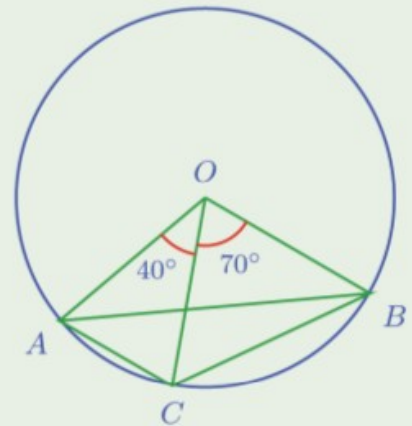
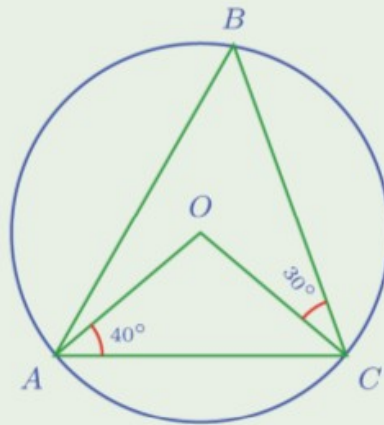
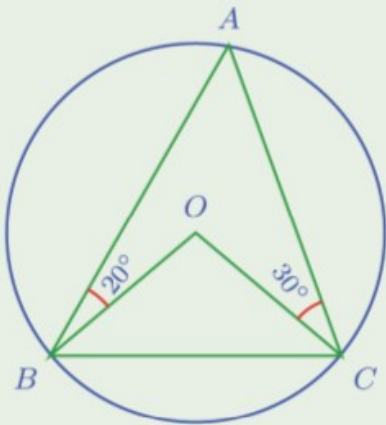


ONLINE MATHS CLASS - X - 25 (07 / 09 / 2020)

Let's do some problems related to the concepts which already we have learned .

- (1) In all the pictures given below, O is the centre of the circle and A, B, C are points on it. Calculate all angles of $\triangle ABC$ and $\triangle OBC$ in each.



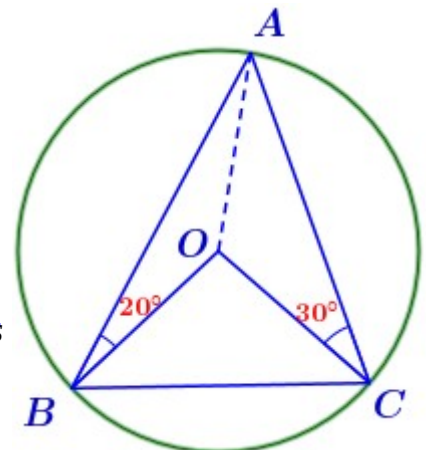
Answer.

a) Join OA .

$$OA = OB = OC \quad (\text{Radii of a circle are equal})$$

In the triangle OAB ,

$$\angle OBA = \angle OAB = 20^\circ \quad (\text{OA} = \text{OB}, \text{OAB is an isosceles triangle})$$



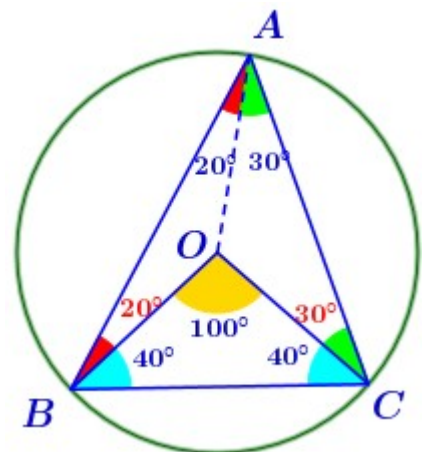
In the triangle OAC ,

$$\angle OCA = \angle OAC = 30^\circ \quad (\text{OA} = \text{OC})$$

$$\angle BAC = \angle OAB + \angle OAC = 20^\circ + 30^\circ = 50^\circ$$

$$\angle BOC = 2 \times 50^\circ = 100^\circ$$

(The angle made by an arc of a circle at the centre is double the angle on the alternate arc .)



In the triangle OBC ,

$$\angle OBC = \angle OCB = \frac{180 - 100}{2} = \frac{80}{2} = 40^\circ \quad (OB = OC)$$

In the triangle ABC ,

$$\angle BAC = 20^\circ + 30^\circ = 50^\circ$$

$$\angle ABC = 20^\circ + 40^\circ = 60^\circ$$

$$\angle ACB = 40^\circ + 30^\circ = 70^\circ$$

b) Join OB .

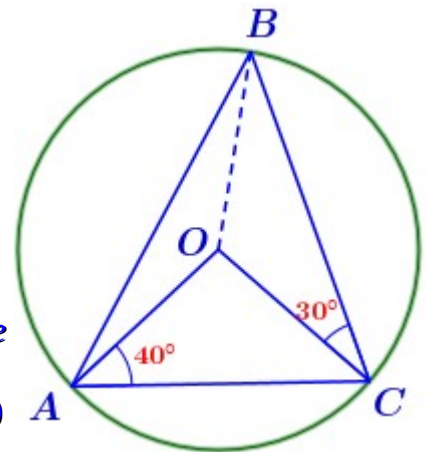
$$OA = OB = OC \quad (\text{Radii of a circle are equal})$$

In the triangle OAC ,

$$\angle OAC = \angle OCA = 40^\circ \quad (OA = OC)$$

$$\angle AOC = 180 - 80 = 100^\circ \quad (\text{Sum of the angles in a triangle is } 180^\circ)$$

$$\angle ABC = \frac{100}{2} = 50^\circ \quad (\text{The angle made by an arc of a circle on the alternate arc is half the angle made at the centre.})$$



on the alternate arc is half the angle made at the centre .)

In the triangle OBC ,

$$\angle OBC = \angle OCB = 30^\circ \quad (OB = OC)$$

$$\angle BOC = 180^\circ - 60^\circ = 120^\circ$$

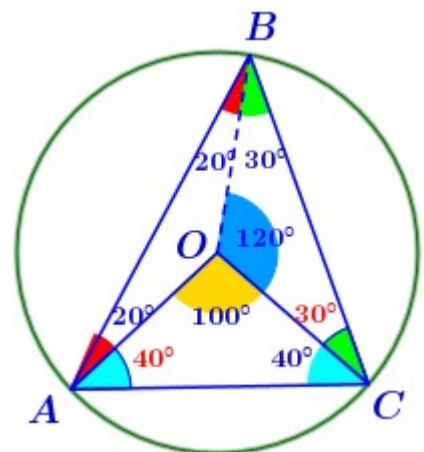
In the triangle OAB ,

$$\angle OBA = 50^\circ - 30^\circ = 20^\circ$$

$$\angle OAB = \angle OBA = 20^\circ \quad (OA = OB)$$

In the triangle ABC ,

$$\angle BAC = 20^\circ + 40^\circ = 60^\circ$$



$$\angle ABC = 20^\circ + 30^\circ = 50^\circ$$

$$\angle ACB = 40^\circ + 30^\circ = 70^\circ$$

c) $OA = OB = OC$ (Radii of a circle are equal)

In the triangle OAC ,

$$\angle OAC = \angle OCA = \frac{180 - 40}{2} = \frac{140}{2} = 70^\circ \quad (OA = OC)$$

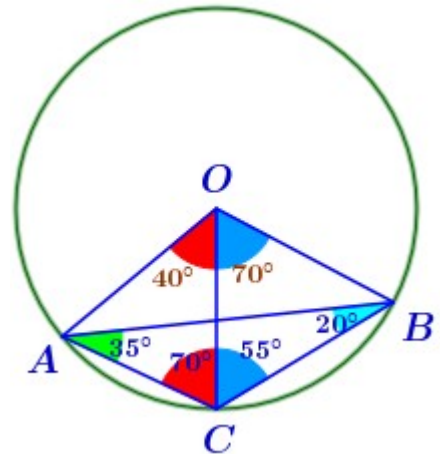
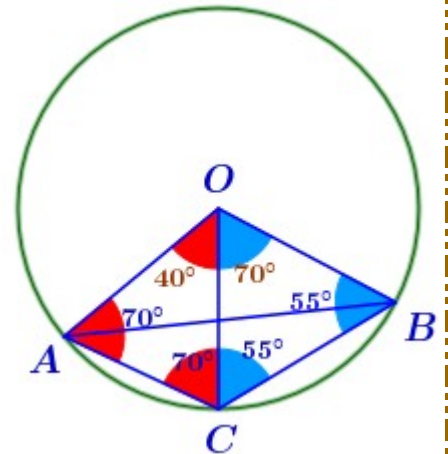
In the triangle OBC ,

$$\angle OBC = \angle OCB = \frac{180 - 70}{2} = \frac{110}{2} = 55^\circ \quad (OB = OC)$$

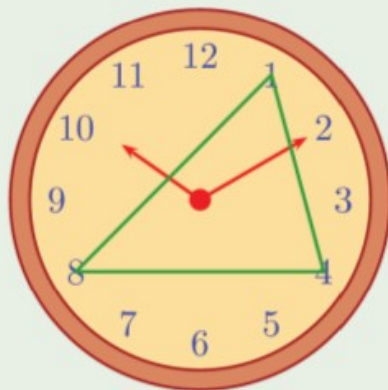
$$\angle ABC = \frac{\angle AOC}{2} = \frac{40^\circ}{2} = 20^\circ$$

$$\angle BAC = \frac{\angle BOC}{2} = \frac{70^\circ}{2} = 35^\circ$$

$$\angle ACB = 70^\circ + 55^\circ = 125^\circ$$



(2) The numbers 1, 4, 8 on a clock's face are joined to make a triangle.



Calculate the angles of this triangle.

Answer .

In a clock's face ,

$$60 \text{ minutes} = 360^\circ$$

$$1 \text{ minute} = \frac{360^\circ}{60} = 6^\circ$$

$$\angle AOC = 90^\circ \quad (15 \text{ minutes} = 15 \times 6 = 90^\circ)$$

$$\angle BOC = 120^\circ \quad (20 \text{ minutes} = 20 \times 6 = 120^\circ)$$

$$\angle AOB = 150^\circ \quad (25 \text{ minutes} = 25 \times 6 = 150^\circ)$$

In the triangle ABC ,

$$\angle BAC = \frac{120^\circ}{2} = 60^\circ$$

$$\angle ABC = \frac{90^\circ}{2} = 45^\circ$$

$$\angle ACB = \frac{150^\circ}{2} = 75^\circ$$

Each angle of an equilateral triangle is 60° .

