1. ARITHEMATIC SEQUANCE

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- Sequence: A set of numbers written as the first, second, third and so on.

  E.g.: Set of prime numbers

- Arithmetic sequence: A sequence got by starting with any number and adding fixed number repeatedly.
- Common difference (d): same number on subtracting from any term immediately preceding it
- For any arithmetic sequence there will be common difference

  E.g.: 4, 7, 10, 13, 16,19, …………… Here common difference is 3

- $x_1, x_2, x_3, x_4, x_5, x_6$,……. Are the terms of an arithmetic sequence and suffix denote position

  E.g.: 4, 7, 10, 13, 16,19, ……………

  4^{th}term= 13; 2^{nd}term=7

  Position of 13=4; Position of 19=6

- The m^{th} term of an arithmetic sequence is x_m and n^{th} term is x_n then

  Common difference (d) = $\frac{Xn-Xm}{n-m}$

- The $X_{(p+q)}$ th term of an arithmetic sequence is $(X_p+q\times d)$

- No of the term in an arithmetic sequence is $\frac{last\ term\ -\ first\ term}{common\ difference}+1$
- To check inclusion of any term X_p in an arithmetic sequence

  Difference of X_p and any term in arithmetic sequence should be a multiple of common difference.

- The sum of three consecutive terms of an arithmetic sequence is three time of middle term

  $x_1+x_2+x_3=3x_2$

- The sum of consecutive terms of an arithmetic sequence is the product of number of terms and middle term

- X,Y,Z are three consecutive terms of an arithmetic sequence where middle term is

  $Y=\frac{x+z}{2}$

- The first term of an arithmetic sequence is 'f' and common difference is 'd'

  n^{th}term is $Xn = f + (n-1)d$

- Algebra of an arithmetic sequence is $Xn = dn + (f-d)$

- Sum of first *n* natural numbers

$$1+2+3+......+n = \frac{n(n+1)}{2}$$

- Sum of first *n* even numbers

$$2+4+6+.......+2n = n(n+1)$$

- Sum of first *n* odd numbers

$$1+3+5+.......+2n-1 = n^2$$

- Sum of the first *n* terms of an arithmetic sequence

$$x_1+ x_2+ x_3+..........+ x_n = S_n= \frac{n}{2}(2f+(n-1)d)$$

$$x_1+ x_2+ x_3+..........+ x_n = S_n= \frac{n}{2}(x_n+x_1)$$

- For the arithmetic sequence $\mathbf{x_n = an + b}$ where **'a = d'** and **'b = f-d'**. the sum first n term is

$$x_1+ x_2+ x_3+..........+ x_n = S_n= \frac{1}{2}an(n+1) + nb$$

- The sum of the first n terms of any arithmetic sequence in algebra is

$S_n= \frac{1}{2}an^2 + (\frac{1}{2}a + b)n$; **'a = d'; 'b = f-d'**

$S_n=Kn^2+Ln$ first term=K+L common difference=2k

- Sum of squares

$$1^2+2^2+3^2+...........+ n^2 = \frac{1}{6}n(n+1)(2n+1)$$

- X_1 — Position of K^{th} line last number = $\frac{k(k+1)}{2}$

  X_2 X_3 — number of terms in K^{th} line = K

  X_4 X_5 X_6 — last term of K^{th} line = $d\left(\frac{k(k+1)}{2}\right) + (x_1 - d)$

  .... — firs term of K^{th} line = last term of K^{th} line – (k – 1) d

  ....

- X_1 — position of K^{th} line last number = k^2

  X_2 X_3 X_4 — number of terms in K^{th} line =2k -1

  X_5 X_6 X_7 X_8 X_9 — last term of K^{th} line= $d\,k^2+(x_1- d)$

  ....

  ....

3. MATHEMATICS OF CHANCE

- Probability of a number.

  $$\text{Probability of favorable condition} = \frac{No.of.favorable\ outcomes}{Total\ No.of\ outcomes}$$

- Total probability always 1

- Geometrical Probability.

  Step1: identify the shapes of shaded part and total figure

  Step2: identify the measures that are equal in these two shapes and denote it Using single letter.

