ARITHMETIC SEQUENCES

Note:- Numbers in an arithmetic sequence are called terms.

The terms can be denoted as

x_1 , x_2 , x_3 , x_4 , x_5 , x_6 , x_7 , ...x_n......

x_1 , x_2 , x_3 , x_4 , x_5 , x_6 , x_7

3 , 5 , 7 , 9 , 11 , 13 , 15 , ...In this arithmetic sequence,

1st term $x_1 = 3$
2nd term $x_2 = 5$
3rd term $x_3 = 7$
4th term $x_4 = 9$

common difference d $= x_2 - x_1 = 5 - 3 = 2$

common difference d $= x_3 - x_2 = 7 - 5 = 2$

common difference d $= x_4 - x_3 = 9 - 7 = 2$

common difference d $= x_5 - x_4 = 11 - 9 = 2$

common difference d $= x_6 - x_5 = 13 - 11 = 2$

Adding the common difference **d** to the 1st term gives the 2nd term

Adding the common difference d to the 2nd term gives the 3rd term

Adding the common difference d to the 3rd term gives the 4th term

Adding the common difference d to the 4th term gives the 5th term

1st term $x_1 = 3$ **common difference d =2**

2nd term $x_2 = x_1 + d = 3 + 2 = 5$

3^{rd} term $x_3 = x_2 + d = 5 + 2 = 7$

4^{th} term $x_4 = x_3 + d = 7 + 2 = 9$

5^{th} term $x_5 = x_4 + d = 9 + 2 = 11$

x_1 , x_2 , x_3 , x_4 , x_5 , x_6 , x_7

3 , 5 , 7 , 9 , 11 , 13 , 15 , ...

+2 +2 +2 +2 +2 +2

To get the 7^{th} tem x_7 to the 1^{st} term x_1 how many times add the common difference d ? <u>**6 times**</u>

That is $x_7 = x_1 + 6d$

$$15 = 3 + 6(2)$$

Adding the common difference d to the 1^{st} term gives the 2^{nd} term

That is $x_2 = x_1 + 1d$

Subtracting 1 time common difference from the 2^{nd} term gives the 1^{st} term

That is $x_1 = x_2 - 1d$

Adding 2 times common difference d to the 3^{rd} term gives the 5^{th} term

$$x_5 = x_3 + 2d$$

Subtracting 2 times common difference from the 5^{th} term gives the 3^{rd} term

$$x_3 = x_5 - 2d$$

Adding 4 times common difference d to the 5th term gives the 9th term

$$x_9 = x_5 + 4d$$

Subtracting 4 times common difference from the 9th term gives the 5th term

$$x_5 = x_9 - 4d$$

Subtracting 2 times common difference from the 5th term gives the 3rd term

$$x_3 = x_5 - 2d$$

Adding 5 times common difference d to the 5th term gives the 10th term

$$x_{10} = x_5 + 5d$$

Subtracting 8 times common difference d from the 9th term gives the 1st term

$$x_1 = x_9 - 8d$$

You will understand the following

$$x_2 = x_1 + 1d$$

$$x_3 = x_1 + 2d$$

$$x_4 = x_1 + 3d$$

$$x_6 = x_1 + 5d$$

$$x_7 = x_1 + 6d$$

$$x_{20} = x_1 + 19d$$

$$x_{31} = x_1 + 30d$$

$$x_2 = x_3 - 1\,d$$

$$x_3 = x_5 - 2\,d$$

$$x_4 = x_{14} - 10\,d$$

$$x_5 = x_{20} - 15\,d$$

$$x_6 = x_{26} - 20\,d$$

$$x_7 = x_{17} - 10\,d$$

$$x_8 = x_{10} - 2\,d$$

$$x_7 - x_5 = 2\,d$$

$$x_{17} - x_7 = 10d$$

$$x_{10} - x_5 = 5\,d$$

$$x_6 - x_2 = 4\,d$$

$$x_7 - x_2 = 5\,d$$

$$x_{15} - x_5 = 10d$$

ARITHMETIC SEQUENCES

To get the n^{th} term of an Arithmetic sequence : -

Add (n-1)times the common difference to the First term

If First term f and common difference d ,

Then n^{th} term(algebraic expression) of an arithmetic sequence is

$$x_n = f + (n-1)\, d$$

$$x_n = f + dn - d$$

$$x_n = dn + f - d$$ (Write like this.)