So the side opposite to each angle makes 120° at the centre of the circle .

120° is equal to 20 minutes .

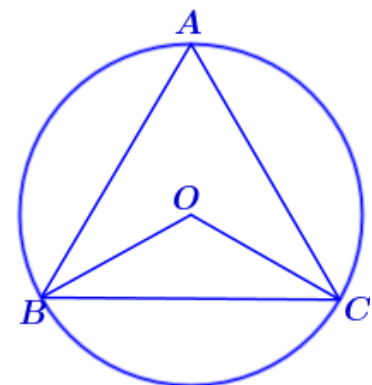
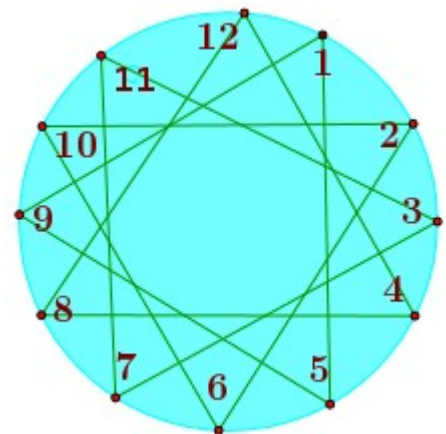
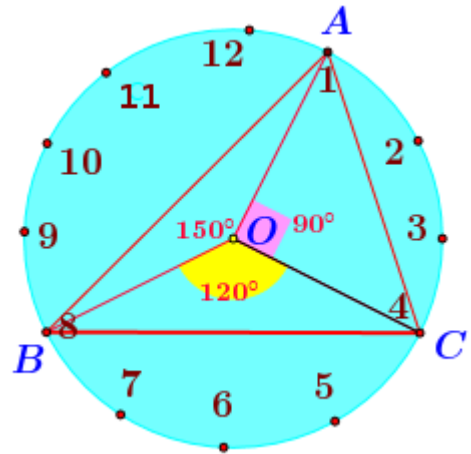
So if we join the numbers 12 , 4 and 8 , we will get an equilateral triangle .

Similarly if we join the group of numbers (1 , 5 , 9) or (2 , 6 , 10) or (3 , 7 , 11) we will get equilateral triangles . So we get 4 equilateral triangles in total .

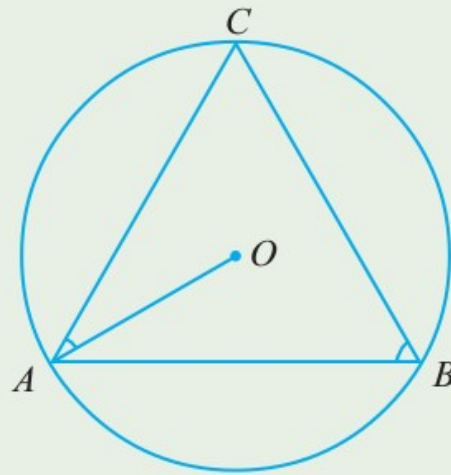
More activities

1. In the figure O is the centre . ABC is an equilateral triangle

Find $\angle BAC$ and $\angle ABO$?



- (2) In the picture, O is the centre of the circle and A, B, C , are points on it. Prove that $\angle OAC + \angle ABC = 90^\circ$.

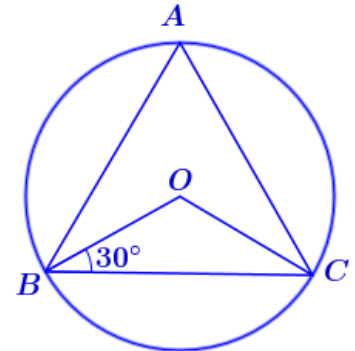


ONLINE MATHS CLASS - X - 25 (07 / 09 / 2020)

WORKSHEET

1. In the figure O is the centre of the circle . $AB = AC$, $\angle OBC = 30^\circ$

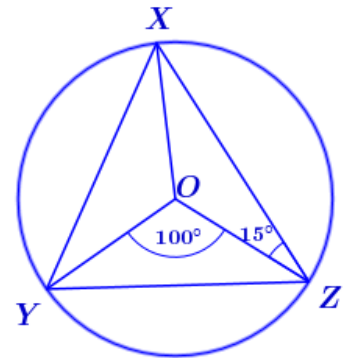
- What is the measure of $\angle OCB$?
- What is the measure of $\angle BOC$?
- What is the measure of $\angle BAC$?
- Prove that triangle ABC is an equilateral triangle ?



2. In the figure O is the centre of the circle . $\angle YOZ = 100^\circ$,

$$\angle OZX = 15^\circ$$

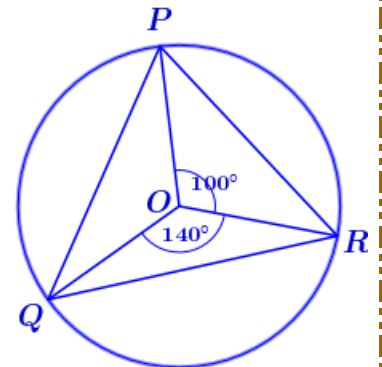
- What is the measure of $\angle YXZ$?
- What is the measure of $\angle OXZ$?
- What is the measure of $\angle OXY$?
- What is the measure of $\angle XYZ$?



3. In the figure O is the centre of the circle . $\angle QOR = 140^\circ$,

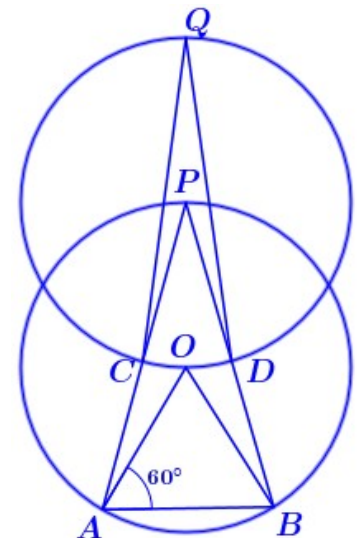
$$\angle POR = 100^\circ$$

- What is the measure of $\angle QPR$?
- What is the measure of $\angle PQR$?
- What is the measure of $\angle POQ$?
- What is the measure of $\angle PRQ$?



4. In the figure O and P are the centres of the circles . $\angle OAB = 60^\circ$

- What is the measure of $\angle OBA$?
- What is the measure of $\angle AOB$?
- What is the measure of $\angle CPD$?
- What is the measure of $\angle CQD$?

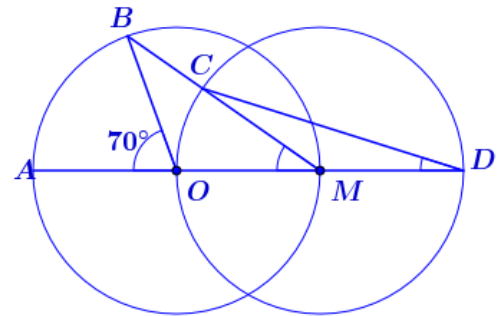


ONLINE MATHS CLASS - X - 26 (08 / 09 /2020)

WORKSHEET

1. In the figure O and M are the centres of the circles . $\angle AOB = 70^\circ$

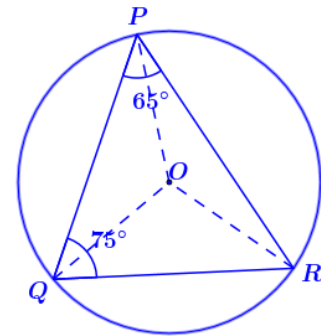
- a) What is the measure of $\angle AMB$?
- b) What is the measure of $\angle CDO$?
- c) Draw an angle of size $32\frac{1}{2}^\circ$?
- d) Draw an angle of size $16\frac{1}{4}^\circ$?



2. In the figure O is the centre of the circle . $\angle PQR = 75^\circ$

and $\angle QOR = 65^\circ$

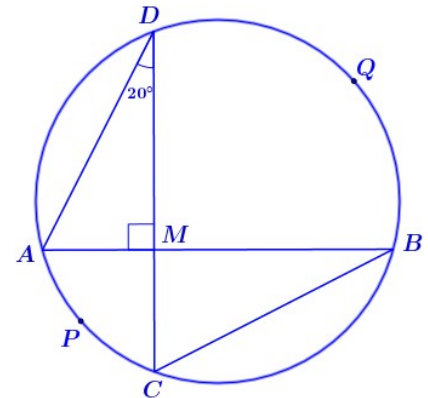
- a) What is the measure of $\angle QOR$?
- b) What is the measure of $\angle POR$?



c) Draw a triangle of circumradius 3 cm and two of the angles are 80° and 70° ?

3. In the figure AB and CD are two perpendicular chords of a circle and they intersect at M . $\angle ADM = 20^\circ$

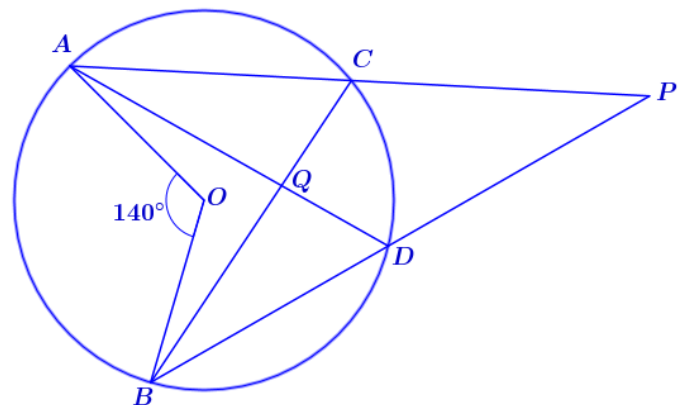
- a) What is the measure of $\angle ABC$?
- b) What is the central angle of the chord APC ?
- c) What is the measure of $\angle BCM$?
- d) What is the sum of the central angles of the chords APC and BQD ?



4. In the figure O is the centre of the circle .

$\angle AOB = 140^\circ$

- a) What is the measure of $\angle ACB$?
- b) What is the measure of $\angle ADB$?
- c) What is the measure of $\angle QCP$?
- d) What is the measure of $\angle QDP$?
- e) $\angle CPD + \angle CQD = 360 - \dots = \dots$

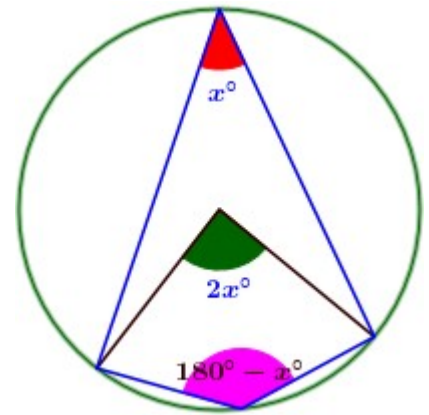


ONLINE MATHS CLASS - X - 27 (11 / 09 /2020)

What did we learn in the previous classes ?