  Step3: find the area of shaded region and area of total shape using that measure.

  Step4: $$\text{Probability of shaded region} = \frac{Area\ Of\ shaded\ Region}{Area\ of\ Total\ shape}$$

  Another method

  Step1: make the full figure into triangles with same area

  Step2: find the no. of triangles in the figure and shaded part

  Step3: $$\text{Probability of shaded region} = \frac{No.of\ triangle\ in\ shaded\ part}{Total\ No.of\ triangle}$$

- Probability Of Pairs

  Total No. of pairs = No. of elements from A x No. of elements from B

  $$\text{Probability of favorable pair} = \frac{No.of.favorable\ pairs}{Total\ No.of\ pairs}$$

4. SECOND DEGREE EQUATIONS

- Take X as the value to be finding.
- Make an equation in the form of $x^2 = a$
- Square completion:

  Make the equation as like $(a + b)^2 = a^2 + 2ab + b^2$ or $(a - b)^2 = a^2 - 2ab + b^2$

  $(x^2 + ax = b)$

  $(x^2 + ax + (\frac{a}{2})^2 = b + (\frac{a}{2})^2)$

  $(x + \frac{a}{2})^2 = b + (\frac{a}{2})^2$

- In a second degree polynomial $p(x) = ax^2 + bx + c$ the number to take as x to get $p(x) = 0$

  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  Here

  If $b^2 - 4ac > 0$ then it has two solutions.
  If $b^2 - 4ac < 0$ then no solution.
  If $b^2 - 4ac = 0$ then it has one solution.

5. TRIGNOMETRY

- The sides of any triangle of angles 45°,45°,90° are in the ratio $1:1:\sqrt{2}$
- The sides of any triangle of angles 30°,60°,90° are in the ratio $1:\sqrt{3}:2$

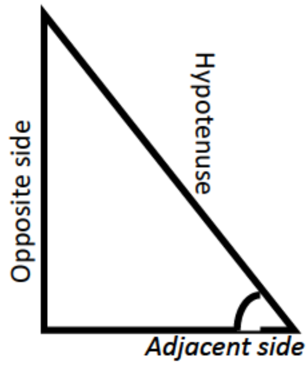
(Length side we have to find = $\frac{ratio\ of\ side}{ratio\ of\ given\ side}$ X length of given side)

	30°	45°	60°
Sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$\text{Sin } x^o = \frac{Opposite\ side}{Hypotenuse}$

$\text{Cos } x^o = \frac{Adjacent\ side}{Hypotenuse}$

$\text{Tan } x^o = \frac{Opposite\ side}{Adjacent\ side}$



- In a circle of radius r, the length of a chord of central angle c^o is $2rSin\left(\frac{a}{2}\right)$
- Sin x = cos (90-x)   sin 50 = cos 40

- In the picture ***r*** is the circumradius of triangle ABC

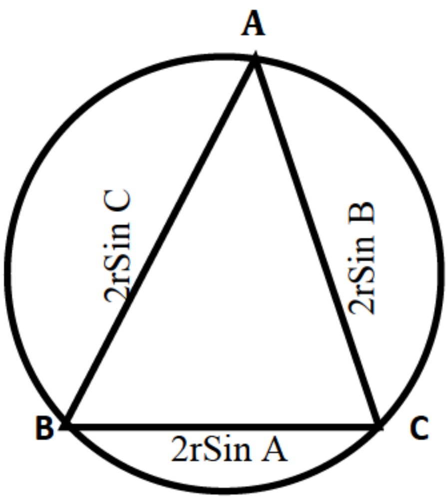
  Length of AB = 2rSinC

  Length of BC = 2rSinA

  Length of AC = 2rsinB

- Circum Diameter of triangle ABC

$$d = \frac{Length\ of\ a\ side}{Sin\ (opposite\ angle\ of\ side)}$$



- Angle of elevation: Angle between straight view and rise view.
- Angle of depression: Angle between straight view and lower view.
- To find the height and distance use only *Tan* ratio.

6. COORDINATES

- Two perpendicular real lines.
- Horizontal real line is named as x-axis and vertical real line is named as y-axis.
- Coordinates of a point in the form of (x, y).

