ONLINE MATHS CLASS - X – 49 (23/ 10 /2020)

5 . TRIGNOMETRY

PREVIOUS KNOWLEDGE

- ***Equal triangles***

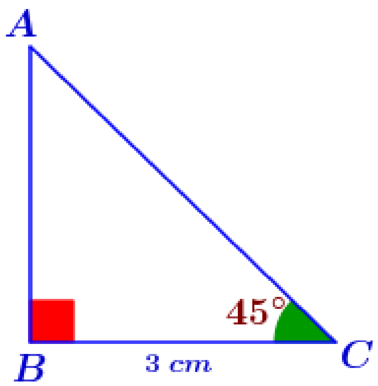
If three sides of a triangle are equal to three sides of another triangle , then the angles opposite to equal sides are equal . Such triangles are known as equal triangles .

- ***Similar triangles***

If three angles of a triangle are equal to three angles of another triangle , then the sides opposite to equal angles are in the same ratio . Such triangles are known as similar triangles .

Trignometry is the study of the relationship between the measure of angles and the length of the sides of a triangle .

Activity 1



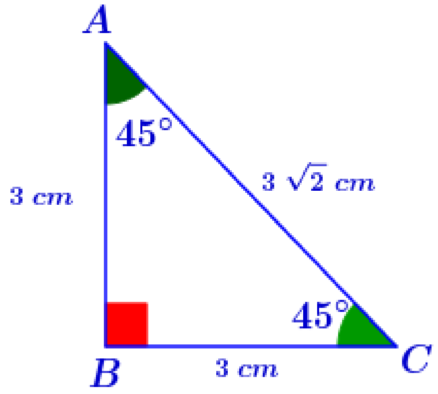
In triangle ABC , $< B = 90^0$ and $< C = 45^0$

Then $< A = 180 - (90 + 45) = 180 - 135 = 45^0$

(*Sum of the angles of a triangle is* 180^0)

If $BC = 3$ *cm* ,

$AB = 3$ *cm* (*The sides opposite to equal angles of a triangle are equal*)



We know that *relation connecting the sides of a right angled triangle is the Pythagoras theorem .*

$$Base^2 + Altitude^2 = Hypotenuse^2$$

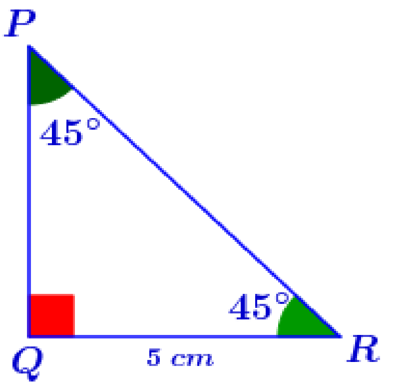
$$AC = \sqrt{BC^2 + AB^2}$$

$$= \sqrt{3^2 + 3^2}$$

$$= \sqrt{9 + 9} = \sqrt{18} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$ സെ.മി

The ratio of the sides opposite to the angles 45^0, 45^0 *and* 90^0 $= 3 : 3 : 3\sqrt{2}$

$$= 1 : 1 : \sqrt{2}$$



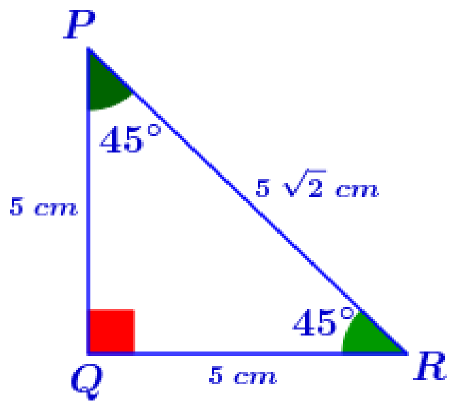
In triangle *PQR* , $\angle$ **Q** = 90^0 *and* $\angle$ **R** = 45^0

Then $\angle$ **P** = 180 - (90 + 45) = 180 – 135 = 45^0

(*Sum of the angles of a triangle is* 180^0)

If QR = 5 *cm* ,

PQ = 5 *cm* (*The sides opposite to equal angles of a triangle are equal*)



$$PR = \sqrt{QR^2 + PQ^2}$$

$$= \sqrt{5^2 + 5^2}$$

$$= \sqrt{25 + 25} = \sqrt{50} = \sqrt{5 \times 5 \times 2} = 5\sqrt{2}$$ സെ.മി

The ratio of the sides opposite to the angles 45^0, 45^0 *and* 90^0 $= 5 : 5 : 5\sqrt{2}$

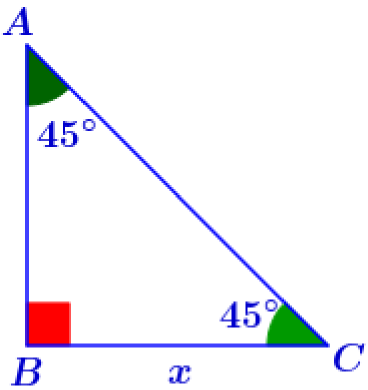
$$= 1 : 1 : \sqrt{2}$$

Is the ratio of the sides of any triangle with angles 45^0, 45^0 ***and*** 90^0, $1 : 1 : \sqrt{2}$?

Let's examine .

Suppose in triangle ABC , $\angle B = 90^0$ *and* $\angle C = 45^0$

Then $\angle A = 180 - (90 + 45) = 180 - 135 = 45^0$



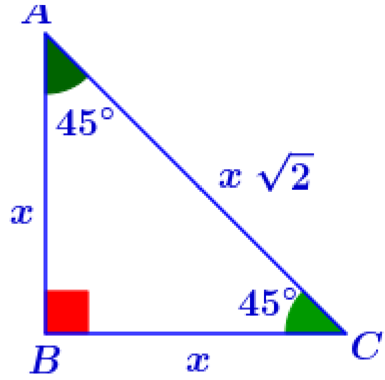
Let's take $BC = x$ *units* ,

Then $AB = x$ *units* .

$$AC = \sqrt{BC^2 + AB^2}$$

$$= \sqrt{x^2 + x^2}$$

$$= \sqrt{2\,x^2} = \sqrt{2 \times x \times x} = x\sqrt{2} \ units$$



The ratio of the sides opposite to the angles 45^0, 45^0 *and* 90^0 $= x : x : x\sqrt{2}$

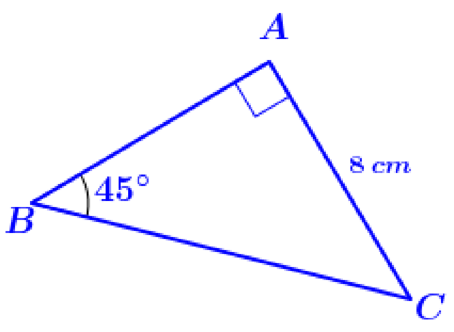
$= 1 : 1 : \sqrt{2}$

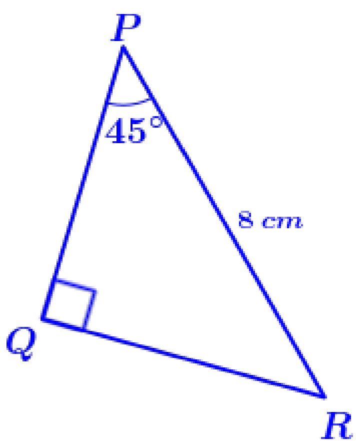
Finding

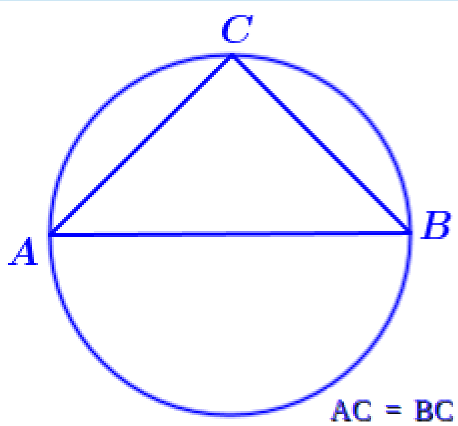
In any triangle of angles 45^0, 45^0, 90^0 ***the sides are in the ratio*** $1 : 1 : \sqrt{2}$

More activities

Find the other two sides of the triangles given below







Diameter $= AB = 7\sqrt{2}$ *cm*

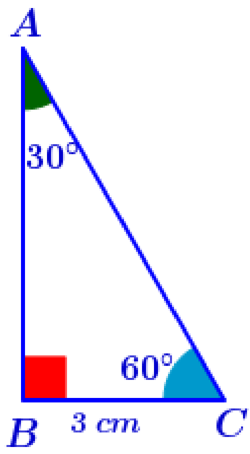
ONLINE MATHS CLASS - X – 50 (27/ 10 /2020)

5 . TRIGNOMETRY - Class 2

What did we learn in the last class ?