An arc makes three types of angles in a circle .

- 1. Angle made by an arc at the centre of the circle*
- 2. Angle on an arc .*
- 3. Angle on its complementary arc .*



What is relation among them ?

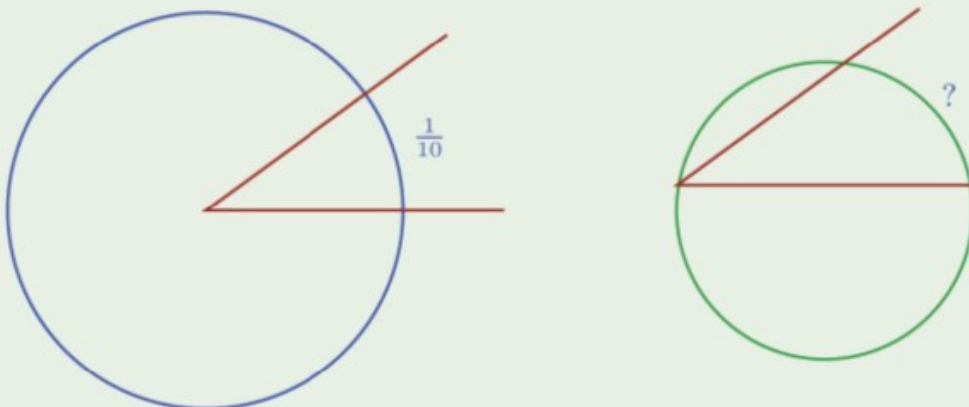
The angle made by an arc of a circle on the alternate arc is half the angle made at the centre .

A pair of an angles on an arc and its alternate are supplementary .

Now let's discuss some problems related to these ideas

1.

A rod bent into an angle is placed with its corner at the centre of a circle and it is found that $\frac{1}{10}$ of the circle lies within it. If it is placed with its corner on another circle, what part of the circle would be within it?

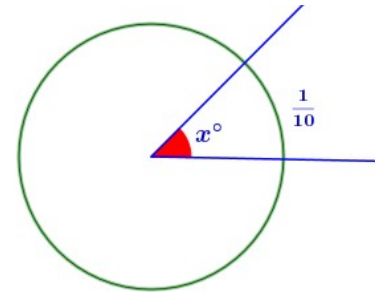


Answer .

$$\frac{\text{Arc length}}{\text{Perimeter of the circle}} = \frac{\text{central angle of the arc}}{360}$$

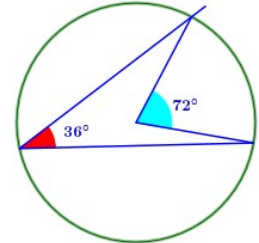
$$\frac{1}{10} = \frac{x}{360}$$

$$x = \frac{360}{10} = 36$$



Angle on the alternate arc = 36°

Central angle of the arc = $2 \times 36 = 72^\circ$



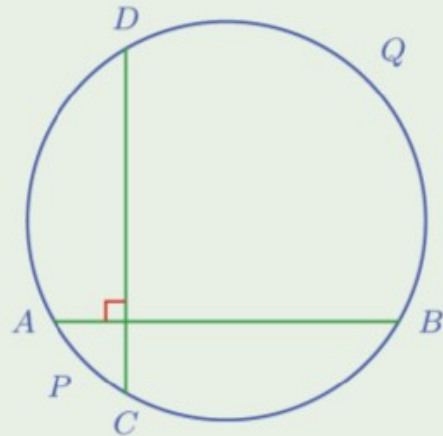
$$\frac{\text{Arc length}}{\text{Perimeter of the circle}} = \frac{\text{central angle of the arc}}{360}$$

$$= \frac{72}{360} = \frac{1}{5}$$

If the rod is placed with its corner on another circle, $\frac{1}{5}$ th part of the circle would be within it .

2.

In the picture, AB and CD are mutually perpendicular chords of the circle. Prove that the arcs APC and BQD joined together would make half the circle.

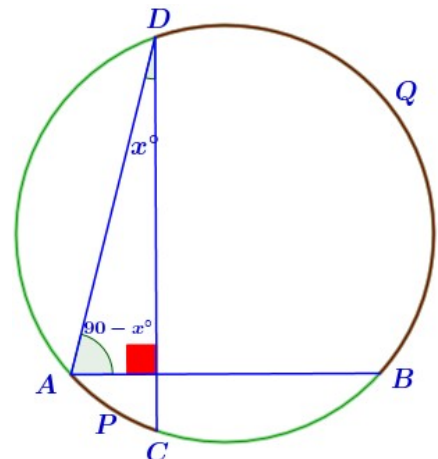


Answer .

Take , $\angle ADC = x^\circ$

$\angle DAB = 90 - x^\circ$

Central angle of the arc $APC = 2 \times \angle ADC$
 $= 2x^\circ$



(The angle made by an arc of a circle on the alternate arc is half the angle made at the centre)

Central angle of the arc $BQD = 2 \times \angle BAD$

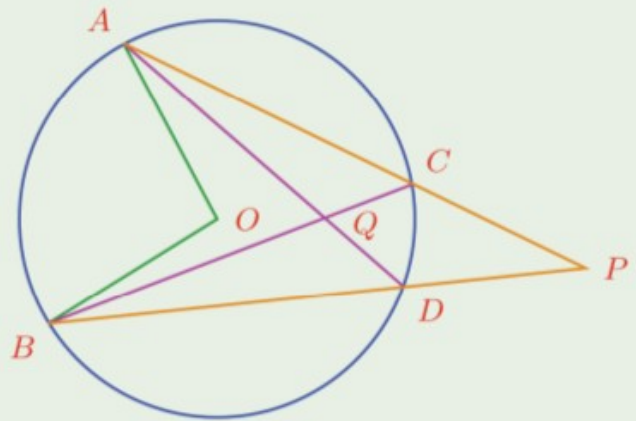
$$= 2 (90 - x^\circ) = 180 - 2x^\circ$$

Central angle of the arc $APC + \text{Central angle of the arc } BQD = 2x^\circ + 180 - 2x^\circ$
 $= 180^\circ$

That is the arcs APC and BQD joined together make a semicircle .

3.

In the picture, A, B, C, D are points on a circle centred at O . The lines AC and BD are extended to meet at P . The lines AD and BC intersect at Q . Prove that the angle which the small arc AB makes at O is the sum of the angles it makes at P and Q .



Answer..

Take , $\angle AOB = 2x^\circ$

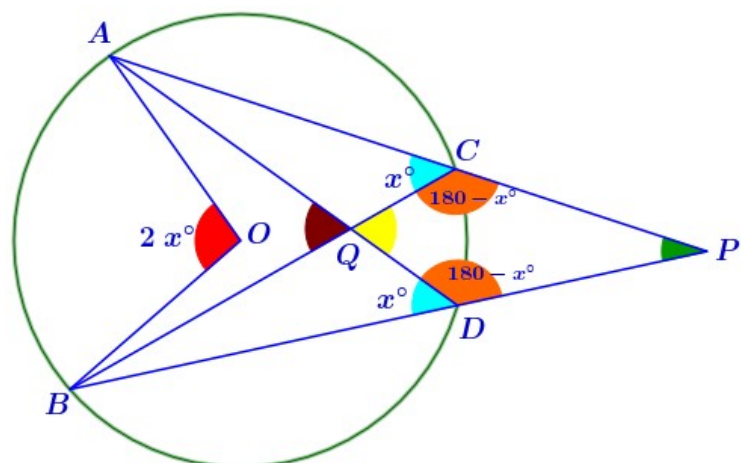
$\angle ACB = \angle ADB = x^\circ$

(The angle made by an arc of a circle on the alternate arc is half the angle made at the centre)

$\angle QCP = 180 - x^\circ$ ($\angle ACB = x^\circ$,

linear pair)

$\angle QDP = 180 - x^\circ$ ($\angle ADB = x^\circ$, linear pair)



In quadrilateral CPDQ ,

$$\begin{aligned}\angle CQD + \angle CPD &= 360 - (180 - x^\circ + 180 - x^\circ) \quad (\text{Sum of the angles of a quadrilateral is } 360^\circ) \\ &= 360 - (360 - 2x^\circ) \\ &= 360 - 360 + 2x^\circ \\ &= 2x^\circ\end{aligned}$$

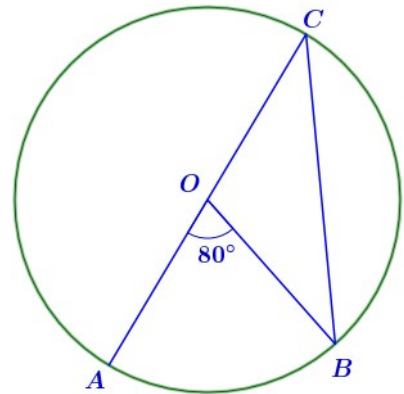
$$\angle AQB = \angle CQD \quad (\text{Opposite angles})$$

$$\angle AQB + \angle CPD = \angle CQD + \angle CPD = 2x^\circ = \angle AOB$$

More activity

In the figure O is the centre of the circle . If $\angle AOB = 80^\circ$

Find the measures of $\angle OCB$ and $\angle OBC$?

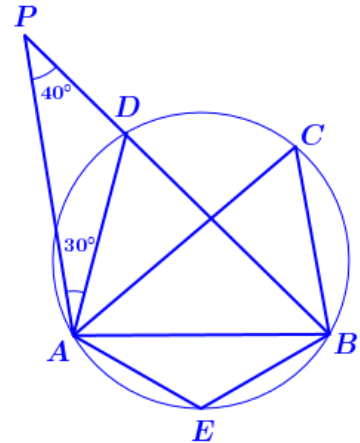


ONLINE MATHS CLASS - X - 27 (09 / 09 /2020)

WORKSHEET

1. In the figure $\angle APD = 40^\circ$ and $\angle PAD = 30^\circ$

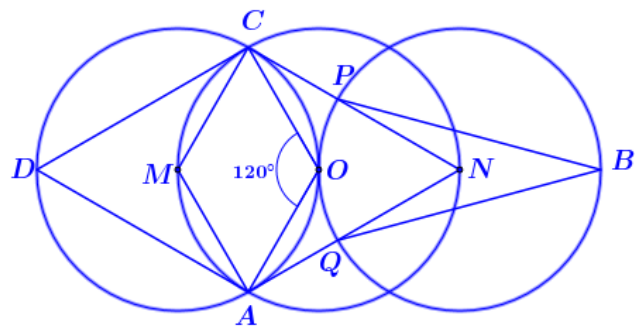
- a) What is the measure of $\angle ADP$?
- b) What is the measure of $\angle ADB$?
- c) What is the measure of $\angle ACB$?
- d) What is the measure of $\angle AEB$?