  X coordinates
- (x_1, y_1) and (x_2, y_2) are the coordinates of two points A and B respectively.

  The distance between AB is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- Distance between a point (x, y) and origin (0, 0) is

$$\sqrt{x^2 + y^2}$$

- If three points A, B and C are on the same line (collinear) then

  AB + BC = AC
- If AB, AC and BC are the three sides of a right triangle then

  $AB^2 + AC^2 = BC^2$ (Pythagoras theorem)
- If (x_1, y_1) and (x_2, y_2) are the coordinates of opposite vertices of rectangle then (x_2, y_1) and (x_1, y_2) are the coordinates of other vertices.

10 POLYNOMIALS

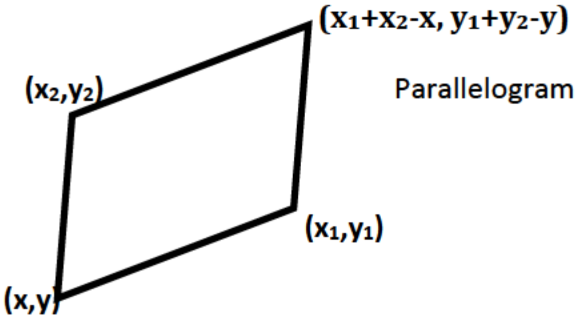
- P(x) is any polynomial and (x – a) is a first degree polynomial
  1. If P(a) = 0 then (x – a) is a factor of P(x)
  2. If P(a) ≠ 0 then (x – a) is not a factor of P(x)
  3. If P(a) = b then b is reminder
- (x – a) and (x – b) are two first degree polynomials
  1. $(x - a)(x - b) = X^2 - (a+b)X + ab$ *(a 2nd degree polynomial as the product of Two first degree polynomials)*
  2. The solutions of the equation p(x) = 0 are x=a and x=b
- P(x) = (x – a)q(x) + b

  q(x) is quotient

  b is reminder
- (x – a) is not a factor of p(x). that is p(a) = b

  The polynomial with factor (x – a) is p(x) + (-b)
- If $X^2 - a^2$ is factor of p(x) then (x + a) and (x – a) are factors of p(x)

GEOMETRY AND ALGEBRA



Parallelogram

- The line joining two points (x_1, y_1) and (x_2,y_2) is divide in the ratio **m:n** at point P the coordinates of P is (X,Y) where

$$X = X_1 + \frac{m}{(m+n)}(X_2 - X_1)$$

$$Y = Y_1 + \frac{m}{(m+n)}(Y_2 - Y_1)$$

- The mid-point of line joining (x_1, y_1) and (x_2,y_2) is

$$\left(\frac{1}{2}(x_1+x_2), \frac{1}{2}(y_1+y_2)\right)$$

- The slope of the line joining points (x_1, y_1) and (x_2,y_2) is

$$\text{Slope} = \frac{Y2 - Y1}{X2 - X1}$$

- Line equation
  1. (x, y) as a point line joining (a, b) and (p, q)

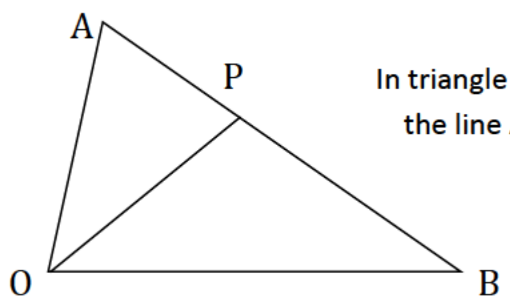
     Slope of (a, b) (p, q) = slope of (p, q) (x, y)
  2. If Equation of a line is in the form $px - qy + r = 0$ then
     - The slope of the line is $\frac{p}{q}$
     - The coordinate of line intersecting point with x-axis is take $(\frac{-r}{p}, 0)$
     - The coordinate of line intersecting point with y-axis is take $(0, \frac{-r}{q})$
- Circle equation
  - A circle with center (a, b) and radius r. (x, y) point on circle the circle equation is