In any triangle of angles 45^0, 45^0, 90^0 *the sides are in the ratio* $1 : 1 : \sqrt{2}$

Activity 1

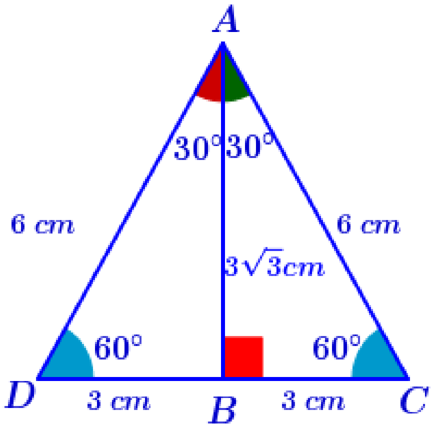


In triangle ABC $\angle B = 90^0$, $\angle BAC = 30^0$

then $\angle C = 180 - (90 + 30) = 180 - 120 = 60^0$

(*Sum of the angles of a triangle is* 180^0)

If BC = 3 cm , *what are the length of the other sides* ?



In the figure triangle ABC is joined with another triangle of same measure . The angles of triangle ADC are 60^0 *each*

AD = AC = DC = 6 *cm*

In right triangle ABC ,

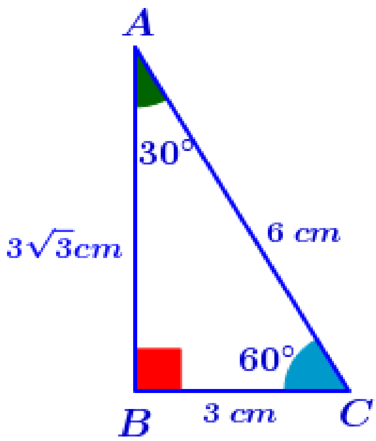
$$BC^2 + AB^2 = AC^2$$

$$3^2 + AB^2 = 6^2$$

$$9 + AB^2 = 36$$

$$AB^2 = 36 - 9 = 27$$

$$AB = \sqrt{27} = \sqrt{3 \times 3 \times 3} = 3\sqrt{3}\ cm$$



The ratio of the sides opposite to the angles $30^0, 60^0, 90^0$ $= 3 : 3\sqrt{3} : 6$

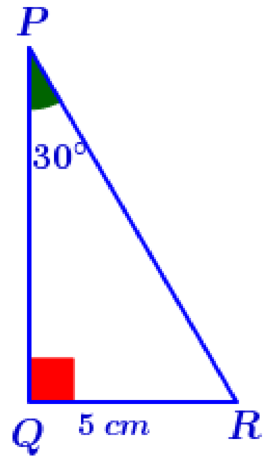
$= 1 : \sqrt{3} : 2$

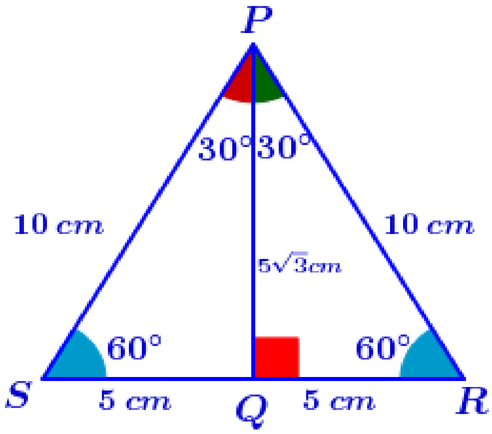
In triangle PQR $\angle Q = 90^0$, $\angle QPR = 30^0$

then $\angle R = 180 - (90 + 30) = 180 - 120 = 60^0$

(*Sum of the angles of a triangle is* 180^0)

If QR = 5 cm , what are the length of the other sides ?





In the figure triangle PQR is joined with another triangle of same measure . The angles of triangle PSR are 60^0 *each .*

PS = PR = SR = 10 *cm*

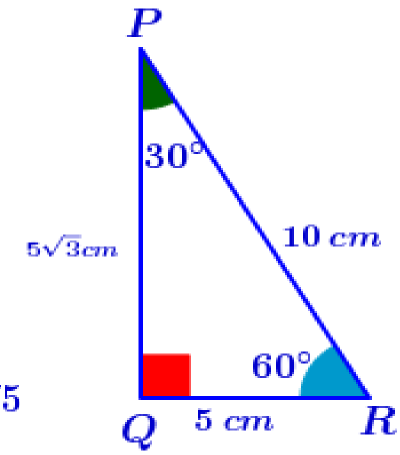
In right triangle PQR ,

$$QR^2 + PQ^2 = PR^2$$

$$5^2 + PQ^2 = 10^2$$

$$25 + PQ^2 = 100$$

$$PQ^2 = 100 - 25 = 75$$



$$PQ = \sqrt{75} = \sqrt{5 \times 5 \times 3} = 5\sqrt{3}\ cm$$

The ratio of the sides opposite to the angles $30^0, 60^0, 90^0$ $= 5 : 5\sqrt{3} : 10$

$= 1 : \sqrt{3} : 2$

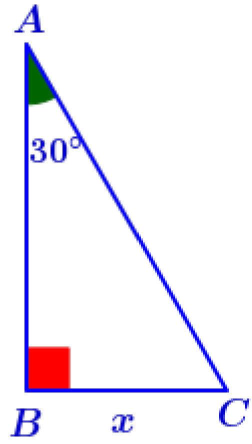
Is the ratio of the sides of any triangle with angles 30^0, 60^0 *and* 90^0, $1 : \sqrt{3} : 2$

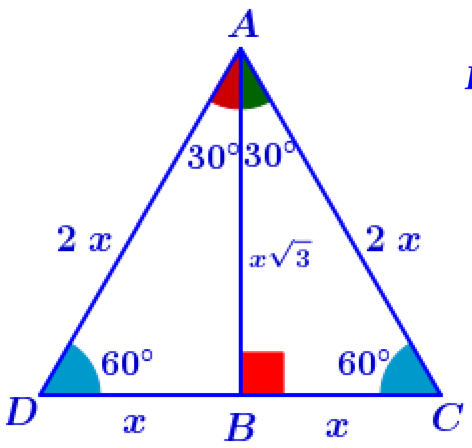
Let's examine .

In triangle ABC , $\angle B = 90^0$, $\angle BAC = 30^0$

then $\angle C = 180 - (90 + 30) = 180 - 120 = 60^0$

(Sum of the angles of a triangle is 180^0 *)*





In the figure triangle ABC is joined with another triangle of same measure . The angles of triangle ADC are 60^0 *each*

If $BC = x$ *units ,* $AD = AC = DC = 2x$ *units*

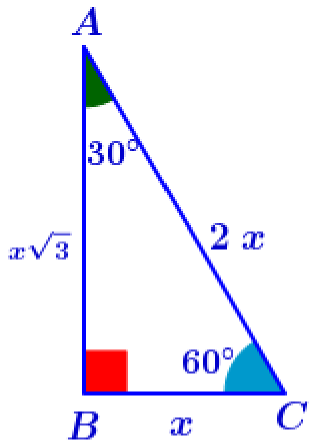
In right triangle ABC ,

$$BC^2 + AB^2 = AC^2$$

$$x^2 + AB^2 = (2x)^2$$

$$x^2 + AB^2 = 4x^2$$

$$AB^2 = 4x^2 - x^2 = 3x^2$$



$$AB = \sqrt{3x^2} = \sqrt{3 \times x \times x} = x\sqrt{3}$$

The ratio of the sides opposite to the angles $30^0, 60^0, 90^0$ $= x : x\sqrt{3} : 2x$

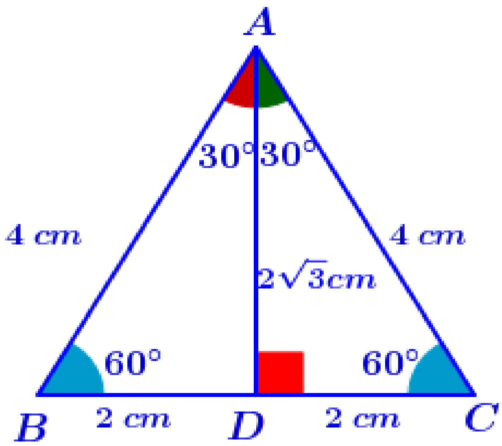
$= 1 : \sqrt{3} : 2$

Finding

In any triangle of angles $30^0, 60^0, 90^0$ ***the sides are in the ratio*** $1 : \sqrt{3} : 2$

Find the area of an equilateral triangle of side 4 cm .