2. In the figure M, O and N are the centres of the circles .

$\angle AOC = 120^\circ$

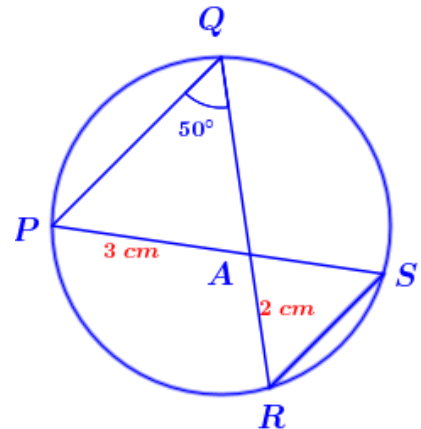
- a) What is the measure of $\angle ANC$?
- b) What is the measure of $\angle PBQ$?
- c) What is the measure of $\angle AMC$?
- d) What is the measure of $\angle ADC$?



3. In the figure chords PQ and RS are parallel .

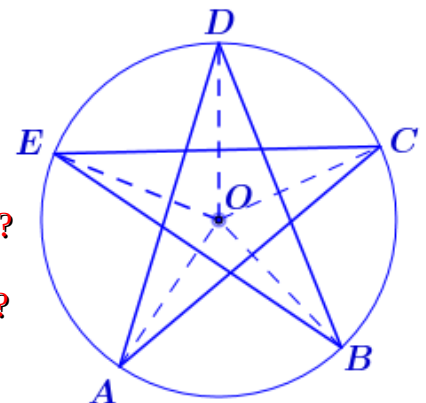
$\angle PQR = 50^\circ$, $AP = 3$ cm and , $AR = 2$ cm

- a) What is the measure of $\angle PSR$?
- b) What is the measure of $\angle QRS$?
- c) What is the measure of $\angle QPS$?
- d) What is the length of the chord QR ?



4. In the figure O is the centre of the circle .

- a) What is the measure of the angle around the point O ?
- b) What is the relation between the measures of $\angle D$ and $\angle AOB$?
- c) What is the relation between the measures of $\angle C$ and $\angle AOE$?
- d) Prove that $\angle A + \angle B + \angle C + \angle D + \angle E = 180^\circ$?



ONLINE MATHS CLASS - X - 28 (11 / 09 /2020)

Quadrilateral with all vertices are on a circle

In the figure A, B, C, D and are four points on the circle .

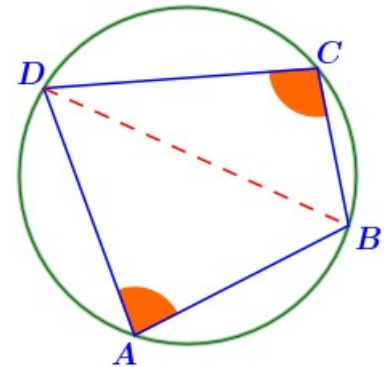
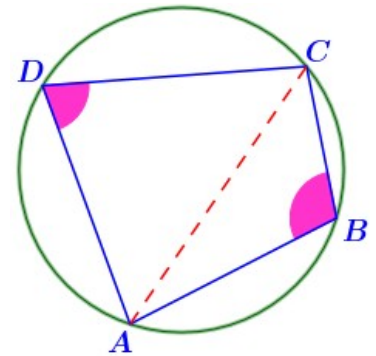
Consider the quadrilateral $ABCD$.

Draw the diagonal AC .

$\angle B + \angle D = 180^\circ$ (The angles made by a chord on either side of it are supplementary or a pair of angles on an arc and its alternate are supplementary)

Draw the diagonal BD .

$\angle A + \angle C = 180^\circ$ (The angles made by a chord on either side of it are supplementary)



If all four vertices of a quadrilateral are on a circle , then its opposite angles are supplementary

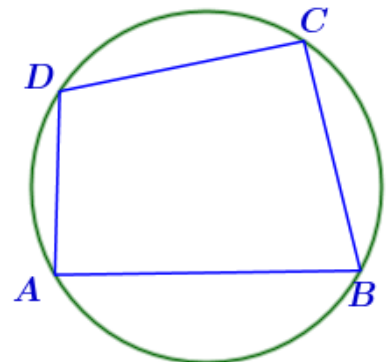
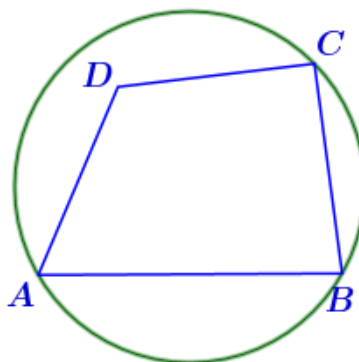
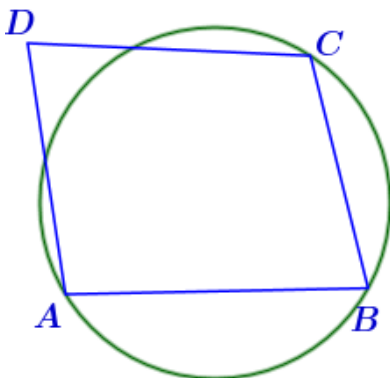
Circle passes through the vertices of a quadrilateral

Can we draw a circle through all four vertices of a quadrilateral ?

We can draw a circle through three vertices of a quadrilateral . (Since we can draw a circle through three vertices of a quadrilateral by forming a triangle using these vertices)

(circumcircle of a triangle)

Three situations may arise .



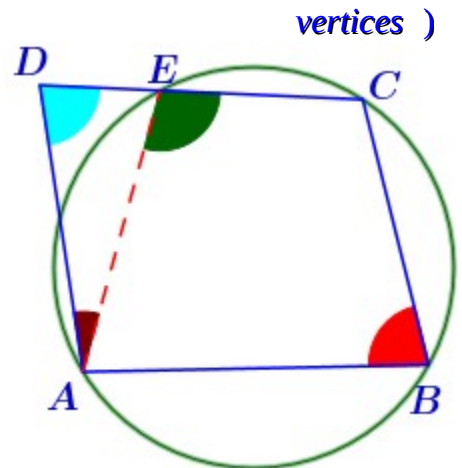
Case 1 (One vertex of a quadrilateral is outside the circle drawn through the other three vertices)

The side CD cuts the circle at E .

Join AE .

In the quadrilateral ABCE ,

$\angle B + \angle AEC = 180^\circ$ (If all four vertices of a quadrilateral are on a circle , then its opposite angles are supplementary)



In the triangle ADE , $\angle AEC = \angle D + \angle DAE$ (The outer angle at a vertex of a triangle is the sum of the inner angles at the other vertices)

That is , the measure of $\angle D$ is less than that of $\angle AEC$.

So $\angle B + \angle D$ is less than 180° . ($\angle B + \angle AEC = 180^\circ$)

If one vertex of a quadrilateral is outside the circle drawn through the other three vertices , then the sum of the angles at this vertex and its opposite vertex is less than 180°

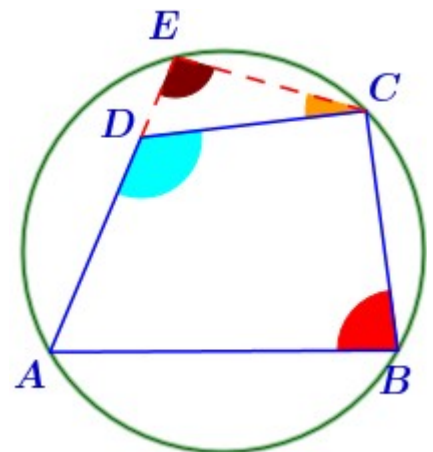
Case 2 (One vertex of a quadrilateral is inside the circle drawn through the other three vertices)

The side AD is extended and meets the circle at E .

Join AE and CE .

In the quadrilateral ABCE ,

$\angle B + \angle E = 180^\circ$ (If all four vertices of a quadrilateral are on a circle , then its opposite angles are supplementary)



In the triangle DEC , $\angle ADC = \angle E + \angle DCE$ (The outer angle at a vertex of a triangle is the sum of the inner angles at the other vertices)

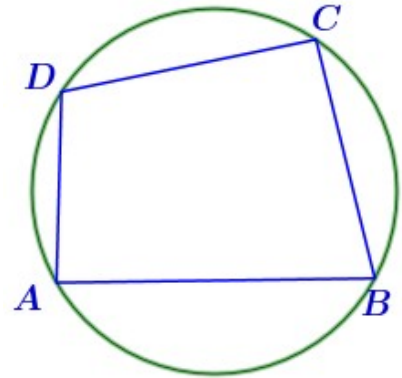
That is the measure of $\angle ADC$ is more than that of $\angle E$.

$\angle B + \angle ADC$ is more than 180° . ($\angle B + \angle AEC = 180^\circ$)

If one vertex of a quadrilateral is inside the circle drawn through the other three vertices ,
then the sum of the angles at this vertex and its opposite vertex is more than 180°

Case 3 (All vertices are on the circle drawn through the vertices of a quadrilateral)

$\angle B + \angle D = 180^\circ$ (If all four vertices of a quadrilateral
are on a circle , then its opposite angles are supplementary)



If all four vertices of a quadrilateral are on a circle , then its opposite angles are

supplementary

NB :

Now suppose in quadrilateral ABCD , we have $\angle B + \angle D = 180^\circ$. Draw the circle through
A, B, C .

Can D be outside the circle , inside the circle or on the circle ?

If D is outside the circle , we must have the sum of $\angle B$ and $\angle D$ less than 180° . So D is
not outside the circle .

If D is inside the circle , we must have the sum of $\angle B$ and $\angle D$ greater than 180° .

So D is not inside the circle .

Since it is neither outside the circle nor inside the circle , D must be on the circle .

If opposite angles of a quadrilateral are supplementary , we can draw a circle passing through
all four of its vertices .