Note:- ***First term*** f , common difference d ,

n^{th} **term** (algebraic expression) of an arithmetic sequence

$$x_n = dn + f - d$$

Note:- *Algebraic form is always* $x_n = an + b$

(a first degree polynomial)

Here common difference d = The number number multiplied by n (that is a)

first term f = the sum of the coefficients $(a + b)$

Question : - **Consider the nth term of an Arithmetic sequence $X_n = 2n+1$. Then**

i) **Find common difference**

ii) **Find the first term**

iii) **Write the sequence**

iv) **Find the 10th term**

Answer : -

$$\boxed{X_n = 2n + 1}$$

i) Common difference d = The number multiplied by n = 2

ii) First term (X_1) OR f = sum of the coefficients

$= 2 + 1$

$= 3$

iii) **Sequence** → **3 , 5 , 7 , 9 , . . .**

iv) **n^{th} term** $X_n = 2n + 1$

10^{th} term $X_{10} = 2(10) + 1$

$= 20 + 1 = 21$

Arithmetic Sequence

Algebraic expression for the arithmetic sequence $2n + 1$ [That is n^{th} term x_n]

2n means multiples of 2. That is 2 , 4 , 6 , 8 , 10,.........

2n + 1 means adding 1 to the multiples of 2. That is 3 , 5 , 7 , 9 , 11,.........

First term $f = 3$

common difference $d = 2$

If algebraic expression is given, there is a trick to see the common difference and the first term without writing the sequence

eg 1 :- algebraic expression $x_n = 2n + 1$

common difference d = The number multiplied by n = 2

First term $f = 2 + 1 = 3$

(Erase n and write numbers only)

Then the sequence is obtained by adding the common difference 2 to the first term 3

That is 3 , 5 , 7 , 9 , 11 ,......

eg 2 :- algebraic expression $x_n = 3n - 1$

common difference d = n നെഗുണിച്ച സംഖ്യ = 3

ആദ്യപദം $f = 3 - 1 = 2$

(Erase n and write numbers only)

Then Sequence = 2 , 5 , 8 , 11 ,.....

From the algebraic form of an Arithmetic Sequence, we can find any terms of the sequence.

1) Consider the algebraic form of an Arithmetic sequence

X_n = 2n+1. Then find its **10th term**?

In algebra , just write 10 instead of n

$$X_n = 2n + 1$$

$$X_{10} = 2(10) + 1$$

$$= 20 + 1$$

$$= 21$$

---

2) Consider the algebraic form of an Arithmetic sequence

X_n = 3n-2. Then find its **5th term**?

In algebra , just write 5 instead of n

$$X_n = 3n - 2$$

$$X_{10} = 3(5) - 2$$

$$= 15 - 2$$

$$= 13$$

3 Questions from the same Concept(From n^{th} term)

1) You can see the position of terms in an arithmetic sequence using the algebraic form.

Question-1

Which term is 99 in the arithmetic sequence

1 , 3 , 5 , 7 ,9, 11 ,... ?

common difference d $= 3 - 1 = 2$

Multiples of d $= 2 = 2, 4, 6, 8, 10, \ldots\ldots\ldots 2n$

Subtracting 1 $= 1, 3, 5, 7, 9 \ldots\ldots\ldots 2n - 1$

Let **n^{th} term** $x_n = 99$

$$2n - 1 = 99$$

$$2n = 99 + 1$$

$$2n = 100$$

$$n = 100/2 = 50$$

That is 50^{th} term is 99

2) You can see how many terms in an arithmetic sequence using the algebraic form.

<u>*Question-2*</u>

How many terms are there in the arithmetic sequence

5 , 8 , 11 , 14, 17 , ...92 **?**

common difference d = 8 - 5 = 3

Multiples of d = 3 = 3 , 6 , 9 , 12 , 15,.........3<u>n</u>

Adding <u>2</u> = 5 , 8 , 11 , 14, 17 , ...<u>3n+2</u>

Let **n^th term** x_n = 92

3n + 2 = 92

3n = 92 - 2

3n = 90

n = 90/3 =30

30^{th} term is 92

That is , there are 30 terms

3) You can check any number is a term in an arithmetic sequence using the algebraic form.