Answer



AB = BC = AC = 4 cm

Draw AD perpendicular to BC .

$AD = 2\sqrt{3}\ cm$

Area of triangle ABC $= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 4 \times 2\sqrt{3}$

$= 4\sqrt{3}\ cm^2$

More activity

Find the area of an equilateral triangle of side 7 cm .

ONLINE MATHS CLASS - X – 51 (30 / 10 /2020)

5 . TRIGNOMETRY - Class 3

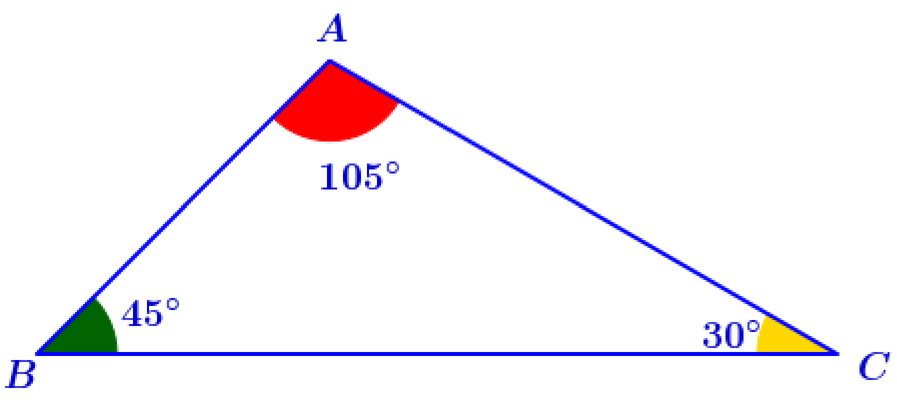
What did we learn in the last class ?

In any triangle of angles 45^0, 45^0, 90^0 *the sides are in the ratio* $1 : 1 : \sqrt{2}$

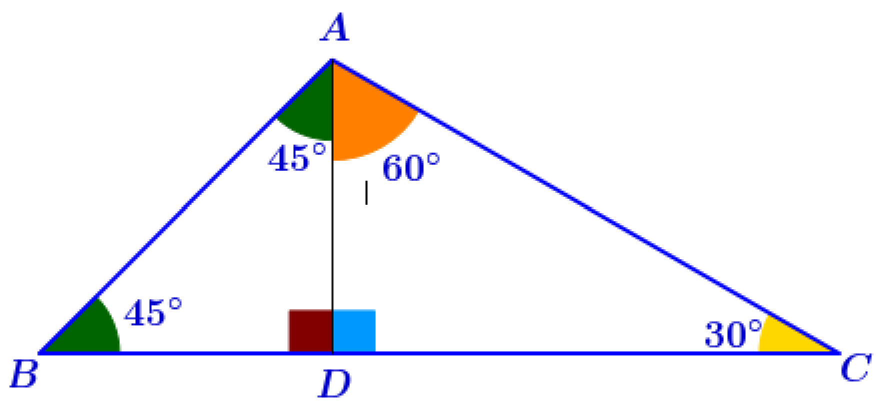
In any triangle of angles 30^0, 60^0, 90^0 *the sides are in the ratio* $1 : \sqrt{3} : 2$

Using these two kinds of triangles , we can compute the ratios of the sides of some non-right triangles also .

Activity 1



In the figure , < A = 105 ^{0} , < B = 45 ^{0} , < C = 30 ^{0}. *Draw AD perpendicular to BC .*



Since AD is perpendicular to BC , $\angle ADB = \angle ADC = 90^0$

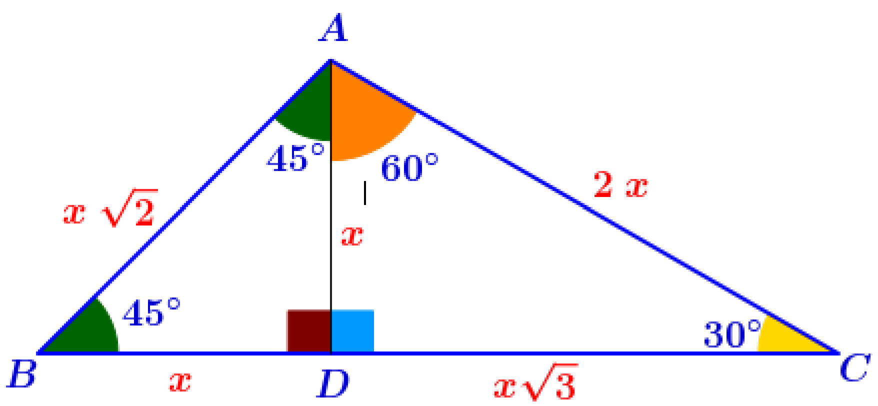
In triangle ADB , $\angle BAD = 180 - (90 + 45) = 180 - 135 = 45^0$

(*Sum of the angles of a triangle is* 180^0)

$\angle DAC = 105 - 45 = 60^0$ ($\angle BAC = 105^0$)

To calculate the ratio of the sides , take x as their common side .Then using the ratios seen earlier , we can write the lengths of the sides .

Take , $AD = x$,



In triangle ADB , $AD = BD = x$, $AB = x\sqrt{2}$

(*In any triangle of angles* 45^0, 45^0, 90^0 *the sides are in the ratio* $1 : 1 : \sqrt{2}$)

In triangle ADC , $AD = x$, $DC = x\sqrt{3}$, $AC = 2x$

(*In any triangle of angles* 30^0, 60^0, 90^0 *the sides are in the ratio* $1 : \sqrt{3} : 2$)

In triangle ABC , $AB = x\sqrt{2}$, $AC = 2x$

$$BC = x + x\sqrt{3} = x(1 + \sqrt{3})$$

The ratio of the sides opposite to the angles $30^0, 45^0, 105^0$ $= AB : AC : BC$

$$= x\sqrt{2} : 2x : x(1+\sqrt{3}) = \sqrt{2} : 2 : 1+\sqrt{3}$$

In any triangle of angles 30^0, 45^0, 105^0 *the sides are in the ratio* $\sqrt{2} : 2 : \sqrt{3}+1$

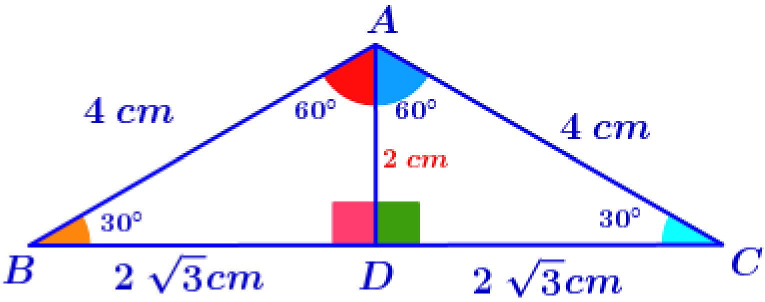
(1) *In the triangle shown , what is the perpendicular distance from the top vertex to the bottom side ? What is the area of the triangle ?*

Answer

In triangle ABC ,

$AB = AC = 4$ *cm*

$< BAC = 120^0$



$$< ABC =< ACB = \frac{180-120}{2} = \frac{60}{2} = 30°$$

(*In a triangle sides opposite to equal angles are equal*)

Draw AD perpendicular to BC

$< ADB = < ADC = 90^0$

BD = CD (In any isosceles triangle , the perpendicular from the point joining equal sides to the opposite side bisects the angle at this point and the side opposite)