    $(x - a)^2 + (y - b)^2 - r^2 = 0$
  - If $x^2+y^2- px - qy + r = 0$ is an equation of a circle, then
    - Coordinate of center is $(\frac{p}{2}, \frac{q}{2})$
    - Radius of circle is $= \sqrt{-r + (\frac{p}{2})^2 + (\frac{q}{2})^2}$

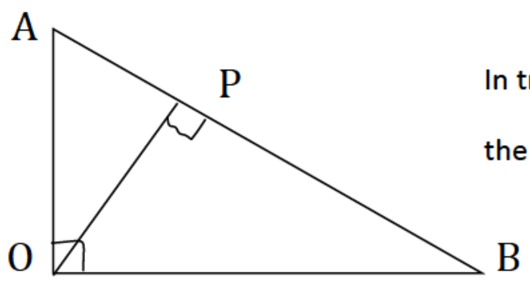
- Coordinates of the centroid of a triangle with vertices (x_1, y_1), (x_2,y_2), (x_3,y_3) is

$$\left(\frac{1}{3}(x_1+x_2+x_3), \frac{1}{3}(y_1+y_2+y_3)\right)$$

- If (x_1, y_1), (x_2,y_2), (x_3,y_3) are three points on a line then x-coordinates and y-coordinates are in arithmetic sequence.

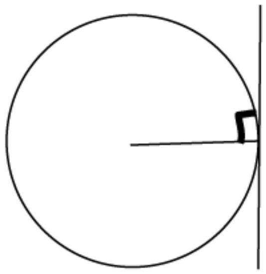
- 

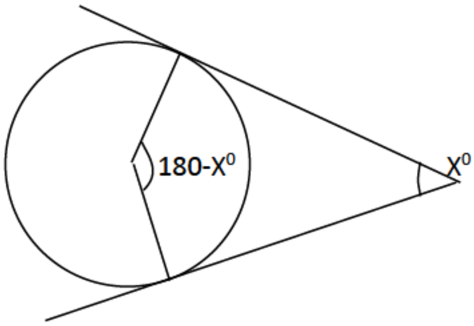
In triangle OAB, If OP bisects angle O then The point P divides the line AB in the ratio OA : OB

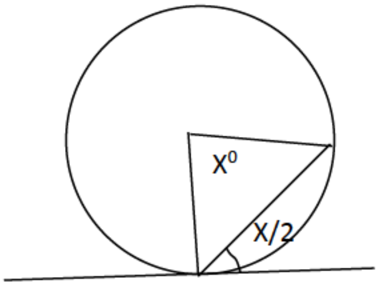
- 

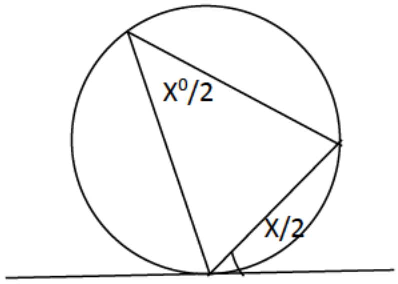
In triangle OAB, The point P divides the line AB in the ratio $OA^2 : OB^2$

