25/9/2020
FRIDAY

MATHEMATICS

STD-X
class-35

Text book pageno. 67, 68

Question no. 1, 5, 6. Answers

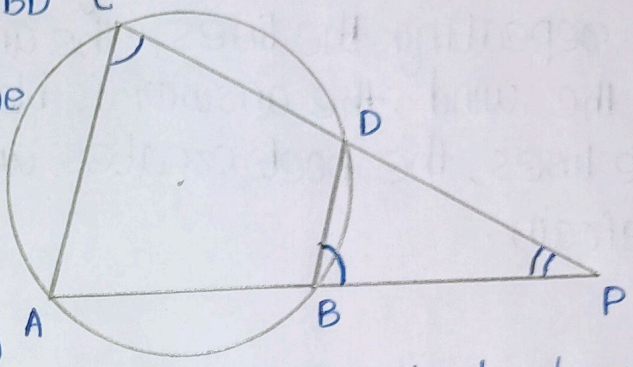
i) In the picture, chords AB and CD of the circle are extended to meet at P.

i) Prove that the angles of ΔAPC and ΔPBD, formed by joining AC and BD, are the same.

ii) Prove that $PA \times PB = PC \times PD$

iii) Prove that if $PB = PD$, then ABCD is an isosceles trapezium.

Ans) i) $\angle C$ of $\Delta APC = \angle B$ of ΔPBD [Any outer angle of a cyclic quadrilateral is equal to the inner angle at the opposite corner.]



$\angle P$ of $\Delta APC = \angle P$ of ΔPBD (same angle)

Two angles of both triangles are equal. so their third angle also will be equal.

ii) Since the three pairs of angles of ΔAPC and ΔDPB are equal, they are similar triangles. Their sides are proportional.

$$\frac{PA}{PC} = \frac{PD}{PB}$$

$$\therefore PA \times PB = PC \times PD$$

iii) If $PB = PD$, $PA \times PD = PC \times PD$

$\therefore PA = PC$

So ΔPAC is isosceles triangle.

Then $\angle A = \angle C$

$\angle ACD + \angle ABD = 180^\circ$ (cyclic quadrilateral)

But $\angle ACD = \angle CAB$

$\therefore \angle CAB + \angle ABD = 180^\circ$

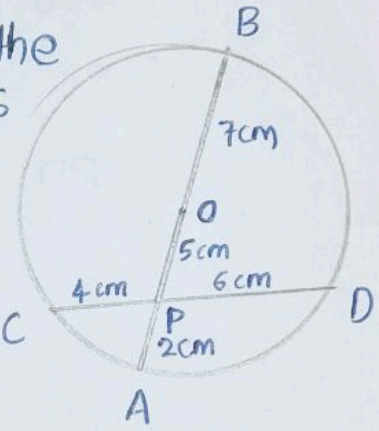
Since a pair of co-interior angles are supplementary, AC and BD are parallel.

$AB = PA - PB = PC - PD = CD \quad (PA = PC,\ PB = PD)$

Since sides AC and BD of quadrilateral ABDC is parallel and the non parallel sides are equal, it is an isosceles trapezium.

5) In the picture, a line through the centre of a circle cuts a chord into two parts:
What is the radius of the circle?

Ans) In the figure O is the centre of the circle. OP is extended to both sides which cuts the circle at A and B. CD is the chord of the circle. If two chords of circle intersect within the circle, then the products of the parts of the two chords are equal.



$\therefore PA \times PB = PC \times PD \longrightarrow (1)$

If $OB = r$, $PB = r + 5$, $PA = r - 5$

From (1), $(r-5)(r+5) = 4 \times 6$

$r^2 - 25 = 24$

$[(a+b)(a-b) = a^2 - b^2]$

$r^2 = 24 + 25 = 49$

$\therefore r = \sqrt{49} = \underline{7}$

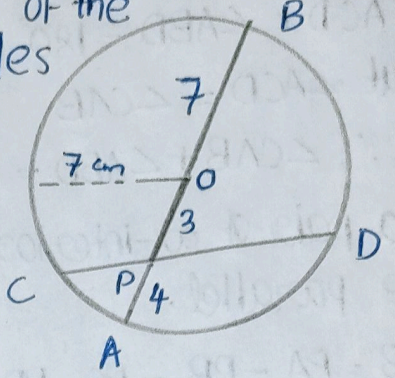
$\therefore$ Radius of the circle = 7cm

6) In the picture, a line through the centre of a circle meets a chord of the circle:

What are the lengths of the two pieces of the chord?

Ans)

In the figure O is the centre of the circle. Extend OP to both sides to cut the circle at A and B.



$OB = 7\text{cm}$ (given)

$PB = 7+3 = \underline{10\text{ cm}}$

$OP = 3\text{cm}$

$\therefore PA = 7-3 = \underline{4\text{ cm}}$

$PA \times PB = PC \times PD$

$4 \times 10 = PC \times PD$

$PC \times PD = 40$

$PC + PD = 13$ (given)

Two numbers whose sum is 13 and product is 40

$= 8, 5$

$\therefore PC = \underline{5\text{ cm}}$

$PD = \underline{8\text{ cm}}$