$< BAD = < CAD = 60^0$

In triangle ADB , $AD : BD : AB = 1 : \sqrt{3} : 2$

(*In any triangle of angles* 30^0, 60^0, 90^0 *the sides are in the ratio* $1 : \sqrt{3} : 2$)

$$AD = 2\ cm \ , \ BD = 2\sqrt{3}\ cm \ , \ AB = 4\ cm$$

In triangle ADC , $AD : CD : AC = 1 : \sqrt{3} : 2$

$$AD = 2\ cm\ ,\ CD = 2\sqrt{3}\ cm\ ,\ AC = 4\ cm$$

In triangle ABC ,

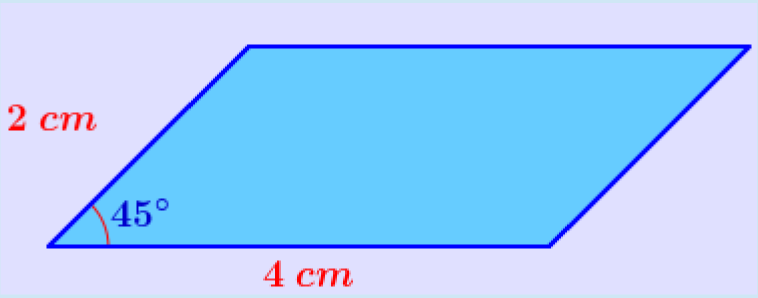
$$BC = 2\sqrt{3} + 2\sqrt{3} = 4\sqrt{3}\ cm$$

Perpendicular distance from the top vertex to the bottom side $= AD = 2\ cm$

Area of the triangle $= \frac{1}{2}\ BC \times AD$

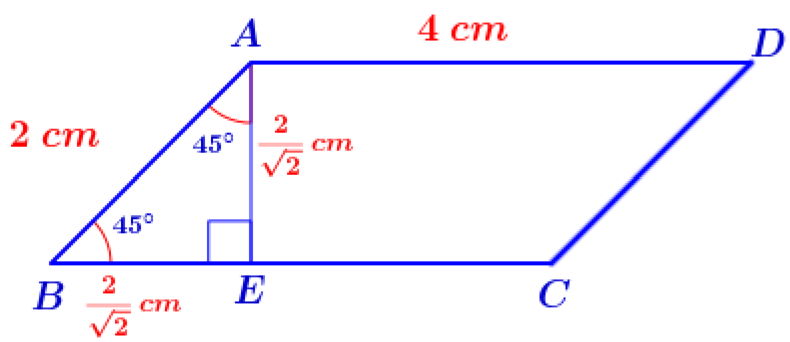
$$= \frac{1}{2}\ 4\sqrt{3} \times 2 = 4\sqrt{3}\ cm^2$$

(2) ***In the parallelogram shown , what is the distance between the top and bottom sides ?***



What is the area of the parallelogram ?

Answer



Draw AE perpendicular to BC .

$< E = 90^0$

$< B = < BAE = 45^0$

In triangle AEB , $BE : AE : AB = 1 : 1 : \sqrt{2}$

(In any triangle of angles 45^0, 45^0, 90^0 *the sides are in the ratio* $1 : 1 : \sqrt{2}$)

Distance between the top and bottom sides $= AE = \frac{2}{\sqrt{2}} = \sqrt{2}\ cm$

Area of the parallelogram $= BC \times AE$

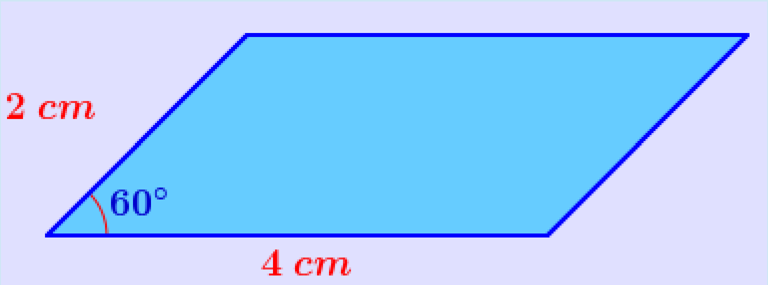
$$= 4 \times \sqrt{2} = 4\sqrt{2}\ cm^2$$

More activity

In the parallelogram shown ,

what is the distance between the top

and bottom sides ?

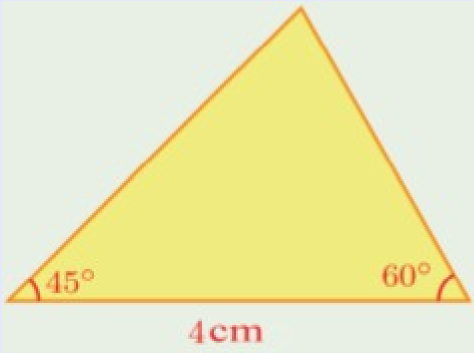


What is the area of the parallelogram ?

ONLINE MATHS CLASS - X – 53 (03 / 11 /2020)

5 . TRIGNOMETRY - Class 5

Calculate the area of the triangle shown



Answer

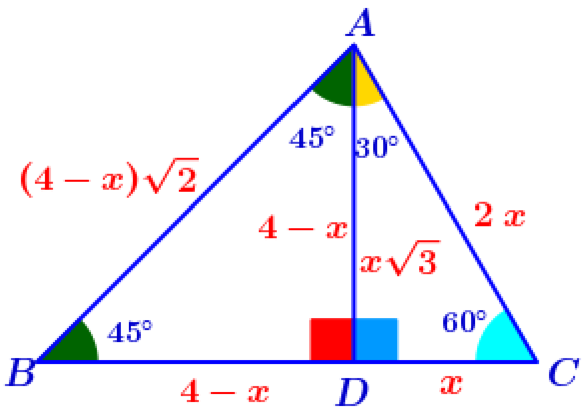
In triangle ABC $\angle B = 45^0$, $\angle C = 60^0$

BC = 4 cm

Draw AD perpendicular to BC .

$\angle ADB = \angle ADC = 90^0$

If $DC = x$, *then* $BD = 4 - x$



In triangle ADB ,

$$BD = AD = 4 - x \quad , \quad AB = (4-x)\sqrt{2}$$

(*In any triangle of angles* 45^0, 45^0, 90^0 *the sides are in the ratio* $1 : 1 : \sqrt{2}$)

In triangle ADC ,

$$DC = x \ , \ AD = x\sqrt{3} \ , \ AC = 2x$$

(*In any triangle of angles* 30^0, 60^0, 90^0 *the sides are in the ratio* $1 : \sqrt{3} : 2$)

Equating the values of AD from the triangles ADB , ADC , we get

$$4 - x = x\sqrt{3}$$

$$4 = x\sqrt{3} + x$$

$$x(\sqrt{3}+1) = 4$$

$$x = \frac{4}{\sqrt{3}+1}$$

$$AD = x\sqrt{3} = \frac{4}{\sqrt{3}+1} \times \sqrt{3} = \frac{4\sqrt{3}}{\sqrt{3}+1}$$

Area of triangle ABC $= \frac{1}{2}\, BC \times AD = \frac{1}{2} \times 4 \times \frac{4\sqrt{3}}{\sqrt{3}+1}$

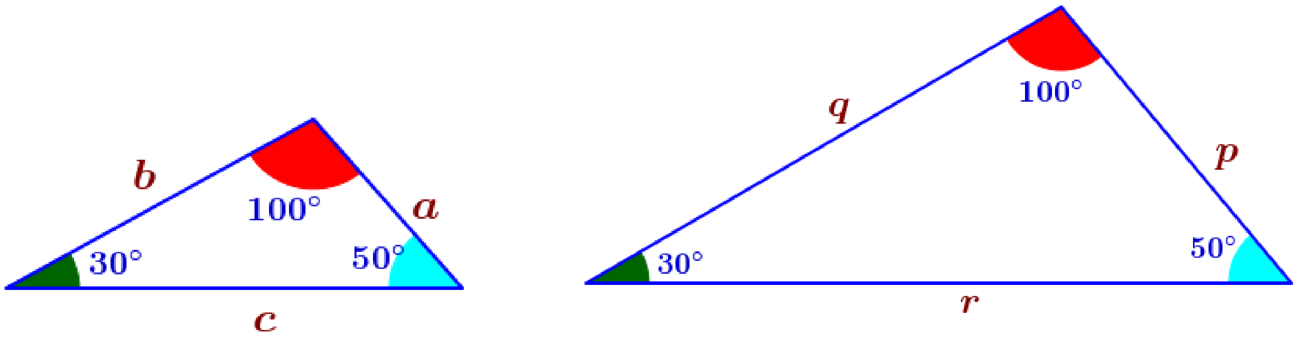
$$= \frac{8\sqrt{3}}{\sqrt{3}+1}\ cm^2$$

New measure of angles

We have calculated the ratios of the sides of some triangles of specific angles .