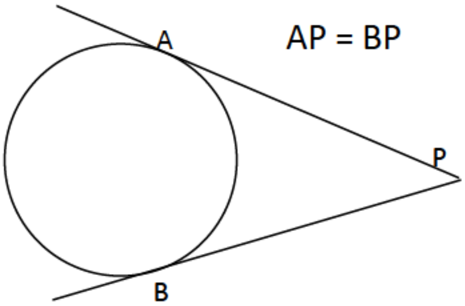
TANGENTS



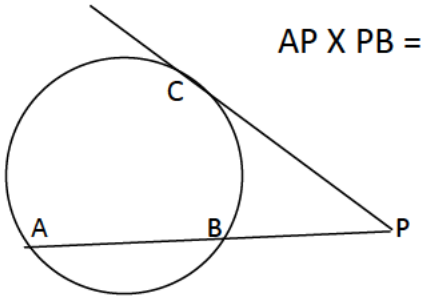




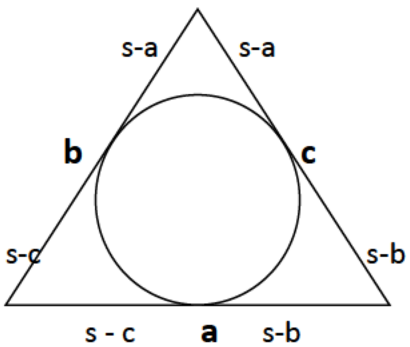




AP = BP



$AP \times PB = PC^2$



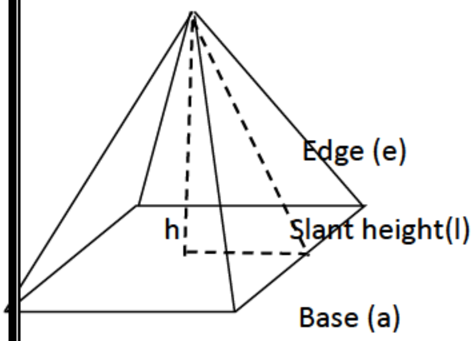
Half the perimeter of triangle= **S**

Radius of the incircle of a triangle = $\frac{A}{S}$

Area of triangle = **A**

- Draw tangents from outside point of a circle.
- Draw a circle with given radius. Draw triangle with given angles and sides are tangent to the circle
- Draw a triangle with given side and draw incircle to the triangle.

SOLIDS

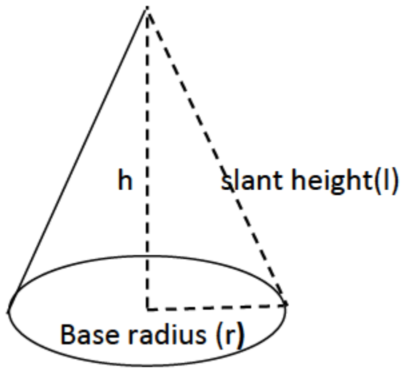


$h = \sqrt{l^2 - (\frac{a}{2})^2}$; Base area = a^2

lateral surface area = $2al$

Total surface area = $a^2 + 2al$

Volume of pyramid = $\frac{1}{3} a^2 h$



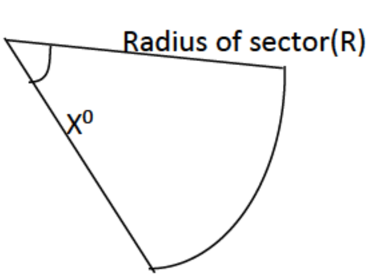
$h = \sqrt{l^2 - r^2}$; base area $= \pi r^2$

curved surface area = πrl

Total surface area = $\pi r (r + l)$

volume of cone = $\frac{1}{3} \pi r^2 h$

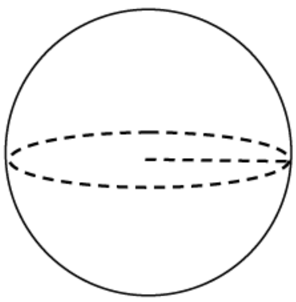
Radius of sector = slant height



$$\frac{r}{R} = \frac{X^0}{360}$$

Radius of sphere = r $surface\ area\ of\ sphere = 4\pi r^2$

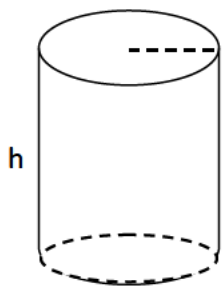
$$Volume\ of\ sphere\ = \frac{4}{3}\pi r^3$$



Curved surface area of hemisphere = $2\pi r^2$

Total surface area of hemisphere = $3\pi r^2$

Volume of hemisphere = $\frac{2}{3} \pi r^3$



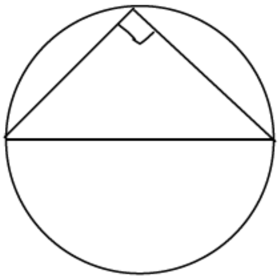
Base area of a cylinder = $2\pi r^2$

curved surface area of a cylinder = $2\pi r h$

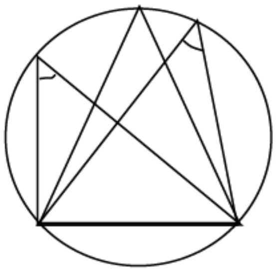
Total surface area of a cylinder = $2\pi r h + 2\pi r^2$