Cyclic quadrilateral

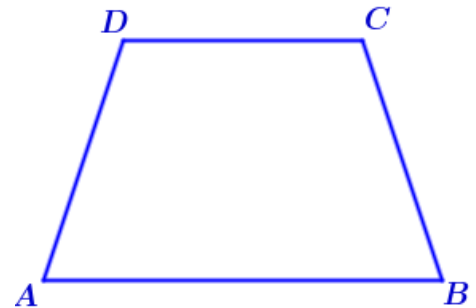
Cyclic quadrilaterals are those quadrilaterals with opposite angles supplementary .

All rectangles are cyclic . (Since each angle of a rectangle is 90° , opposite angles are supplementary)

More activity

1. In the figure ABCD is an isosceles trapezium .

Prove that ABCD is cyclic ?

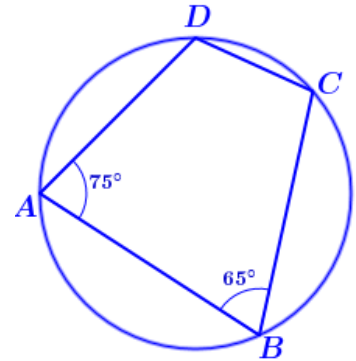


ONLINE MATHS CLASS - X - 28 (11 / 09 /2020)

WORKSHEET

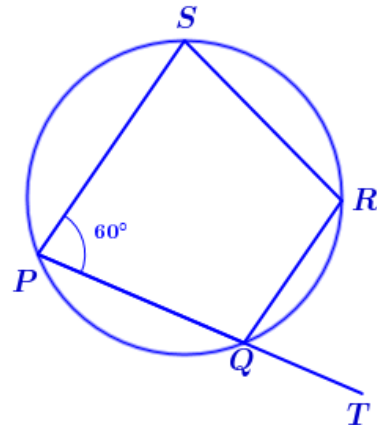
1. In the figure $\angle A = 75^\circ$ and $\angle B = 65^\circ$

- What is the measure of $\angle C$?
- What is the measure of $\angle D$?



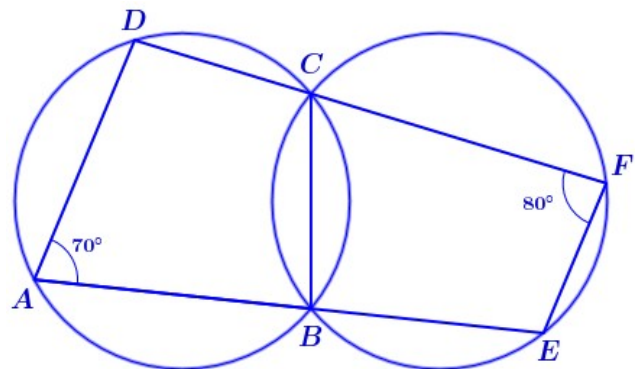
2. In the figure $\angle P = 60^\circ$, PS is parallel to QR

- What is the measure of $\angle R$?
- What is the measure of $\angle RQT$?
- What is the measure of $\angle PQR$?
- What is the measure of $\angle S$?



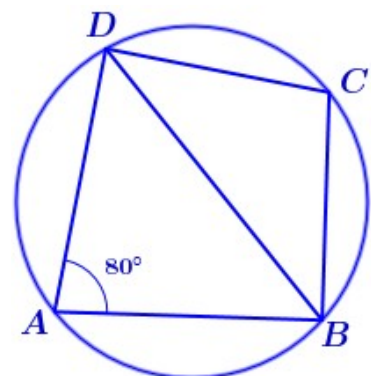
3. In the figure $\angle A = 70^\circ$, $\angle F = 80^\circ$

- What is the measure of $\angle BCD$?
- What is the measure of $\angle BCF$?
- What is the measure of $\angle E$?
- What is the measure of $\angle CBE$?
- What is the measure of $\angle ABC$?
- What is the measure of $\angle D$?



4. In the figure $\angle A = 80^\circ$, $AD = AB$ and $BC = CD$

- What is the measure of $\angle C$?
- What is the measure of $\angle ADB$?
- What is the measure of $\angle BDC$?
- What is the measure of $\angle ABC$?



ONLINE MATHS CLASS - X - 29 (14 / 09 /2020)

What did we learn in the last class ?

Cyclic quadrilateral

Cyclic quadrilaterals are those quadrilaterals with opposite angles supplementary .

- If one vertex of a quadrilateral is outside the circle drawn through the other three vertices , then the sum of the angles at this vertex and its opposite vertex is less than 180°
- If one vertex of a quadrilateral is inside the circle drawn through the other three vertices , then the sum of the angles at this vertex and its opposite vertex is more than 180°
- If all four vertices of a quadrilateral are on a circle , then its opposite angles are supplementary
- If opposite angles of a quadrilateral are supplementary , we can draw a circle passing through all four of its vertices .

NB : In the figure ABCD is an isosceles trapezium .

Prove that $\angle A = \angle B$?

Answer

In the given isosceles trapezium ABCD

$AD = BC$. Also AB is parallel to DC .

A line drawn through C parallel to AD cuts AB at E .

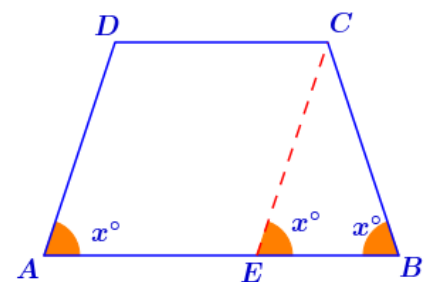
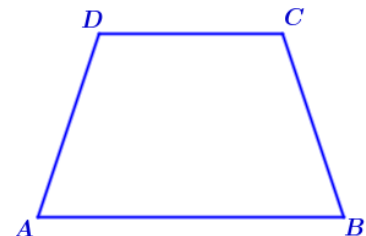
ADCE is a parallelogram . (AE is parallel to DC and

AD is parallel to CE)

$\angle A = \angle BEC = x^\circ$

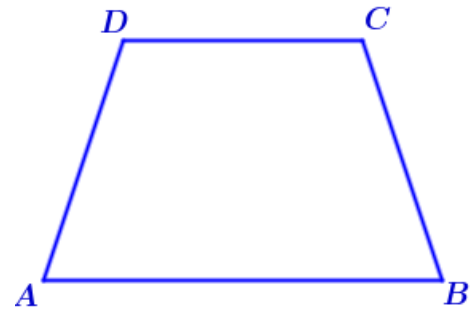
In triangle BCE , $\angle BEC = \angle B = x^\circ$ ($CE = BC$, $AD = BC$)

That is , $\angle A = \angle BEC = \angle B \implies \angle A = \angle B$



1. In the figure $ABCD$ is an isosceles trapezium .

Prove that $ABCD$ is cyclic ?



Answer

[In the given isosceles trapezium $ABCD$

$$AD = BC$$

AB is parallel to DC . Also $\angle A = \angle B$]

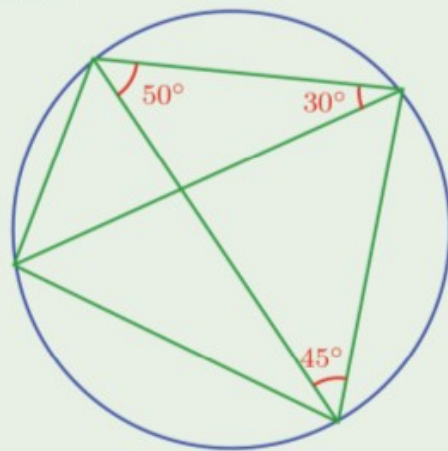
$\angle A + \angle D = 180^\circ$ (AB is parallel to DC , co-interior angles are supplementary)

$\angle B + \angle D = 180^\circ$ ($\angle A = \angle B$)

Since the opposite angles are supplementary , $ABCD$ is cyclic .

2.

Calculate the angles of the quadrilateral in the picture and also the angles between their diagonals:



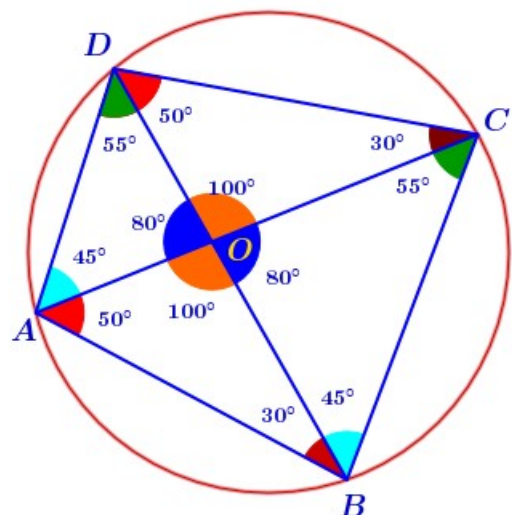
Answer

$\angle BDC = \angle BAC = 50^\circ$ (All angles made by an arc on the alternate arc are equal)

$\angle ACD = \angle ABD = 30^\circ$

$\angle CBD = \angle CAD = 45^\circ$

$\angle ADB = 55^\circ$ ($\angle ABC + \angle ADC = 180^\circ$, A pair of



angles on an arc and its alternate arc are supplementary)

$$\angle ADB = \angle ACB = 55^\circ$$

In triangle AOB , $\angle AOB = 180 - (50 + 30) = 180 - 80 = 100^\circ$ (Sum of the angles of a triangle is 180°)

$$\angle AOB = \angle COD = 100^\circ \text{ (opposite angles)}$$

$$\angle AOD = 180 - \angle AOB = 180 - 100 = 80^\circ \text{ (linear pair)}$$

$$\angle AOD = \angle BOC = 80^\circ \text{ (opposite angles)}$$

Angles of the quadrilateral

$$\angle DAB = 45 + 55 = 95^\circ$$

$$\angle ABC = 30 + 45 = 75^\circ$$

$$\angle BCD = 55 + 30 = 85^\circ$$

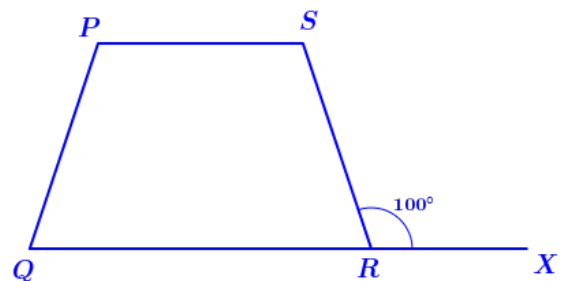
$$\angle CDA = 50 + 55 = 105^\circ$$

More activities

1. In the figure $PQRS$ is an isosceles trapezium and

QR is extended to X .