<u>Question-3</u>

<u>Is 61 a term</u> in the sequence 4 , 7 , 10 , 13 , ...?

common difference d = 7 – 4 = 3

d = multiples of 3 = 3 , 6 , 9 , 12 , 15,.........3<u>n</u>

Adding 1 = 4 ,7 , 10 , 13 , 16......... 3<u>n+1</u>

Let n^{th} x_n = 61

$$3n + 1 = 61$$

$$3n = 61 - 1$$

$$3n = 60$$

$$n = 60/3 = 20$$

n is a natural number

So 61 is a term of the sequence

If we know any 2 terms in an arithmetic sequence, we can find the common difference

If we divide the term difference by the position difference , we get the common difference

Consider the arithmetic sequence : 3 , 5 , 7 , 9 , 11 , 13 , 15 , 17, ...

	1^{st} term	2^{nd} term	3^{rd} term	4^{th} term	5^{th} term	6^{th} term	7^{th} term	....
Terms	3 ,	5 ,	7 ,	9 ,	11 ,	13 ,	15,	....
Positions	1^{st} place	2^{nd} place	3^{rd} place	4^{th} place	5^{th} place	6^{th} place	7^{th} place	....

Note the positions and the terms

7^{th} position's term = 15

2^{nd} position's term = 5

Term difference = 15 - 5 = 10

Position difference = 7 - 2 = 5

$$\text{common difference } d = \frac{\text{Term difference}}{\text{Position difference}}$$

$$d = \frac{10}{5} = 2$$

(3 , 5 , 7 , 9 , 11 , 13 ..... Here common difference is 2)

That is, the difference between any two terms in the arithmetic sequence divided by their position difference gives the common difference

Note:- If you know the first term (x_1) and the last term (x_n) of an arithmetic sequence , Another way to find the number of terms **n**.

To find the number of terms in an arithmetic sequence ,

$$n = \frac{Lastterm - Firstterm}{commondifference} + 1$$

Question:-

How many terms are in the sequence

3,5,7,9,..................103 ?

Answer

$$\text{Number of terms } n = \frac{(\text{Last term} - \text{First term})}{\text{common difference}} + 1$$

$$\text{Number of terms } n = \frac{(103 - 3)}{2} + 1$$

$$= \frac{100}{2} + 1$$

$$= 50 + 1 = 51$$

There are 51 terms in the sequence

That is , 51st term is 103

ARITHNETIC SEQUENCES   Note - 7

1 മുതൽ തുടർച്ചയായ നിശ്ചിത എണ്ണൽ സംഖ്യകളുടെ തുക

(The sum of fixed number of consecutive natural natural numbers starting with 1)

1 മുതൽ n വരെയുള്ള തുടർച്ചയായ എണ്ണൽ സംഖ്യകളുടെ തുക $= \boxed{\dfrac{n(n+1)}{2}}$

eg:-

1 മുതൽ 10 വരെയുള്ള എണ്ണൽ സംഖ്യകളുടെ തുക

$$= \frac{10 \times (10+1)}{2}$$

$$= \frac{10 \times 11}{2}$$

$$= \frac{110}{2} = 55$$

1 മുതൽ 100 വരെയുള്ള എണ്ണൽ സംഖ്യകളുടെ തുക

$$= \frac{100 \times (100+1)}{2}$$

$$= \frac{100 \times 101}{2}$$

$$= \frac{10100}{2} = 5050$$

Sequence: 1,3,5,7,9,11,13,15.....

In the arithmetic sequences ,the sum of the positions is equal to the sum of the pairs of terms

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1,	3,	5,	7,	9,	11,	13,	15.....

No. of terms = 8 ,That is 4 pairs.

Pair as shown above

$x_1 + x_8$ ⇨ sum of the positions $= 1 + 8 = 9$

$x_2 + x_7$ ⇨ sum of the positions $= 2 + 7 = 9$

$x_3 + x_6$ ⇨ sum of the positions $= 3 + 6 = 9$

$x_4 + x_5$ ⇨ sum of the positions $= 4 + 5 = 9$

$x_1 + x_8$ (sum of the pairs of terms) $= 1 + 15 = 16$

$x_2 + x_7$ (sum of the pairs of terms) $= 3 + 13 = 16$

$x_3 + x_6$ (sum of the pairs of terms) $= 5 + 11 = 16$

$x_4 + x_5$ (sum of the pairs of terms) $= 7 + 9 = 16$

Note:- the sum of the positions is equal to the sum of the pairs of terms.

