD is perpendicular to the diametre ,then $PA \times PB = PC^2$

b) This relation is used to construct a square with same area of a rectangle.It can be used to draw the lines of irrational lengths.

$\boxed{\text{Worksheet 26}}$

1) AB is the diametre of a semicircle.The lines PQ and RS are perpendicular to AB.If $PQ = RS$ then
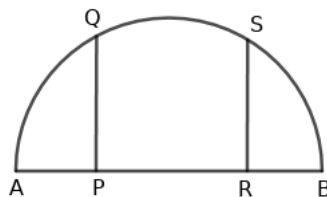
a) What is the relation between the lengths PA, PB and PQ ?
b) What is the relation between the lenghths AR, BR and RS
c) Prove that $PA = BR$

Answers

a) $PA \times PB = PQ^2$

b) $AR \times BR = RS^2$

c) Since $RS = PQ$,$RS^2 = PQ^2$
$PA \times PB = AR \times BR$
$PA(PR + BR) = BR(PA + PR)$
$PA \times PR + PA \times BR = PA \times BR + BR \times PR$
Cancel $PA \times BR$ from both sides
$PA \times PR = PR \times BR$
$PA = BR$

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2)   a) Draw an equilateral triangle of altitude 3 cm

       b) What is the lenght of one side ?

       c) What is the radius of its incircle?

Answers

    a) Draw a line AD of length 4cm. Draw a circle with diametre AD.Mark a point P at a distance 3cm from A. From P draw a line perpendicular to AD. This line cut the circle at C, B .Draw triangle ABC.
$PC^2 = 3 \times 1, PC = \sqrt{3}, BC = 2\sqrt{3}$cm
$AC = \sqrt{3^2 + \sqrt{3}^2} = \sqrt{12} = 2\sqrt{3}$cm.
$AB = 2\sqrt{3}$cm . $AB = BC = AC = 2\sqrt{3}$cm

    b) $AB = AC = BC = 2\sqrt{3}$

3) Draw a rectangle of sides 5cm and 3cm .Construct a square whose area is same as the area of the rectangle

Answers

★ Draw the quadrilateral $ABCD$. $AB = 5$cm , $BC = 3$cm.

★ Produce AB and mark the point E such that $BC = BE$

★ Draw a semicircle of diametre AE.Produce BC, meet the semixircle at F.

★ $BA \times BE = BF^2$ can be written as $AB \times BC = BF^2$. $AB \times BC$ is the area of the rectangle

★ Draw a square of side BF.Area of rectangle is equal to the area of the square as per the relation $AB \times BC = BF^2$

4)   a) Draw a semicircle of suitable diametre .Draw a line of length $\sqrt{12}$cm whose one end on AB and other end on the semicircle.

   b) Draw a chord of length $\sqrt{48}$ cm by make the semicircle as the circle

Answers

★ Take two numbers with product 12, draw a circle with diameter as the sum of these numbers .$12 = 6 \times 2$, diametre $= 6 + 2 = 8$cm .
Draw a line of $AB = 8$cm. Draw a circle with this line as the diametre
Mark a point P at the diatance 6cm from A, draw a perpendicular line from P to the diametre .This line cut the semicircle at C.
$PC^2 = PA \times PB = 6 \times 2 = 12$
$PC = \sqrt{12}$cm

★ Make semicircle as the circle . Produce CP in such a way as the meet the circle at D.
$CD = 2\sqrt{12} = \sqrt{2^2 \times 12} = \sqrt{48}$cm

5) AB is the diametre of a semicircle. $PQ = \sqrt{14}$cm $RS = \sqrt{18}$ cm . These lines are perpendicular to the diametre . Find the length of AB?
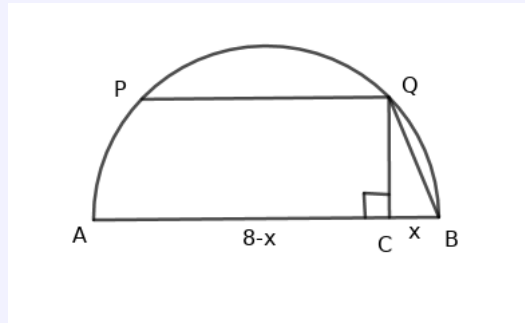
Answers

★ $PQ = \sqrt{14} = \sqrt{7 \times 2}$
Length of AP is 2cm . length of BP cm 7 .
Then $AB = 9$cm.

★ $RS = \sqrt{18} = \sqrt{6 \times 3}$
Length of AR is 6cm ,length of BR is 3cm .
Then $AB = 9$cm.

★ In both cases $AB = 6 + 3 = 9$ and $AB = 2 + 7 = 9$. $AB = 9$ cm

6) AB is the diametre of a semicircle , PQ is parallel to the diametre
If $AB = 8$cm , $BQ = 2$ cm then find the langth PQ.

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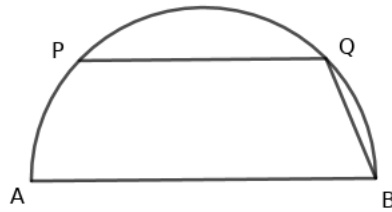
Answers

- ★ Draw a perpendicular from Q to AB. Draw QC, If $CB = x$ then $CA = 8 - x$

- ★ $CB \times CA = CQ^2$
  $x(8 - x) = CQ^2$

- ★ In the right triangle QCB , $CQ^2 = 2^2 - x^2 = 4 - x^2$

- ★ $x(8 - x) = 4 - x^2, 8x - x^2 = 4 - x^2, 8x = 4, x = \frac{4}{8} = \frac{1}{2}$

- ★ Draw a perpendicular from P to AB ,
  $AD = \frac{1}{2}$ ആകുന്നു.
  $PQ = 8 - (\frac{1}{2} + \frac{1}{2}) = 7$ cm

</div>

7) Draw an equilateral triangle of one side $\sqrt{18}$cm

- ★ $18 = 6 \times 3, 6 + 3 = 9$
  Draw a line AB of length 9cm

- ★ Draw a semicircle with diameter AB. Mark a point P at the diatance 6cm from A. Draw a perpendicular from P to AB. This line cut the circle at C. $PC = \sqrt{18}$.

- ★ Draw an equilateral triangle with PC as side

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