If $\angle SRX = 100^\circ$, find all angles of $PQRS$?



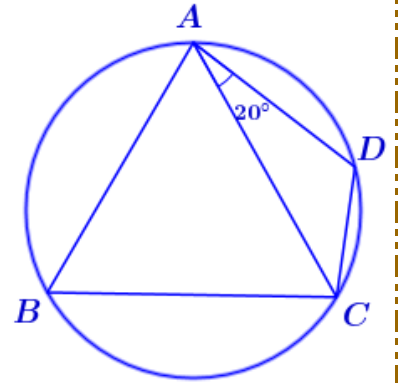
2. Prove that any non- isosceles trapezium is not cyclic ?

ONLINE MATHS CLASS - X - 29 (14 / 09 / 2020)

WORKSHEET

1. In the figure ABC is an equilateral triangle . $\angle CAD = 20^\circ$

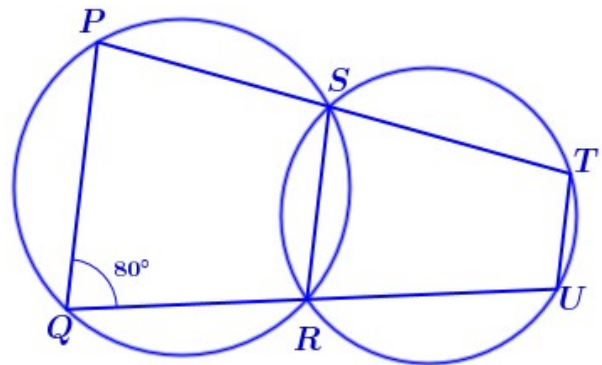
- What is the measure of $\angle B$?
- What is the measure of $\angle D$?
- What is the measure of $\angle BAD$?
- What is the measure of $\angle BCD$?



2. In the figure two circles intersect at the points S and R . PQ is parallel to RS and

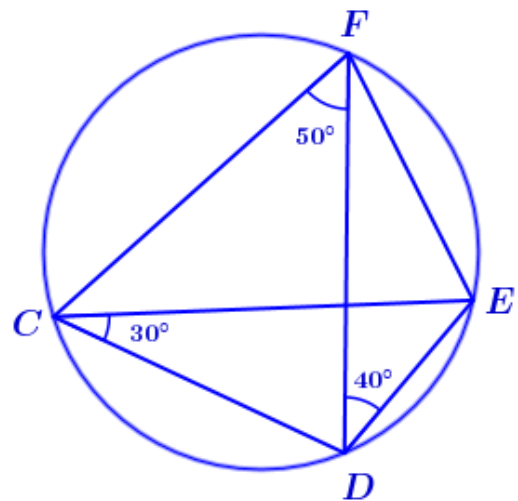
$$\angle Q = 80^\circ$$

- What is the measure of $\angle PSR$?
- What is the measure of $\angle RST$?
- What is the measure of $\angle U$?
- What is the measure of $\angle SRU$?
- What is the measure of $\angle T$?
- What is the measure of $\angle P$?

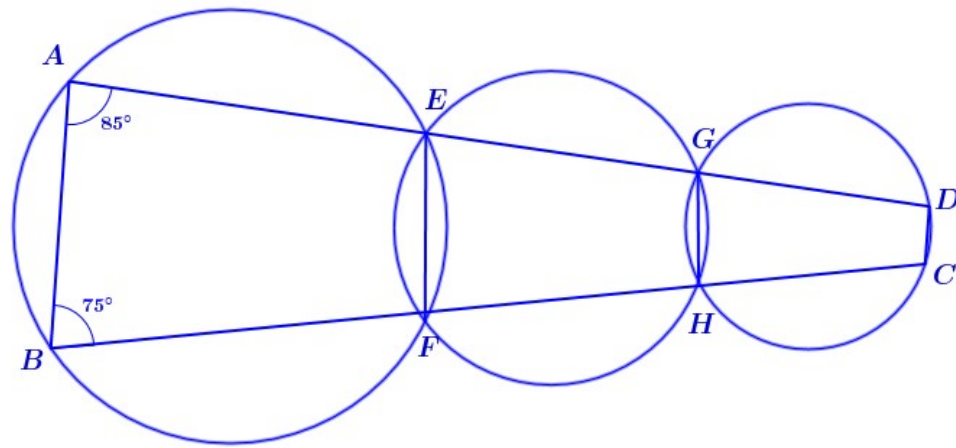


3. In the figure $\angle DCE = 30^\circ$, $\angle CFD = 50^\circ$ and $\angle EDF = 40^\circ$

- What is the measure of $\angle DFE$?
- What is the measure of $\angle CED$?
- What is the measure of $\angle ECF$?
- What is the measure of $\angle CDF$?
- What is the measure of $\angle DEF$?
- What is the measure of $\angle CEF$?



4.



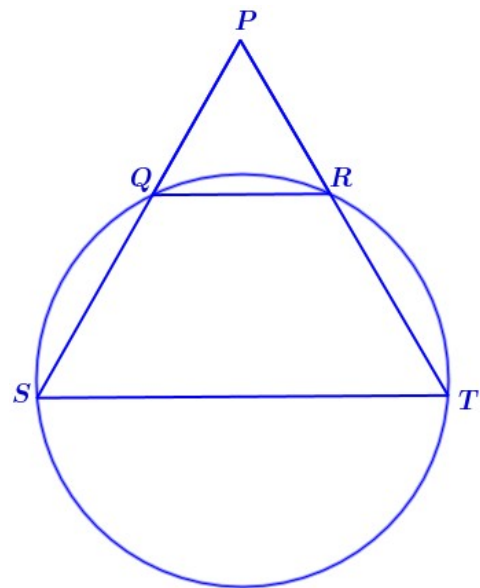
4. In the figure three circles intersect at the points E, F, G and H .

$$\angle A = 85^\circ, \angle B = 75^\circ$$

- What is the measure of $\angle AEF$?
- What is the measure of $\angle FEG$?
- What is the measure of $\angle FHG$?
- What is the measure of $\angle D$?
- What is the measure of $\angle BFE$?
- What is the measure of $\angle C$?

5. In the figure PQR is an equilateral triangle .

- What is the measure of $\angle P$?
- What is the measure of $\angle SQR$?
- What is the measure of $\angle T$?
- What is the measure of $\angle QRT$?
- Prove that PST is an equilateral triangle ?



ONLINE MATHS CLASS - X - 30 (15 / 09 /2020)

What did we learn in the last class ?

Cyclic quadrilateral

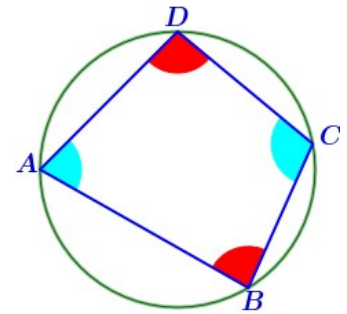
If all the four vertices of a quadrilateral are on the circle , then it is called a cyclic quadrilateral .

Opposite angles of a cyclic quadrilateral are supplementary .

In the cyclic quadrilateral ABCD ,

$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$



If opposite angles of a quadrilateral are supplementary , then it is cyclic

Cyclic quadrilaterals which we have already learned

Rectangle , Square , isosceles trapezium

Let's solve some problems related to these ideas .

1. Prove that any non- isosceles trapezium is not cyclic ?

Answer .

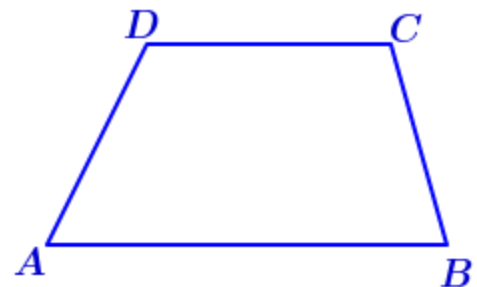
In the figure ABCD is a non - isosceles trapezium

$\angle A + \angle D = 180^\circ$ (AB is parallel to DC , co-interior angles are supplementary)

Also

$$\angle A \neq \angle B$$

$$\angle B + \angle D \neq 180^\circ$$



Since the opposite angles are not supplementary, $ABCD$ is not cyclic.

That is, any non-isosceles trapezium is not cyclic.

1 Prove that any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite vertex.

Answer.

In the figure $\angle CBE$ is an outer angle at the vertex B of a cyclic quadrilateral $ABCD$.

Take, $\angle CBE = x^\circ$

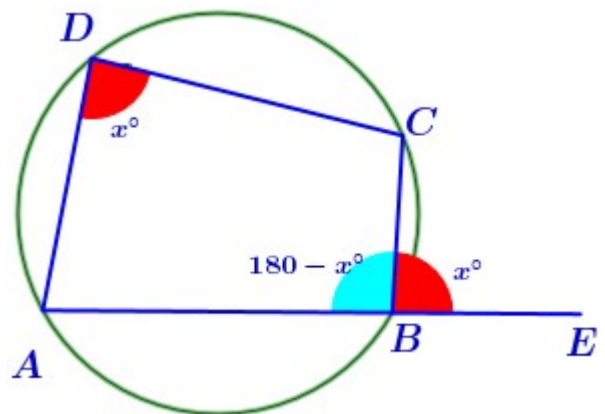
$\angle ABC = 180 - x^\circ$ (linear pair)

$\angle ADC = 180 - (180 - x^\circ)$

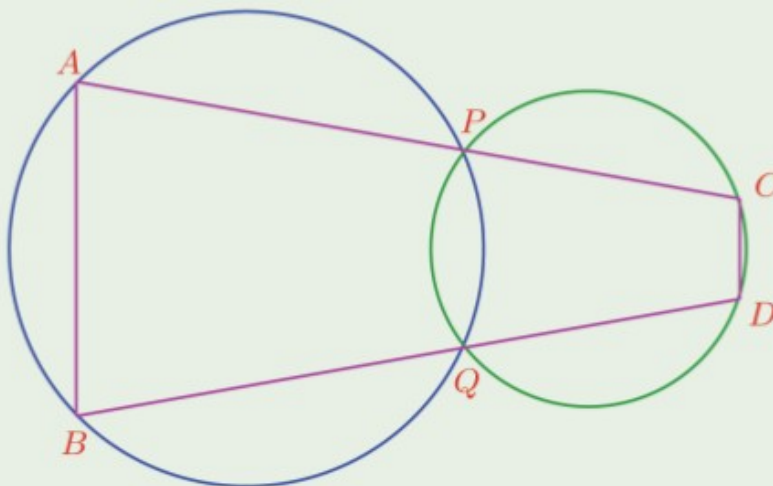
$= 180 - 180 + x^\circ = x^\circ = \angle CBE$

($\angle ABC + \angle ADC = 180$, Opposite angles of

a cyclic quadrilateral are supplementary)



2 The two circles below intersect at P, Q and lines through these points meet the circles at A, B, C, D . The lines AC and BD are not parallel. Prove that if these lines are of equal length, then $ABDC$ is a cyclic quadrilateral.