ARITHMETIC SEQUENCES

Question: - 5,10,15,20,..... In this arithmetic sequences, find the sum of the first 4 terms.

Term positions	1	2	3	4	
	x_1	x_2	x_3	x_4	
	5 ,	10 ,	15 ,	20	,..........

Number of terms = 4 ,That is 2 pairs

Sum of the 1st pair $= x_1 + x_4 = 5 + 20 = 25$

Sum of the 2nd pair $= x_2 + x_3 = 10 + 15 = 25$

Total $2 \times 25 = 50$

Number of pairs $= 2$

Sum of one pair $= 25$

If the number of terms in an arithmetic sequence is an even number ,

Sum = Number of pairs X Sum of one pair

Question: - 1,3,5,7,9,11,13,15..... In this arithmetic sequences, find the sum of the first 8 terms.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1,	3,	5,	7,	9,	11,	13,	15.....

Number of terms = 8 ,That is 4 pairs

Sum of the 1st pair = $x_1 + x_8$ = 1 + 15 = 16

Sum of the 2nd pair = $x_2 + x_7$ = 3 + 13 = 16

Sum of the 3rd pair = $x_3 + x_6$ = 5 + 11 = 16

Sum of the 4th pair = $x_4 + x_5$ = 7 + 9 = 16

Total 4 X 16 = 64

Number of pairs = 4

Sum of one pair = 16

If the number of terms in an arithmetic sequence is an even number ,

Sum = Number of pairs X Sum of one pair

Another Feature

sum of the pairs of terms	sum of the positions
$x_1 + x_8 = 1 + 15 = 16$	$1 + 8 = 9$
$x_2 + x_7 = 3 + 13 = 16$	$2 + 7 = 9$
$x_3 + x_6 = 5 + 11 = 16$	$3 + 6 = 9$
$x_4 + x_5 = 7 + 9 = 16$	$4 + 5 = 9$

$x_1 + x_8 \Rightarrow$ sum of the positions $= 1 + 8 = 9$

$x_2 + x_7 \Rightarrow$ sum of the positions $= 2 + 7 = 9$

$x_3 + x_6 \Rightarrow$ sum of the positions $= 3 + 6 = 9$

$x_4 + x_5 \Rightarrow$ sum of the positions $= 4 + 5 = 9$

$x_1 + x_8$ (sum of the pairs of terms) $= 1 + 15 = 16$

$x_2 + x_7$ (sum of the pairs of terms) $= 3 + 13 = 16$

$x_3 + x_6$ (sum of the pairs of terms) $= 5 + 11 = 16$

$x_4 + x_5$ (sum of the pairs of terms) $= 7 + 9 = 16$

Note:- the sum of the positions is equal to the sum of the pairs of terms.

x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 . . .

5 , 10 , 15 , 20 , 25 , 30 , 35 , 40, . . .

Here

1^{st} and 8^{th} (1 + 8 = 9)

2^{nd} and 7^{th} (2 + 7 = 9)

3^{rd} and 6^{th} (3 + 6 = 9)

4^{th} and 5^{th} (4 + 5 = 9)

Make pairs the terms and add

$$x_1 + x_8 = 5 + 40 = 45$$

$$x_2 + x_7 = 10 + 35 = 45$$

$$x_3 + x_6 = 15 + 30 = 45$$

$$x_4 + x_5 = 20 + 25 = 45$$

Note : -

In an arithmetic sequence ,
the sum of the positions is equal to
the sum of the pairs of terms.

The sum of the positions is equal to the sum of the pairs of terms.

eg : -

2 4,6,8,10,12,... is an arithmeticsequence.
Find the sum of the first 6 terms.
Sum of 1st term + 6th term = 2 + 12 = 14
Sum of 2nd term + 5th term = 4 + 10 = 14
Sum of 3rd term + 4th term = 6 + 18 = 14

Here write the terms as pairs.
6 terms = 3 pairs of terms
Sum of each pair = 14
∴ Sum of 6 terms = 3 x 14 = 42