Volume of cylinder = $\pi r^2 h$

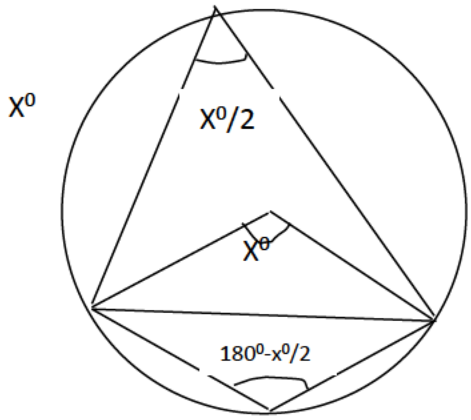
CIRCLES



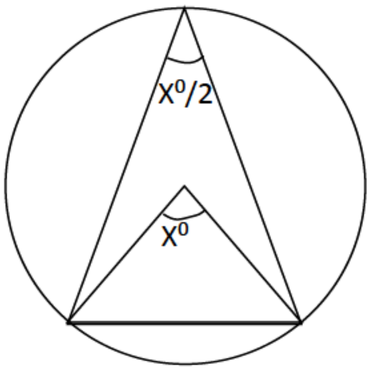
Angle on semicircle is 90^0



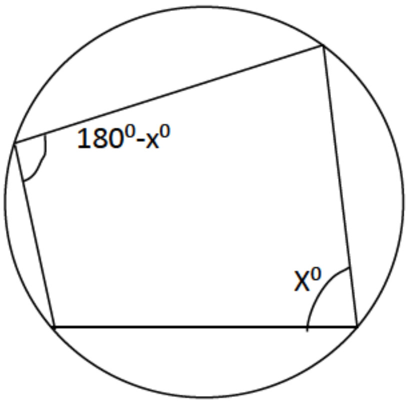
All angles on same side are equal.



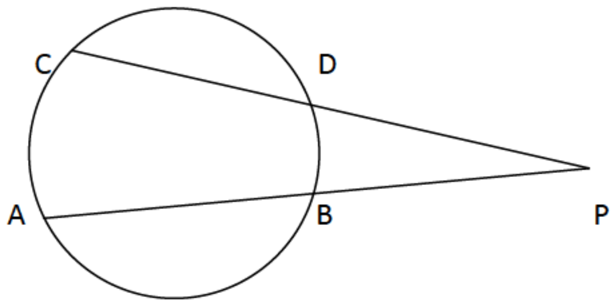
Angle on small circle part of chord is half the central angle of the chord subtracted from 180^0.



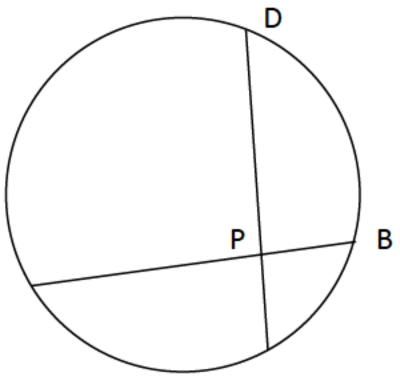
Angle on the large circle part is half the central angle of the chord.



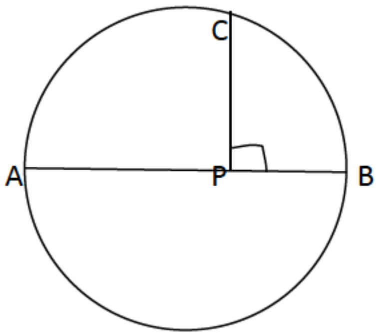
- A circle passes through the four vertices of a quadrilateral is called cyclic quadrilateral.
- Opposite angles of a cyclic quadrilateral are supplementary



AP x PB = PC x PD



$$AP \times PB = PC \times PD$$



$$AP \times PB = PC^2$$

- Draw a circle of given radius. draw a triangle with given angles and all its vertices on the circle
- Draw a rectangle of the same area that of a rectangle
- Draw a square of the same area of a rectangle.
- Draw a square of given side.