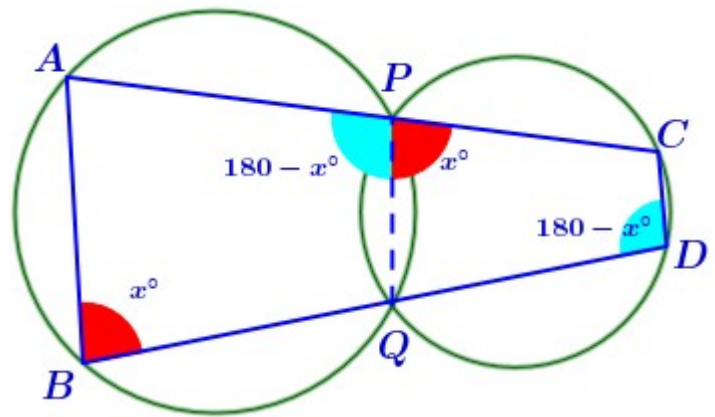
Answer.

Join PQ

Take , $\angle ABQ = x^\circ$

$\angle APQ = 180 - x^\circ$

(ABQP is a cyclic quadrilateral)



$\angle QPC = x^\circ$ ($\angle APQ = 180 - x^\circ$, linear pair)

$\angle CDQ = 180 - x^\circ$ (PQDC is a cyclic quadrilateral)

AB is parallel to CD ($\angle ABQ = x^\circ$, $\angle CDQ = 180 - x^\circ$, co-interior angles are supplementary)

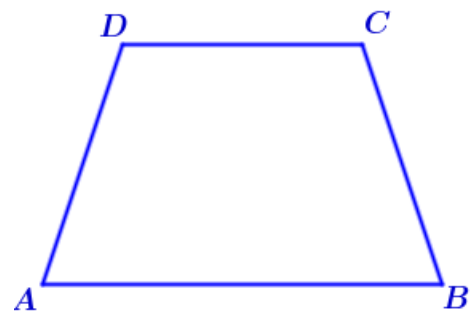
ABCD is an isosceles trapezium (AB is parallel to CD and AC = BD)

So ABCD is a cyclic quadrilateral (Any isosceles trapezium is cyclic)

NB : (To remember)

In the figure ABCD is an isosceles trapezium .

Prove that ABCD is cyclic ?



Answer

[In the given isosceles trapezium ABCD

$$AD = BC$$

AB is parallel to DC . Also $\angle A = \angle B$]

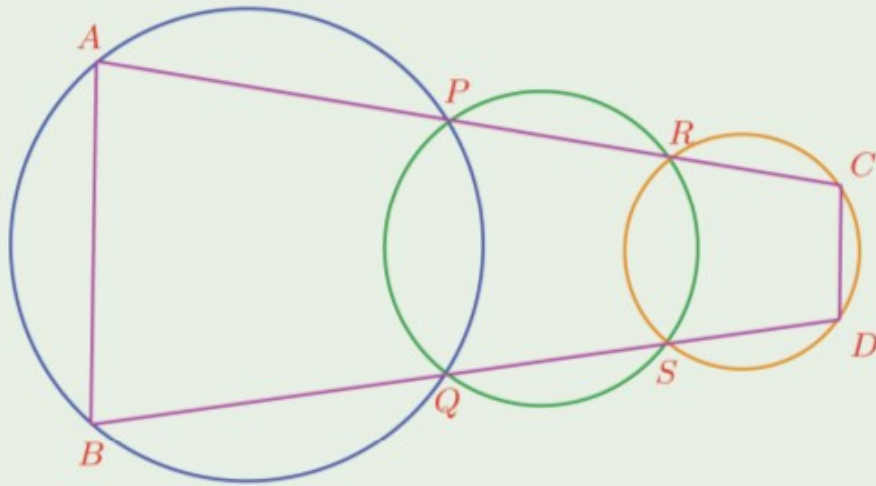
$\angle A + \angle D = 180^\circ$ (AB is parallel to DC , co-interior angles are supplementary)

$\angle B + \angle D = 180^\circ$ ($\angle A = \angle B$)

Since the opposite angles are supplementary , ABCD is cyclic .

More activity

In the picture, the circles on the left and right intersect the middle circle at P, Q, R, S ; the lines joining them meet the left and right circles at A, B, C, D . Prove that $ABDC$ is a cyclic quadrilateral.

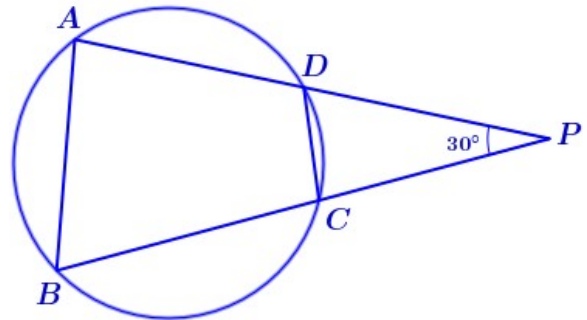


ONLINE MATHS CLASS - X - 30 (15 / 09 /2020)

WORKSHEET

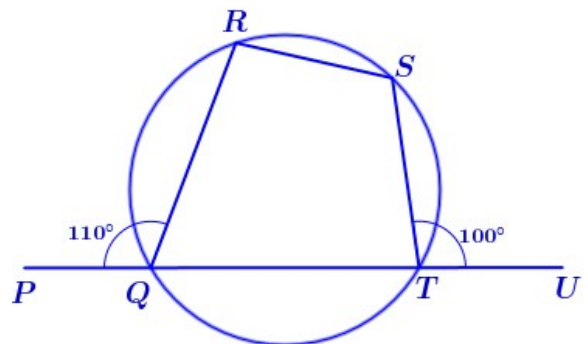
1. In the figure $DP = CP$, $\angle CPD = 30^\circ$

- a) What is the measure of $\angle PCD$?
- b) What is the measure of $\angle BCD$?
- c) What is the measure of $\angle A$?
- d) What is the measure of $\angle ADC$?
- e) What is the measure of $\angle B$?



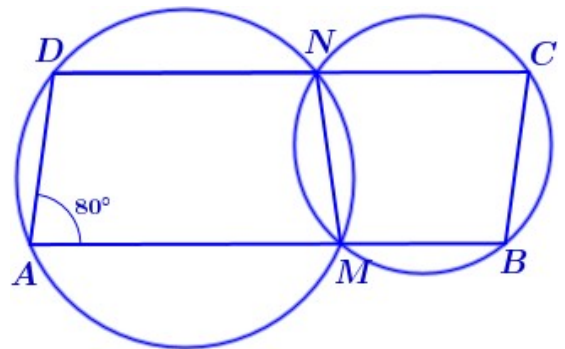
2. In the figure $\angle PQR = 110^\circ$, $\angle STU = 100^\circ$

- a) What is the measure of $\angle RQT$?
- b) What is the measure of $\angle S$?
- c) What is the measure of $\angle STQ$?
- d) What is the measure of $\angle R$?



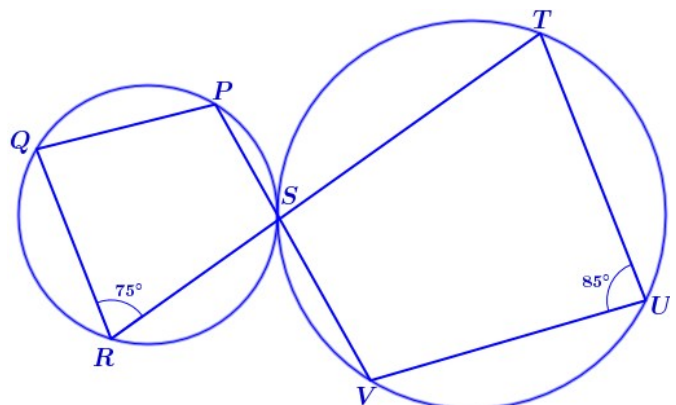
3. In the figure lines AB and DC are parallel . $\angle A = 80^\circ$

- a) What is the measure of $\angle DNM$?
- b) What is the measure of $\angle D$?
- c) What is the measure of $\angle AMN$?
- d) What is the measure of $\angle B$?
- e) What is the measure of $\angle C$?



4. In the figure lines QR and TU are parallel . $\angle R = 75^\circ$, $\angle U = 85^\circ$

- a) What is the measure of $\angle P$?
- b) What is the measure of $\angle T$?
- c) What is the measure of $\angle V$?
- d) What is the measure of $\angle VST$?
- e) What is the measure of $\angle Q$?



ONLINE MATHS CLASS - X - 31 (17 / 09 / 2020)

What did we learn in the last classes ?

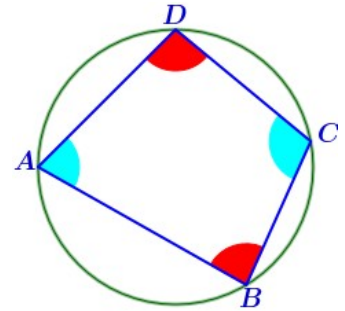
Cyclic quadrilateral

If all the four vertices of a quadrilateral are on the circle, then it is called a cyclic quadrilateral. Opposite angles of a cyclic quadrilateral are supplementary.

In the cyclic quadrilateral ABCD,

$$\angle A + \angle C = 180^\circ$$

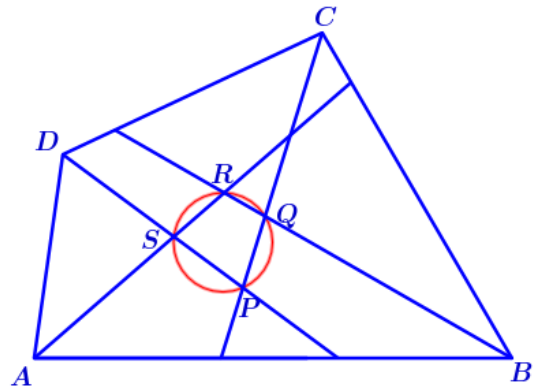
$$\angle B + \angle D = 180^\circ$$



If opposite angles of a quadrilateral are supplementary, then it is cyclic.