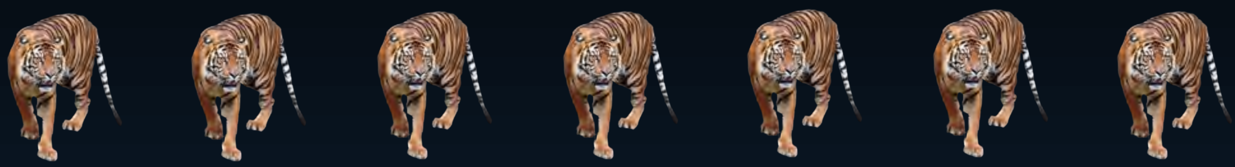
Add the positions in those pairs of terms.
Then we get 7

1st place + 6th place = 1 + 6 = 7
2nd place + 5th place = 2 + 5 = 7
3rd place + 4th place = 3 + 4 = 7
That is

In an arithmetic sequence,
The sum of the positions is equal to the sum of the pairs of terms.

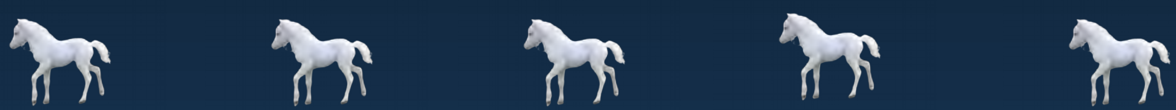
ARITHMETIC SEQUENCE

Odd Numbers **1, 3 ,5 ,7 , 9 ,11, 13............**



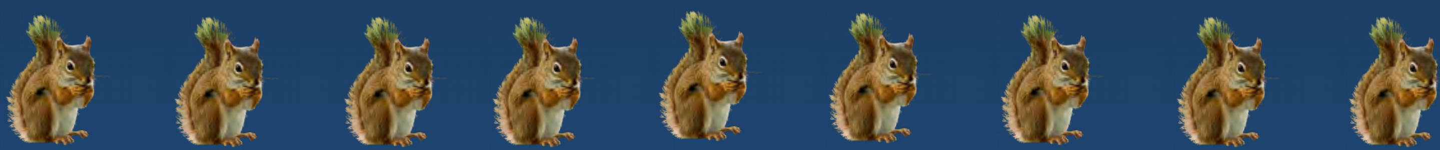
How many tigers are there in the picture? 7
What is the place of the tiger in the middle?

Tiger in the 4th place (Hint:- 7 odd number , Adding 1 = 8, Half 4)



How many horses are there in the picture? 5
What is the place of the horse in the middle?

Horse in the 3rd place (Hint:- 5 odd number , Adding 1 = 6, Half 3)



How many squirrels are there in the picture? 9
What is the place of the squirrel in the middle?

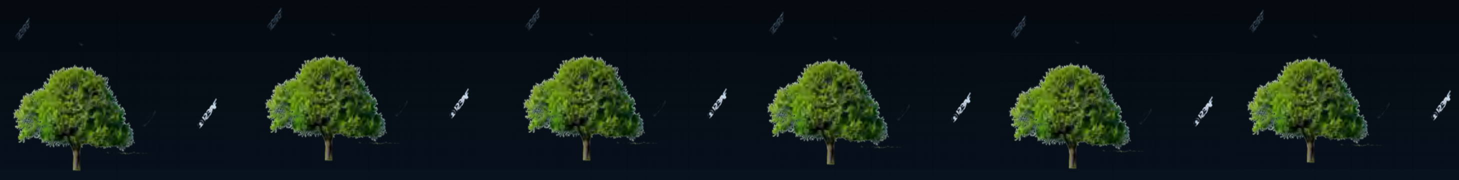
Squirrel in the 5th place (Hint:- 9 odd number , Adding 1 = 10, Half 5)

Note:- **If the total number is odd , what is the place in the middle?**

Add 1 to the number . Find half of it .

ARITHMETIC SEQUENCE

Even Numbers *2, 4 ,6 ,8 ,10 ,12 ,14..............*



1 2 3 4 5 6

How many trees are there in the picture? 6

What is the place of the tree in the middle?