Do the angles of any triangle determine the ratio of its sides ? Let's see

Consider the following triangles



They have same angles . Let's write the sides of the small triangle as a, b, c *in increasing size and those of the larger as* p, q, r *. Then we have ,*

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$$

(The sides of triangle with the same angles , taken in the order of size , are in the same ratio)

Let $$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k$$

Then we get ,

$$\frac{a}{p} = k \quad ==> \quad a = kp$$

$$\frac{b}{q} = k \quad ==> \quad b = kq$$

$$\frac{c}{r} = k \quad ==> \quad c = kr$$

So ,

$$a : b : c = kp : kq : kr$$

$$= p : q : r$$

Finding

In triangles of the same angles drawn in different sizes , the lengths of the sides are different , but their ratios are same

Conclusion

The angles of a triangle determines the ratio of its sides

sine and cosine of angles

It has been found that , for a right triangle of one angle 40°, the side opposite to this angle is approximately 0.6428 times the hypotenuse and the other perpendicular side is approximately 0.7660 times the hypotenuse . These numbers have special names .

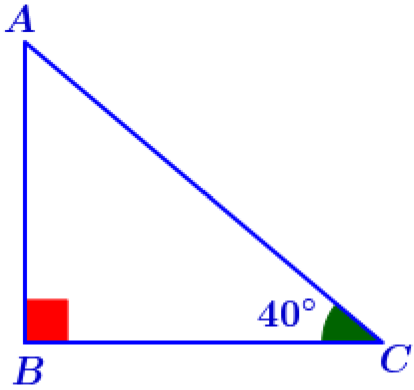
The number 0.6428 shows how much of the hypotenuse is the side opposite to the 40° angle . It is called sine of 40° and written sin 40°

$$\sin 40^\circ = \frac{\text{opposite side of } 40^\circ \text{ angle}}{\text{hypotenuse}}$$

That is ,

In triangle ABC , $\angle B = 90^0$, $\angle C = 40^0$, then

$$\sin 40^\circ = \frac{\text{side opposite to } 40^\circ \text{ angle}}{\text{hypotenuse}} = \frac{AB}{AC}$$



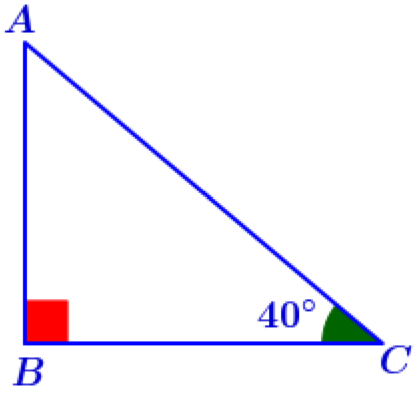
The number 0.7660 shows how much of the hypotenuse is the adjacent side to the 40° (the other side of the 40° angle) . It is called cosine of 40° and written cos 40°

$$\cos 40^\circ = \frac{\text{adjacent side of } 40^\circ \text{ angle}}{\text{hypotenuse}}$$

That is ,

In triangle ABC , < B = 90^0 , < C = 40^0 , then

$$\cos 40^\circ = \frac{\text{adjacent side of } 40^\circ \text{ angle}}{\text{hypotenuse}} = \frac{BC}{AC}$$



Like this we can find the sin and cos values of other angles too .

More activity

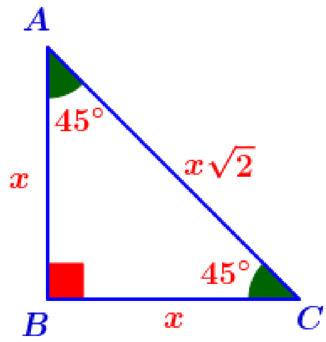
Find the sin and cos values of the following angles from the table given in the text book

0° , 30° , 45° , 60° , 90°

ONLINE MATHS CLASS - X – 54 (05 / 11 /2020)

5 . TRIGNOMETRY - Class 6

Activity 1



In triangle ABC , $\angle B = 90^0$, $\angle A = \angle C = 45^0$

$$AB : BC : AC = 1 : 1 : \sqrt{2}$$

(*In any triangle of angles* 45^0, 45^0, 90^0 *the sides are in the ratio* $1 : 1 : \sqrt{2}$)

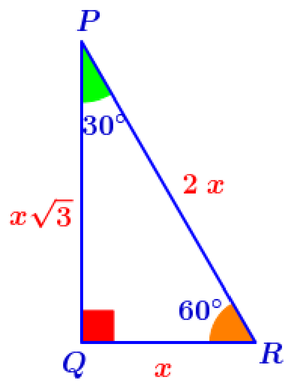
If $AB = x$, *then* $BC = x$, $AC = x\sqrt{2}$

$$\sin 45° = \frac{\text{opposite side of } 45° \text{ angle}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{\text{adjacent side of } 45° \text{ angle}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\sin 45° = \frac{1}{\sqrt{2}}$	$\cos 45° = \frac{1}{\sqrt{2}}$

Activity 2



In triangle PQR , $\angle Q = 90^0$, $\angle P = 30^0$, $\angle R = 60^0$

$$QR : PQ : PR = 1 : \sqrt{3} : 2$$

(*In any triangle of angles* 30^0, 60^0, 90^0 *the sides are in the ratio* $1 : \sqrt{3} : 2$)

If $QR = x$, *then* $PQ = x\sqrt{3}$, $PR = 2x$

$$\sin 30^\circ = \frac{\text{opposite side of } 30^\circ \text{ angle}}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{x}{2x} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{adjacent side of } 30^\circ \text{ angle}}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$$

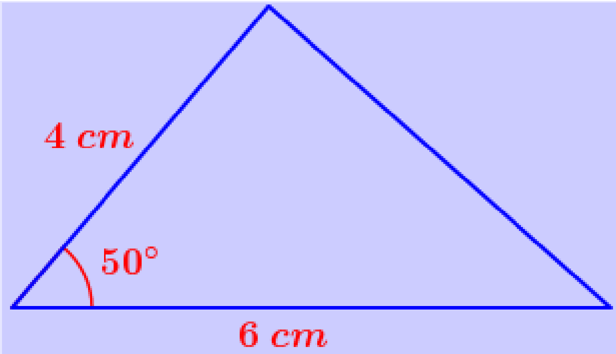
$$\sin 60^\circ = \frac{\text{opposite side of } 60^\circ \text{ angle}}{\text{hypotenuse}} = \frac{PQ}{PR} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{\text{adjacent side of } 60^\circ \text{ angle}}{\text{hypotenuse}} = \frac{QR}{PR} = \frac{x}{2x} = \frac{1}{2}$$

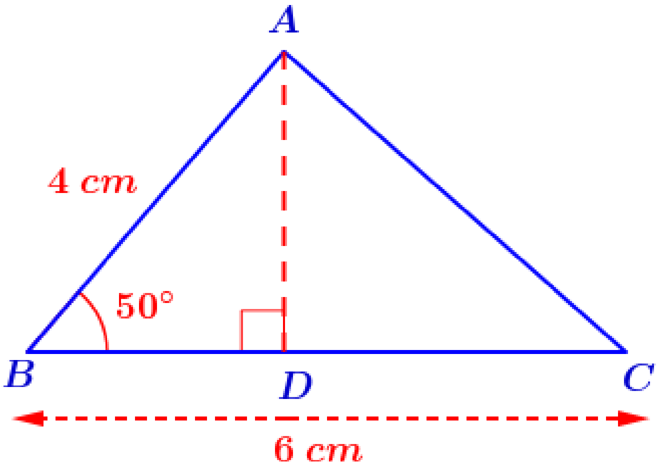
$sin\ 30^\circ = \frac{1}{2}$	$cos\ 30^\circ = \frac{\sqrt{3}}{2}$
$sin\ 60^\circ = \frac{\sqrt{3}}{2}$	$cos\ 60^\circ = \frac{1}{2}$

$Angle$	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$

(1) *Calculate the area of the triangle shown in the figure*



Answer



Draw AD perpendicular to BC .

