MENSURATION, TRIGONOMETRY, HEIGHT AND DISTANCE Pre-Class Notes

Mensuration

It is the branch of mathematics which deals with the study of Geometric shapes , Their area , Volume and different parameters in geometric objects.

Some important mensuration formulas are:

a. Area of rectangle (A) = length(l) * Breath(b). A = l×b
b. Perimeter of a rectangle (P) = 2 * (Length(l) + Breath(b)).P= 2 × (l×b)
c. Area of a square (A) = Length (l) * Length (l).A= l^2
d. Perimeter of a square (P) = 4 * Length (l). p = 4l
e. Area of a parallelogram(A) = Length(l) * Height(h).A= l×h
f. Perimeter of a parallelogram (P) = 2 * (length(l) + Breadth(b). P= 2×(l×b)
g. Area of a triangle (A) = (Base(b) * Height(b)) / 2. A= ½ (b× h)
   And for a triangle with sides measuring "a" , "b" and "c" , Perimeter = a+b+c
   And s = semi perimeter = perimeter / 2 = (a+b+c)/2
   Also.Area of triangle = A $\sqrt{s(s-a)(s-b)(s-c)}$
   This formulas is also knows as **"Heroe's formula".**
h. Area of isosceles triangle =, $\frac{b}{4}\sqrt{4a^2 - b^2}$Where , a = length of two equal side , b= length of base of isosceles triangle.
i. Area of trapezium (A) = ½ (a +b),Where , "a" and "b" are the length of parallel sides.
j. Perimeter of a trapezium (P) = sum of all sides
k. Area f rhombus (A) = Product of diagonals / 2
l. Perimeter of a rhombus (P) = 4 * l, where l = length of a side
m. Area of quadrilateral (A) = 1/2 * Diagonal * (Sum of offsets)
n. Area of a Kite (A) = 1/2 * product of it's diagonals
o. Perimeter of a Kite (A) = 2 * Sum on non-adjacent sides
p. Area of a Circle (A) =∏r^2, Where , r= radius of the circle
q. Circumference of a Circle =2∏r, r= radius of circle,
r. Total surface area of cuboids =, 2(lb ×bh× lh)where , l= length , b=breadth , h=height
s. Total surface area of, cuboids =6l^2, where , l= length
t. Length of diagonal of cuboids = $\sqrt{l^2 + b^2 + h^2}$
u. Length of diagonal of cube = $\sqrt{3l}$
v. Volume of cuboids = l * b * h
w. Volume of cube = $(L)^3$, l= side of cube
x. Area of base of a cone = ∏r^2
y. Curved surface area of a cone =∏rl, Where, r = radius of base, l = slanting height of cone
z. Total surface area of a cone = ∏r(r + l)
aa. Volume of right circular cone =, 1/3 ∏r^2 h, Where , r = radius of base of cone , h= height of the cone (perpendicular to base)
bb. Surface area of triangular prism = (P * height) + (2 * area of triangle), Where, p = perimeter of base

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cc. Surface area of polygonal prism = (Perimeter of base * height) + (Area of polygonal base *2)

dd. Lateral surface area of prism = Perimeter of base * height

ee. Volume of Triangular prism = Area of the triangular base * height

ff. Curved surface area of a cylinder = $2\pi rh$, Where , r = radius of base , h = height of cylinder

gg. Total surface area of a cylinder = $2\prod r(r + h)$

hh. Volume of a cylinder = $\prod r h^2$

ii. Surface area of sphere =,$4\prod r^2 = \prod d^2$ where , r= radius of sphere , d= diameter of sphere

jj. Volume of a sphere = $\frac{4}{3}\prod r^3$

kk. Volume of hollow cylinder = $\prod rh (R^2 – r^2)$,where , R = radius of cylinder , r= radius of hollow , h = height of cylinder

ll. Surface area of a right square pyramid = $a\sqrt{4b^2 - a^2}$, Where , a = length of base , b= length of equal side ; of the isosceles triangle forming the slanting face.