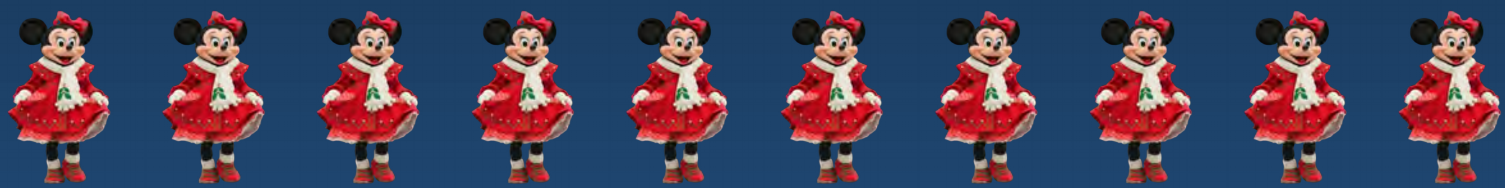
3rd and 4th (Hint:- 6 even number , Half 3 ,Adding 1 = 4)

1 2 3 4 5 6 7 8

How many circles are there in the picture? 8

What is the place of the circle in the middle?

4th and 5th (Hint:- 8 even number , Half 4 ,Adding 1 = 5)



1 2 3 4 5 6 7 8 9 10

How many Minni mouses are there in the picture? 10

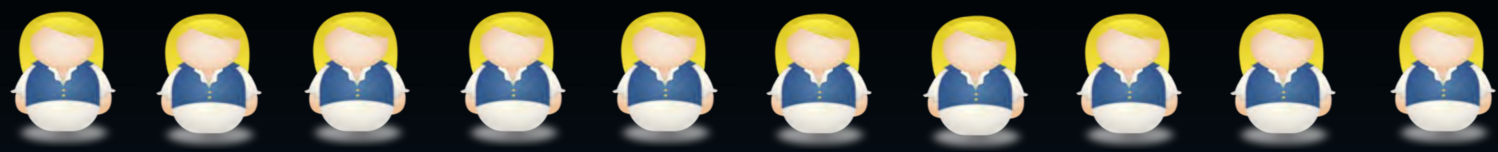
What is the place of the Minni mouse in the middle?

5th and 6th (Hint:- 10 even number , Half 5 ,Adding 1 = 6)

Note:- **If the total number is even , what is the place in the middle?**

Find half of the number . Add 1 to it .These two are in the middle.

സമാന്തരശ്രേണി



1 2 3 4 5 6 7 8 9 10

What is the place of the child between 1 and 5 in the middle?

Adding 1 and 5 ,sum = 6 . Half 3 . So 3rd child

What is the place of the child between 5and 9 in the middle?

Adding 5 and 9 ,sum = 14 . Half 7 . So 7th child

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

What is the place of the hen between 1 and 7 in the middle?

Adding 1 and 7 , sum = 8 . Half 4 . So 4th hen.

What is the place of the first 7 hens , in the middle?

Adding 1 and 7 , sum = 8 . Half 4 . So 4th hen.

What is the place of the hen between 5 and 15 in the middle?

Adding 5 and 15 , sum = 20 . Half 10 . So 10th hen.

What is the place of the hen between 2 and 4 in the middle?

Adding 2 and 4 , sum = 6 . Half 3 . So 3rd hen.

What is the place of the first 3 hens , in the middle?

Adding 1 and 3 , sum = 4 . Half 2 . So 2nd hen.

Question: Find the sum of the first 5 terms of the arithmetic sequence 3,5,7,9,11,13,15,...

middle term

x_1	x_2	x_3	x_4	x_5
3,	5,	7,	9,	11

No. of terms = 5 (**Odd no.**)

Middle term x_3 = 7

If the number of trems in an arithmetic sequence is an odd number

Sum of terms = Middle term X No.of terms

= 7 X 5

= 35

Note:- **If you know the first term (x_1) and the last term (x_n) of an arithmetic sequence ,The formula for finding the sum (S_n) of the terms**

$$S_n = \frac{n}{2} \text{ (First term + Last term)}$$

$$S_n = \frac{n}{2} (x_1 + x_n)$$

....

Question:- 3,5,7,9,.........In ths arithmetic sequence , Find the sum of the first 20 terms.