Area of triangle ABC $= \frac{1}{2} BC \times AD$

In triangle ADB ,

$$\sin 50^\circ = \frac{\text{opposite side of } 50^\circ \text{ angle}}{\text{hypotenuse}} = \frac{AD}{AB}$$

$$\sin 50^\circ = \frac{AD}{4}$$

$$4 \times \sin 50^\circ = AD$$

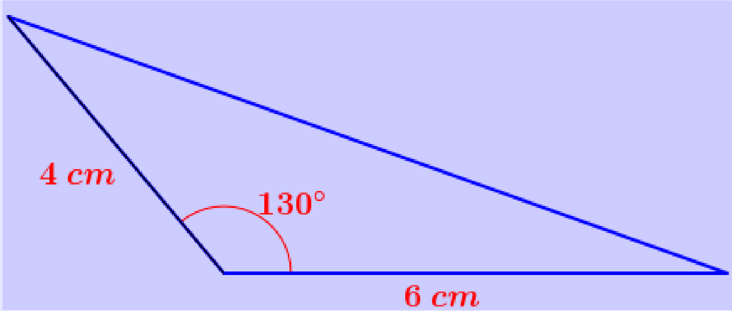
$AD = 4 \times 0.7660$ സെ.മി

Area of triangle ABC $= \frac{1}{2} BC \times AD = \frac{1}{2} \times 6 \times 4 \times 0.7660$

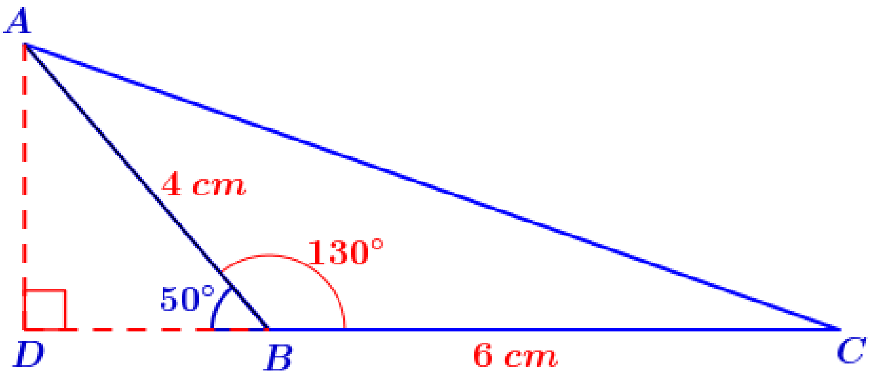
$= 9.192 \ cm^2$

SARATH .A .S , HST ,GHS ANCHACHAVADI

(2) *Calculate the area of the triangle shown in the figure .*



Answer



AD is the perpendicular drawn from A to the side BC .

< ABD = 180 – 130 = 50 ^{0}

Area of triangle ABC $= \frac{1}{2} BC \times AD$

In triangle ADB ,

$$sin\ 50° = \frac{\text{opposite side of } 50^{0} \text{ angle}}{\text{hypotenuse}} = \frac{AD}{AB}$$

$$sin\ 50° = \frac{AD}{4}$$

$$4 \times sin\ 50° = AD$$

$$AD = 4 \times 0.7660 \text{ സെ.മി}$$

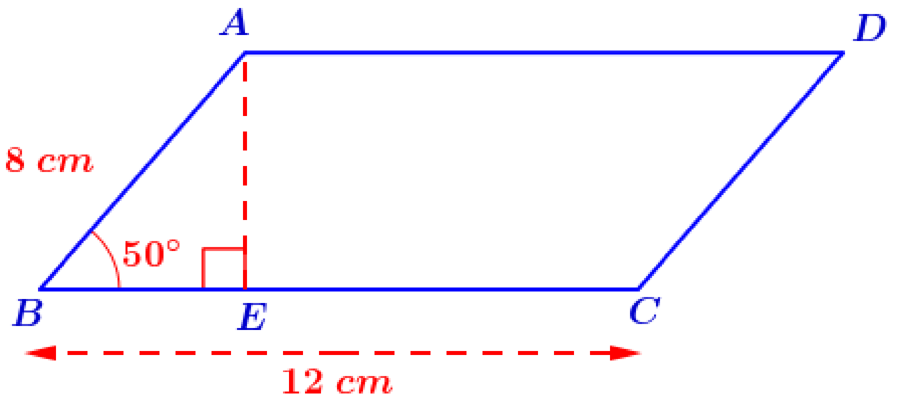
Area of triangle ABC $= \frac{1}{2} BC \times AD = \frac{1}{2} \times 6 \times 4 \times 0.7660$

$= 9.192\ cm^2$

(3)*The sides of a parallelogram are 8 cm and 12 cm and the angle between them is* 50^0 .

Calculate its area .

Answer



Draw AE perpendicular to BC .

Area of the parallelogram $= BC \times AE$

In triangle AEB ,

$$\sin 50^\circ = \frac{\text{opposite side of } 50^0 \text{ angle}}{\text{hypotenuse}} = \frac{AE}{AB}$$

$$\sin 50^\circ = \frac{AE}{8}$$

$$8 \times \sin 50^\circ = AE$$

$$AE = 8 \times 0.7660 \text{ സെ.മീ}$$

Area of the parallelogram $= BC \times AE = 12 \times 8 \times 0.7660$

$$= 73.536 \ cm^2$$

More activity

Angles of 50^0 *and* 60^0 *are drawn at the ends of a 5 cm long line , to make a triangle .*

Calculate its area .

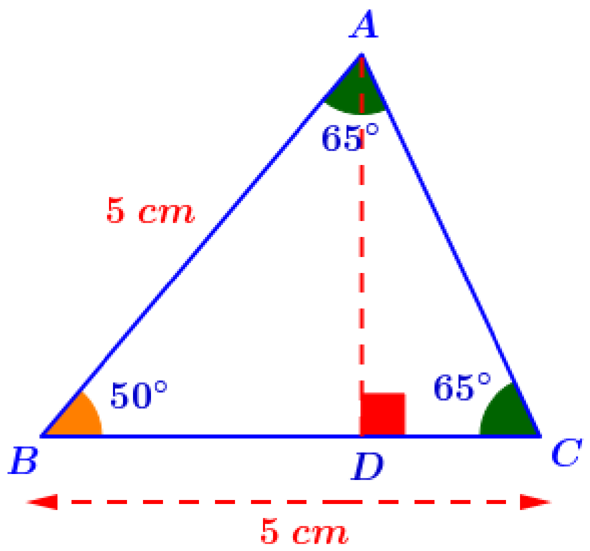
ONLINE MATHS CLASS - X – 55 (06 / 11 /2020)

5 . TRIGNOMETRY - Class 7

(1) *Angles of* 50° and 65° *are drawn at the ends of a 5 cm long line , to make a triangle .*

Calculate its area .

Answer



In triangle ABC , < B = 50°, < C = 65°

BC = 5 cm

< BAC = 180 - (50 + 65) = 180 – 115 = 65°

(Sum of the angles of a triangle is 180°)

BC = AB = 5 cm (< BAC = < C = 65°,

Sides opposite to equal angles of a triangle are equal)

Draw AD perpendicular to BC .

Area of triangle ABC $= \frac{1}{2} BC \times AD$

In right triangle ADB ,

$$\sin 50° = \frac{\text{opposite side of } 50° \text{ angle}}{\text{hypotenuse}} = \frac{AD}{AB}$$

$$\sin 50° = \frac{AD}{5}$$

$$5 \times \sin 50° = AD$$

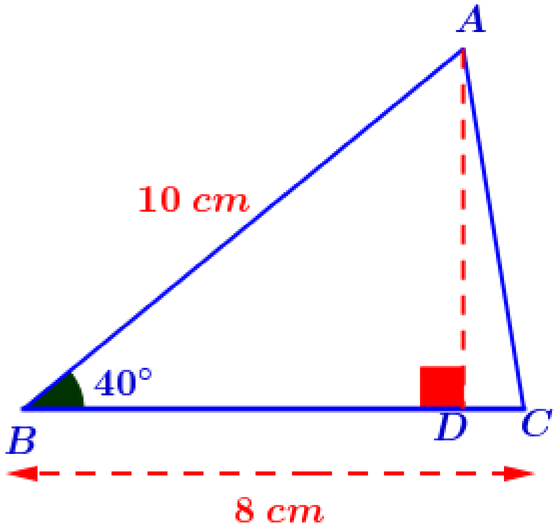
$$AD = 5 \times 0.7660 \ cm$$

Area of triangle ABC $= \frac{1}{2} BC \times AD = \frac{1}{2} \times 5 \times 5 \times 0.7660$

$$= 9.575 \ cm^2$$

(2) *The length of two sides of a triangle are 8 cm and 10 cm and the angle between them is 40^0 . Calculate its area*

What is the area of the triangle with sides of the same length , but angle between them 140^0 ?