Let's solve some problems related to these ideas.

1. In the picture bisectors of adjacent angles of the quadrilateral ABCD intersect at P, Q, R, S. Prove that PQRS is a cyclic quadrilateral.



Answer.

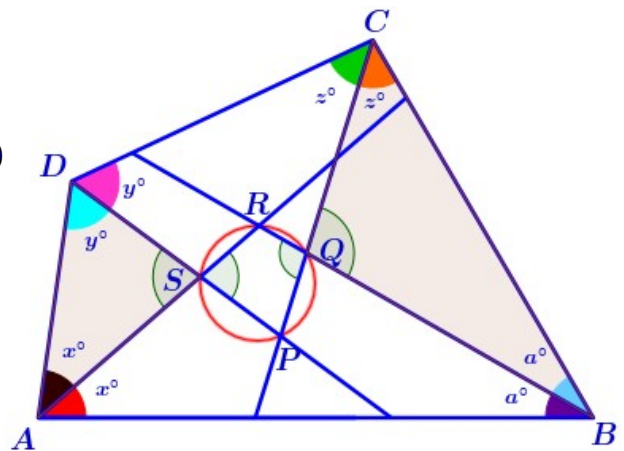
Take,

$$\angle BAS = \angle DAS = x^\circ \quad (\text{AS is the bisector of } \angle DAB)$$

$$\angle ADS = \angle CDS = y^\circ \quad (\text{DS is the bisector of } \angle ADC)$$

$$\angle DCQ = \angle BCQ = z^\circ \quad (\text{CQ is the bisector of } \angle BCD)$$

$$\angle CBQ = \angle ABQ = a^\circ \quad (\text{BQ is the bisector of } \angle ABC)$$



$(x+x) + (y+y) + (z+z) + (a+a) = 360^\circ$ (Sum of the angles of the quadrilateral ABCD)

$$2x + 2y + 2z + 2a = 360^\circ$$

$$2(x + y + z + a) = 360^\circ$$

$$x + y + z + a = \frac{360^\circ}{2} = 180^\circ$$

In the triangle ADS, $\angle ASD = 180 - (x + y)$ (Sum of the angles of a triangle is 180°)

$$\angle ASD = \angle PSR = 180 - (x + y) \quad (\text{Opposite angles are equal})$$

In the triangle BQC, $\angle BQC = 180 - (z + a)$ (Sum of the angles of a triangle is 180°)

$$\angle BQC = \angle PQR = 180 - (z + a) \quad (\text{Opposite angles are equal})$$

$$\angle PSR + \angle PQR = 180 - (x + y) + 180 - (z + a)$$

$$= 180 - x - y + 180 - z - a$$

$$= 180 + 180 - (x + y + z + a)$$

$$= 360 - (x + y + z + a)$$

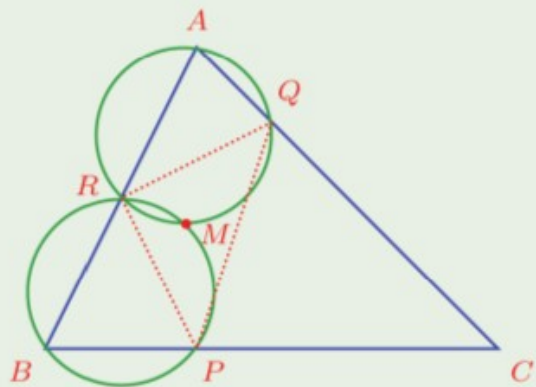
$$= 360 - 180 = 180^\circ$$

Since the opposite angles are supplementary, PQRS is a cyclic quadrilateral.

2.

In the picture, points P, Q, R are marked on the sides BC, CA, AB of $\triangle ABC$ and the circumcircles of $\triangle AQR$ and $\triangle BRP$ are drawn. M is a point where these circles intersect.

Prove that the circumcircle of $\triangle CPQ$ also passes through M .



Answer .

Draw the lines PM, QM, RM .

Take $\angle A = x^\circ$, $\angle B = y^\circ$

In triangle ABC ,

$$\angle C = 180 - (x + y)$$

In the cyclic quadrilateral $AQMR$,

$$\angle QMR = 180 - x \quad (\text{Opposite angles of a cyclic quadrilateral are supplementary})$$

In the cyclic quadrilateral $BPMR$,

$$\angle PMR = 180 - y \quad (\text{Opposite angles of a cyclic quadrilateral are supplementary})$$

$$\angle PMQ = 360 - (180 - x + 180 - y) \quad (\text{Angle around a point is } 360^\circ)$$

$$= 360 - (360 - x - y)$$

$$= 360 - 360 + x + y$$

$$= x + y$$

$$\angle PCQ + \angle PMQ = 180 - (x + y) + x + y = 180^\circ$$

Since the opposite angles are supplementary , $PCQM$ is a cyclic quadrilateral .

That is , the circumcircle of triangle CPQ passes through M .

More activity

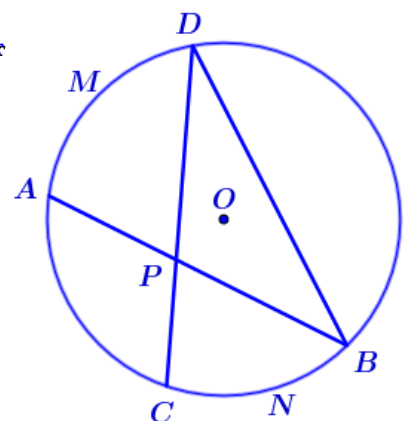
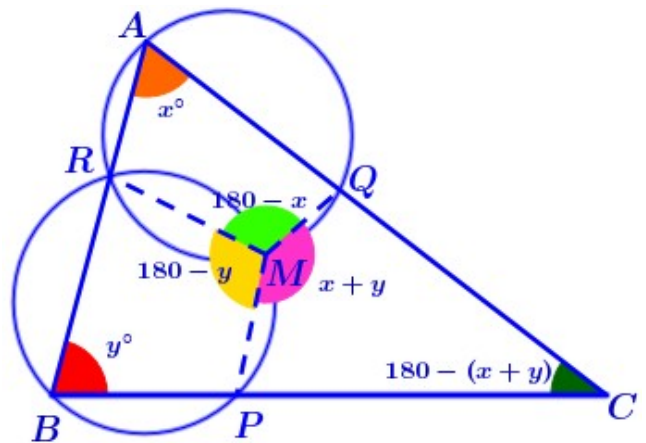
In the figure the length of the arc CNB is $\frac{1}{5}$ of the perimeter of

the circle and the length of the arc AMD is $\frac{1}{6}$ of the perimeter

of the circle .

a) What is the measure of the central angle of the arc CNB ?

b) Find the measures of $\angle CDB$, $\angle ABD$ and $\angle APD$?



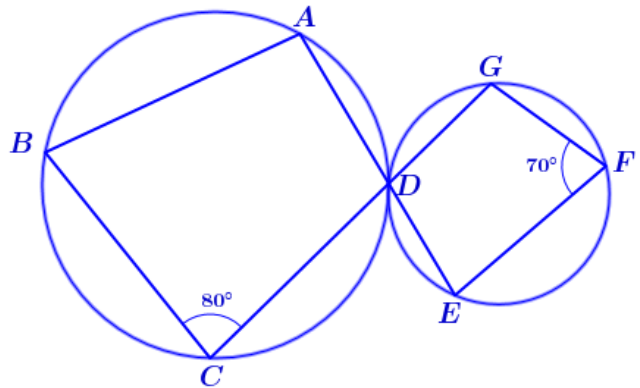
ONLINE MATHS CLASS- X - 31 (17 / 09 /2020)

WORKSHEET

1. In the figure two circle intersect at D

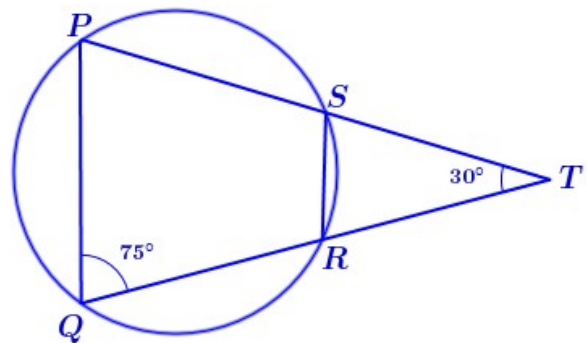
$\angle BCD = 80^\circ$, $\angle EFG = 70^\circ$

- a) What is the measure of $\angle BAD$?
- b) What is the measure of $\angle EDG$?
- c) What is the measure of $\angle ADC$?
- d) What is the measure of $\angle ABC$?



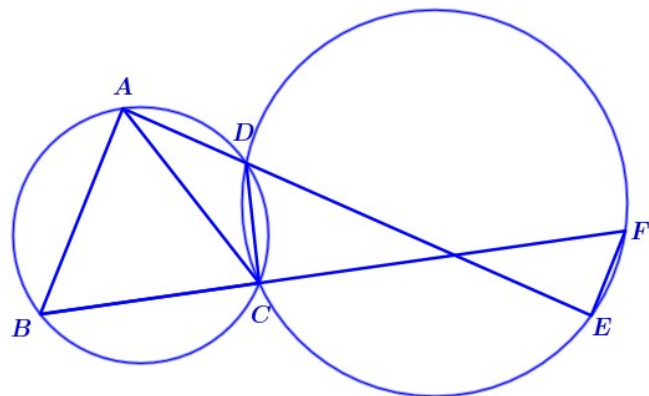
2. In the figure $\angle PQR = 75^\circ$, $\angle STR = 30^\circ$

- a) What is the measure of $\angle PSR$?
- b) What is the measure of $\angle QPS$?
- c) What is the measure of $\angle QRS$?
- d) Prove that STR is an isosceles triangle ?



3. In the figure ABC is an equilateral triangle . $AD = CD$

- a) What is the measure of $\angle ABC$?
- b) What is the measure of $\angle ADC$?
- c) What is the measure of $\angle CFE$?
- d) What is the measure of $\angle DEF$?



4. In the figure two circles intersect at P .

Lines QR and XY are parallel . $\angle SPQ = 65^\circ$, $\angle PXY = 75^\circ$

- a) What is the measure of $\angle SRQ$?
- b) What is the measure of $\angle PQR$?
- c) What is the measure of $\angle PSR$?
- d) What is the measure of $\angle XYZ$?
- e) What is the measure of $\angle PZY$?