Answer: Number of terms $n = 20$

common difference $d = 2$

First term $x_1 = 3$

Last term $x_{20} = x_1 + 19d$

$= 3 + 19(2)$

$= 3 + 38 = 41$

Sum of n terms $S_n = \frac{n}{2}\text{(first term +last term)}$

$$= \frac{20}{2} (3 + 41)$$

$$= 10 (44) = 440$$

Question :- 3,5,7,9,..........21 in this arithmetic sequence

i) **How many terms?**

ii) Find the sum of all terms

ഉത്തരം

i) Number of terms n = $\frac{(\text{Last term} - \text{First term})}{\text{common difference}} + 1$

Number of terms n = $\frac{(21 - 3)}{2} + 1$

$= \frac{18}{2} + 1$

$= 9 + 1$

$= 10$

There are 10 words in the range, i.e. 10th term 2

1ii) Number of terms n = 10First term $x_1 = 3$

Last term $x_{10} = 21$

Sum of S_n $= \frac{n}{2}$ (**First term** + Last term)

$= \frac{10}{2}(3 + 21)$

$= 5(24) = 120$

Note:- The algebraic expression of the sum of an arithmetic sequence is always $S_n = an^2 + bn$.

Note:- If the sum of the first n terms is of the form $S_n = an^2 + bn$,

common difference = twice the coefficient of $n^2 = 2a$

First term = The sum of the coefficients = a+b

Question :- . If the algebraic expression of the sum of an arithmetic sequence is $3n^2 + 2n$

i) What is the common difference ?

ii) What is first term ?

Answer

The number multiplied by $n^2 = 3$

common difference = twice the coefficient of $n^2 = 2(3) = 6$

First term = The sum of the coefficients = 3 + 2 = 5

Question :- . The Algebraic expression of the sum of an arithmetic sequence is $5n^2 - 3n$

i) Find the sum of the first 10 terms

ii)Find the sum of the first 5 terms

Answer

i)The sum of the first n terms $S_n = 5n^2 - 3n$

The sum of the first 10 terms $S_{10} = 5(10)^2 - 3(10)$

$= 5(100) - 30$

$= 500 - 30$

$=470$

ii) The sum of the first n terms $S_n = 5n^2 - 3n$

The sum of the first 5 terms $S_5 = 5(5)^2 - 3(5)$

$= 5(25) - 15$

$= 125 - 15$

$=110$

ARITHMETIC SEQUENCES

Natural Numbers : 1, 2, 3, 4, 5 ,6, 7 ,8 , 9,10....

Even Numbers : 2, 4, 6, 8, 10, 12,....

Odd Numbers : 1 ,3, 5, 7, 9,11,......

The sum of the first n natural numbers $= \dfrac{n(n+1)}{2}$

The sum of the first 3 natural numbers $= \dfrac{3(3+1)}{2}$

$= \dfrac{3(4)}{2} = \dfrac{12}{2} = 6$

(**The sum of the first 3 natural numbers** $= 1+2+3 = 6$)

The sum of the first n even numbers $= n(n+1)$

The sum of the first 3 even numbers $= 3(3+1) = 3(4) = 12$

(**The sum of the first 3 even numbers** $= 2 + 4 + 6 = 12 = 3 \times 4$)

The sum of the first n odd numbers $= n^2$

The sum of the first 3 odd numbers $= 3^2 = 9$

(**The sum of the first 3 odd numbers** $= 1 + 3 + 5 = 9 = 3^2$)

If the terms are large numbers, fractions or negative integers of an arithmetic sequence

Algebraic expression (n^{th} term) $x_n = dn + f - d$

It will be convenient to find n^{th} term using this formula.

Question:- 101 ,108 ,115 ,122 ,.....

Write the algebraic expression of this arithmetic sequence

Answer:-

1^{st} term $f = 101$

common difference $d = 108 - 101 = 7$

Algebraic expression (n^{th} term) $x_n = dn + f - d$

$$x_n = 7n + 101 - 7$$

$$x_n = 7n + 94$$

Note:- Algebraic expression $x_n = 7n + 94$, Then

1^{st} term f =sum of the coefficients $= 7 + 94 = 101$

common difference d = coefficient of $n = 7$

In an arithmetic sequence ,

Algebraic expression of sum $S_n = \frac{d}{2} n^2 + \left(f - \frac{d}{2}\right) n$

Can be find using this formula

Question:- 5,8,11,14,.....