Answer

a)

In triangle ABC , AB = 10 cm ,

BC = 8 cm and *< B = 40^0*

Draw AD perpendicular to BC .

Area of triangle ABC $= \frac{1}{2} BC \times AD$

In right triangle ADB ,

$$\sin 40° = \frac{\text{opposite side of } 40^0 \text{ angle}}{\text{hypotenuse}} = \frac{AD}{AB}$$

$$\sin 40° = \frac{AD}{10}$$

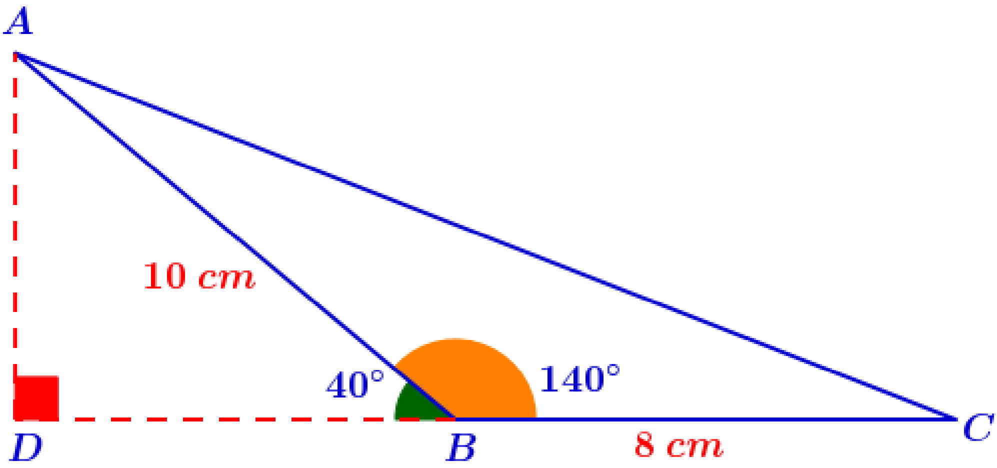
$$10 \times \sin 40° = AD$$

$$AD = 10 \times 0.6428 \ cm$$

Area of triangle ABC $= \frac{1}{2} BC \times AD = \frac{1}{2} \times 8 \times 10 \times 0.6428$

$$= 25.712 \ cm^2$$

b)



In triangle ABC , AB = 10 cm , BC = 8 cm , < ABC = 140 ⁰

AD is the perpendicular drawn from the vertex A to the side BC .

Area of triangle ABC $= \frac{1}{2} BC \times AD$

In right triangle ADB ,

$$\sin 40° = \frac{\text{opposite side of } 40^0 \text{ angle}}{\text{hypotenuse}} = \frac{AD}{AB}$$

$$\sin 40° = \frac{AD}{10}$$

$$10 \times \sin 40° = AD$$

$$AD = 10 \times 0.6428 \ cm$$

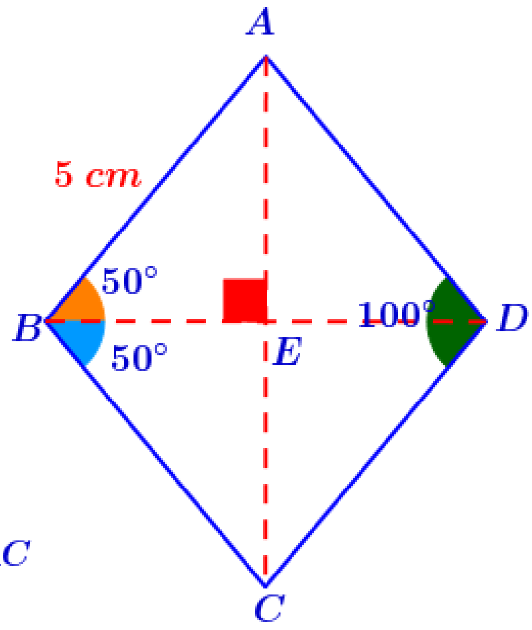
Area of triangle ABC $= \frac{1}{2} BC \times AD = \frac{1}{2} \times 8 \times 10 \times 0.6428$

$$= 25.712 \ cm^2$$

For any two triangles , if the two sides are equal and angles between them are supplementary , then their areas are equal

(3) *The sides of a rhombus are 5 cm long and one of its angles is* 100^0 *. Compute its area*

Answer



In rhombus ABCD , AB = 5 cm , < ABC = 100^0

The diagonals of the rhombus intersect at E .

< AEB = 90 (Diagonals of a rhombus bisect each other at right angles)

< ABE = < CBE = 50^0 *(Diagonals of a rhombus bisect its angles)*

Area of the rhombus $= \frac{1}{2} BD \times AC$

In right triangle AEB ,

$$\sin 50° = \frac{\text{opposite side of } 50^0 \text{ angle}}{\text{hypotenuse}} = \frac{AE}{AB}$$

$$\sin 50° = \frac{AE}{5}$$

$$5 \times sin\ 50° = AE$$

$$AE = 5 \times 0.7660\ cm$$

$$AC = 2 \times AE = 2 \times 5 \times 0.7660 = 7.660\ cm$$

(AE = CE Diagonals of a rhombus bisect each other at right angles)

$$\cos 50° = \frac{\text{adjacent side of } 50^0 \text{ angle}}{\text{hypotenuse}} = \frac{BE}{AB}$$

$$\cos 50° = \frac{BE}{5}$$

$$5 \times \cos 50° = BE$$

$$BE = 5 \times 0.6428\ cm$$

$$BD = 2 \times BE = 2 \times 5 \times 0.6428 = 6.428\ cm$$

Area of the rhombus $= \frac{1}{2} BD \times AC$

$$= \frac{1}{2} \times 6.428 \times 7.660 = 24.62\ cm^2$$

More activity

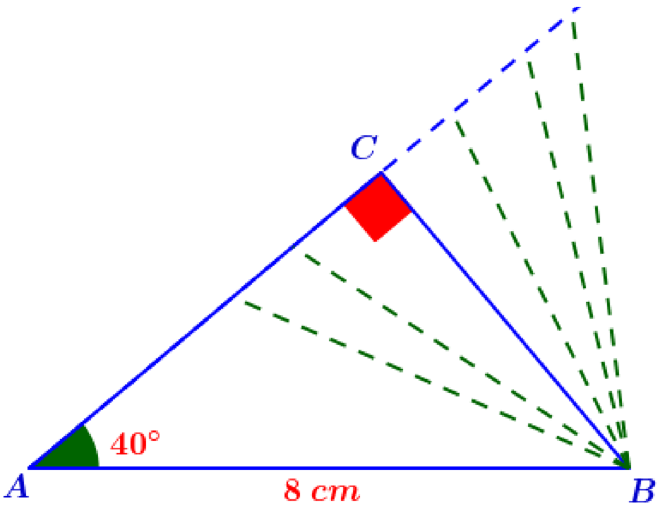
A triangle is to be drawn with one side 8 *cm and an angle on it is* 40^0 . *What should be the minimum length of the side opposite this angle* ?

ONLINE MATHS CLASS - X – 56 (09 / 11 /2020)

5 . TRIGNOMETRY - Class 8

A triangle is to be drawn with one side 8 cm and an angle on it is 40^0 . What should be the minimum length of the side opposite this angle ?

Answer



We can draw so many triangles with these measures as shown in the figure . Among these triangles , the minimum length of the side opposite to 40° is the perpendicular distance from B to its opposite side .

In triangle ABC , AB = 8 cm , < A = 40° , < C = 90°

$$\sin 40° = \frac{\text{opposite side of } 40^0 \text{ angle}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\sin 40° = \frac{BC}{8}$$

$$8 \times \sin 40° = BC$$

$$BC = 8 \times 0.6428 = 5.1424 \ cm$$

Length of an arc

The length of an arc of a circle can be computed from its central angle .

The length of an arc of a circle is that fraction of the perimeter as the fraction of 360° that its central angle is .

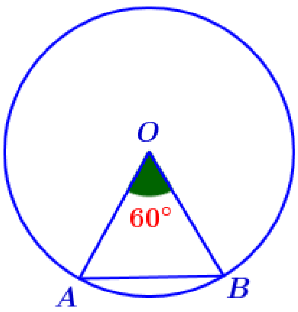
In a circle of radius r , the length of an arc of central angle x^0

$$= 2\pi r \times \frac{x}{360}$$

Length of a chord

Length of a chord of central angle 60^0

In the figure , chord AB makes an angle 60^0 at the centre of the circle and O is the centre .

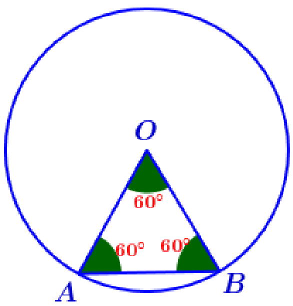


OA = OB (Radii of a circle are equal)

$$< OAB = < OBA = \frac{180 - 60}{2} = \frac{120}{2} = 60^\circ$$

(The sides opposite to equal angles of a triangle are equal)

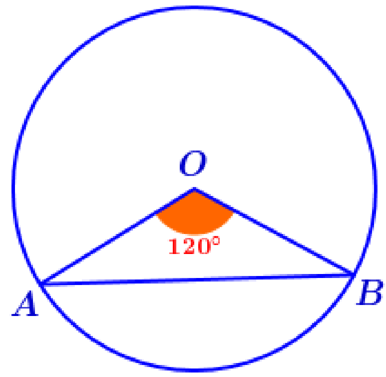
Since all the angles of the triangle ABC are equal , it is an equilateral triangle . That is , AB = OA = OB



The length of a chord of a circle of central angle 60° is equal to the radius .

Length of a chord of central angle 120^0

In the figure , chord AB makes an angle 120^0 at the centre of the circle and O is the centre .

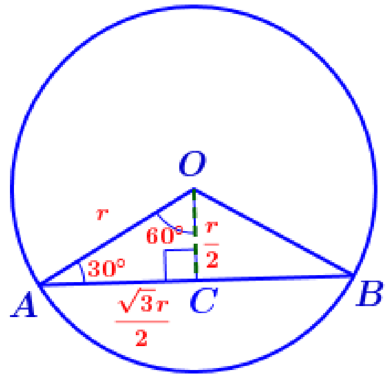


Draw OC perpendicular to AB .

$$< AOC = < BOC = \frac{120}{2} = 60°$$

AC = BC

(In any isosceles triangle the perpendicular from the point joining equal sides to the opposite side bisects the angle at this point and the opposite side) .



In right triangle OCA ,

If OA = r , then $OC = \frac{r}{2}$, $AC = \sqrt{3} \times \frac{r}{2}$

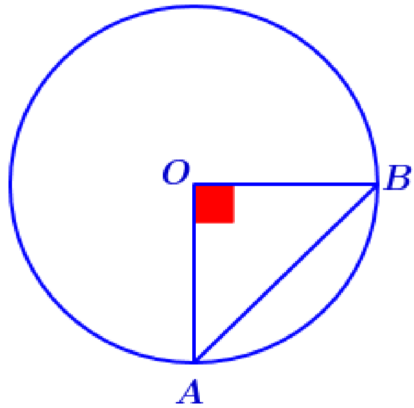
(In any triangle of angles 30^0, 60^0 , 90^0 the sides are in the ratio $1 : \sqrt{3} : 2$)

Length of the chord AB $= 2\,AC = 2 \times \sqrt{3} \times \frac{r}{2} = \sqrt{3}\,r$

The length of a chord of a circle of central angle 120° is $\sqrt{3}$ times the radius .

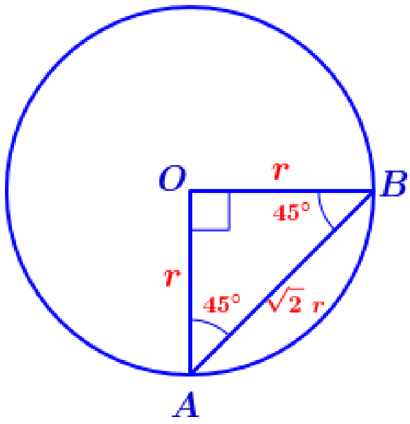
Length of a chord of central angle 90°

In the figure , chord AB makes an angle 90^0 at the centre of the circle and O is the centre .



OA = OB (Radii of a circle are equal)

$$\angle OAB = \angle OBA = \frac{180 - 90}{2} = \frac{90}{2} = 45°$$



(The sides opposite to equal angles of a triangle are equal)

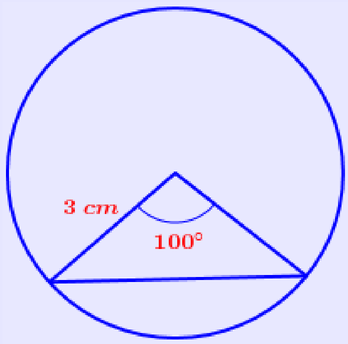
If OA = OB = r ,

$$AB = \sqrt{2}\, r$$

(In any triangle of angles 45^0, 45^0, 90^0 the sides are in the ratio $1 : 1 : \sqrt{2}$)

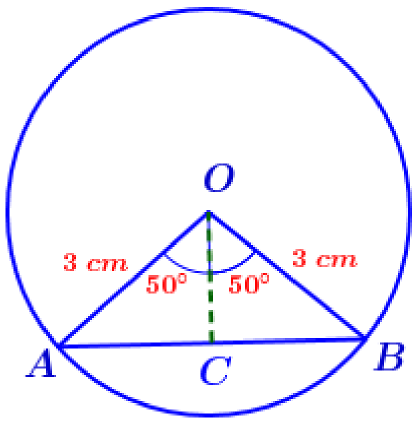
The length of a chord of a circle of central angle $90°$ is $\sqrt{2}$ times the radius .

What is the length of the chord shown in the picture ?



In the figure , chord AB makes an angle 100^0 at the centre of the circle and O is the centre .

Draw OC perpendicular to AB .



$$\angle AOC = \angle BOC = \frac{100}{2} = 50°$$

AC = BC

(In any isosceles triangle the perpendicular from the point joining equal sides to the opposite side bisects the angle at this point and the opposite side)

In right triangle OCA ,

$$\sin 50° = \frac{\text{opposite side of } 50^0 \text{ angle}}{\text{hypotenuse}} = \frac{AC}{OA}$$

$$\sin 50° = \frac{AC}{3}$$

$$3 \times sin50° = AC$$

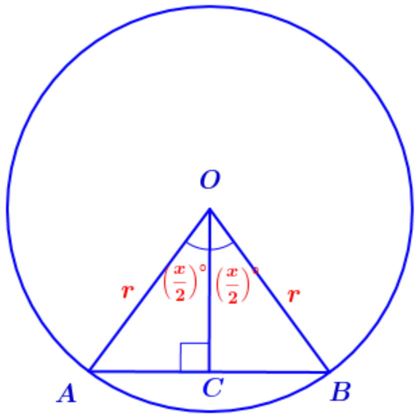
$$AC = 3 \times 0.7660\ cm$$

Length of the chord AB $= 2 \times AC = 2 \times 3 \times 0.7660 = 4.596\ cm$

Length of a chord of central angle $x°$

In the figure , chord AB makes an angle 60^0 at the centre of the circle and O is the centre .

Draw OC perpendicular to AB .



$$< AOC = < BOC = \left(\frac{x}{2}\right)^{\circ}$$

AC = BC

(In any isosceles triangle the perpendicular from the point joining equal sides to the opposite side bisects the angle at this point and the opposite side)

In right triangle OCA , ,

$$sin\left(\frac{x}{2}\right)^{\circ} = \frac{\text{opposite side of } \left(\frac{x}{2}\right)^{\circ} \text{ angle}}{\text{hypotenuse}} = \frac{AC}{OA}$$

$$\sin\left(\frac{x}{2}\right)^\circ = \frac{AC}{r}$$

$$r \times \sin\left(\frac{x}{2}\right)^\circ = AC$$

Length of the chord AB $= 2\,AC = 2 \times r \times \sin\left(\frac{x}{2}\right)^\circ$

$$AB = 2r \times \sin\left(\frac{x}{2}\right)^\circ$$

In a circle , the length of any chord is double the product of the radius and sin of the half the central angle .

More activity

Raju and Babu are standing at the starting point A of a circular track of radius 20 metres . Raju walks through the arc AB and Babu walks through the chord AB to reach B . If the central angle of the arc is 160 °, how much distance did Raju walk more than Babu ?