Write the algebraic expression of this arithmetic sequence

Answer:-

1^{st} term $f = 5$

common difference $d = 8 - 5 = 3$

തുകയുടെ ബീജഗണിത രൂപം $S_n = \frac{d}{2} n^2 + \left(f - \frac{d}{2}\right) n$

$$S_n = \frac{3}{2} n^2 + \left(5 - \frac{3}{2}\right) n$$

$$S_n = \frac{3}{2} n^2 + \left(\frac{10}{2} - \frac{3}{2}\right) n$$

$$S_n = \frac{3}{2} n^2 + \left(\frac{10 - 3}{2}\right) n$$

$$S_n = \frac{3}{2} n^2 + \frac{7}{2} n$$

1. Number Sequences

A set of numbers written as the first , second , third and so on , according to a particular rule is called a **sequence.**

eg : -

- 1 , 2 , 3 , 4 , . . . (next 5)
- 1 , 4 , 9 , 16 , . . .(next 25)
- 10 , 100 , 1000 , 10000 , . . .(next 100000)
- 2 , 4 , 8 , 16 , . . . (next 32)

2. Algebra Of Sequences

The generally used mathematical principle in such a sequences of numbers can be written in algebraic expressions.

eg : -

- 1 , 2 , 3 , 4 , . . . (algebraic expression n)
- 1 , 4 , 9 , 16 , . . .(algebraic expression n^2)
- 10 , 100 , 1000 , 10000 , . . .(algebraic expression 10^n)
- 2 , 4 , 8 , 16 , . . . (algebraic expression 2^n)

3. Arithmetic Sequences

When writing the numbers consecutively , if a particular number is added or subtracted to get the next number such sequences are called Arithmetic Sequences.

eg:-

- 1,2,3,4,..... (Add **1** to get the next one)
- 10,20,30,40,.....(Add **10** to get the next one)
- 6,12,18,24,..... (Add **6** to get the next one)
- 100,90,80,... (Subtract **10** to get the next one)
- 56,52,48,..... (Subtract **4** to get the next one)

4. Terms

Numbers in arithmetic sequence are called terms

eg:-

10,20,30,40,..... in this arithmetic sequence ,

First term $x_1 = 10$

Second term $x_2 = 20$

Third term $x_3 = 30$ ....

n^{th} term can be written as x_n (algebraic expression)

5. Common Difference

The difference between 2 consecutive terms 0f an Arithmetic sequence is called the Common difference.

It is denoted by the letter d

eg : -

Consider the sequence 6 , 10 , 14 , 18 , . . .

Common difference = 2nd term - 1st term ($X_2 - x_1$)

=3rd term - 2nd term ($X_3 - x_2$) . . .

NOTE : - In an arithmetic sequence , term difference is proportional to position difference ; and the constant of proportionality is the common difference.

In the arithmetic sequence 6 , 10 , 14 , 18 , ... ,

Position	➡	1	2	3	4	. . .	n
Terms	➡	6	10	14	18	. . .	n

Dividing term difference of any 2 terms by the position difference, will get Common difference of an arithmetic sequence.

eg : -

Position	1	2
Terms	6	10

$$Common difference = \frac{term difference}{position difference}$$

$$= \frac{10 - 6}{2 - 1}$$

$$= \frac{4}{1} = 4$$

6. Using some terms of an arithmetic Sequence,we can find another terms.

eg : -

The 4^{th} term of an arithmetic sequence is 22 and the 10^{th} term is 46. Wht is the 20^{th} term?

Answer

Between 4^{th} term and 10^{th} term

Term difference = 46 – 22 = 24

Position difference = 10 – 4 = 6

∴ Common difference = 24 / 6 = 4

Adding 10 times the common difference to the 10^{th} term , we get the 20^{th} term.

$$
\begin{aligned}
x_{20} &= x_{10} + 10\,d \\
&= 46 + 10(4) \\
&= 46 + 40 \\
&= \underline{\underline{86}}
\end{aligned}
$$

7. To check the term of an arithmetic sequence

eg : -

Check 37 is a term of the sequence 5,19,13,... , and Is 42 a term ?

Sequence = 5,19,13,...

∴ common difference d = 9 - 5 = 4

When First term (5) is divided by common difference 4, remainder = 1

When 37 is divided by common difference 4, remainder = 1

Here remainders are same .

∴37 is a term of the sequence.

When 42 is divided by common difference 4, remainder = 2

Here remainders are different .

So 42 is not a term of the sequence.

All the terms of the arithmetic sequence have the same remainder On division by the common difference.