mm. Area of a regular hexagon = $\frac{3\sqrt{3}a^2}{2}$ area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

nn. Curved surface area of a Frustums = $\pi h(r_1 + r_2)$

oo. Total surface area of a Frustums = $\pi(r_1^2 + h(r_1 + r_2) + r_2^2)$

pp. Curved surface area of a Hemisphere = $2\pi r^2$

qq. Total surface area of a Hemisphere = $3\pi r^2$

rr. Volume of a Hemisphere = $\frac{2}{3}\pi r^3 = \frac{1}{12}\pi d^3$

ss. Area of sector of a circle = $\frac{\theta r^2 \pi}{360}$ where , θ= measure of angle of the sector , r= radius of the sector

Trigonometry:

Trigonometry is the study of the relationship between the angle & the side of a triangle.

Angle Measure:-

Angle are measured in many units, i.e degree, minute, seconds, radians. We have 1degree = 60 minutes, 1 minute = 60 seconds, $\prod$ radians = 180º

Trigonometric measures of certain angle:-

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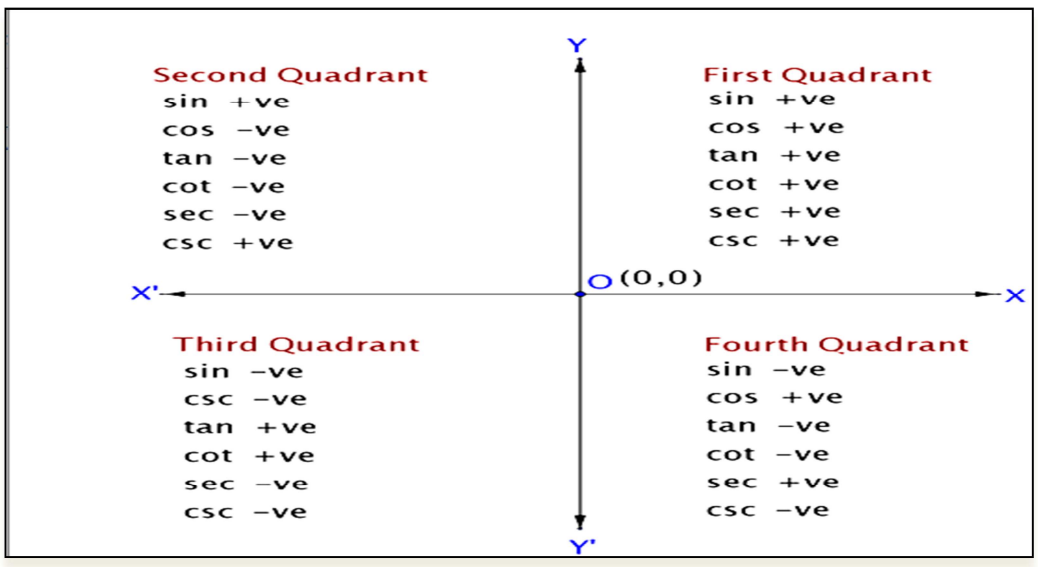
A	sin A	cos A	tan A	cot A	sec A	cosec A
0°	0	1	0	∞	1	∞
30 °	1 / 2	$\sqrt{3}$ / 2	1 / $\sqrt{3}$	$\sqrt{3}$	2 / $\sqrt{3}$	2
45°	$\sqrt{2}$ / 2	$\sqrt{2}$ / 2	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\sqrt{3}$ / 2	1 / 2	$\sqrt{3}$	1 / $\sqrt{3}$	2	2 / $\sqrt{3}$
90°	1	0	∞	0	∞	1

Signs of Trigonometric ratios:-



Sine and Cosine Laws in Triangles:-

In any triangle we have:
The sine law : sin A / a = sin B / b = sin C / c

The cosine laws
$a^2 = b^2 + c^2 - 2\,b\,c \cos A$
$b^2 = a^2 + c^2 - 2\,a\,c \cos B$
$c^2 = a^2 + b^2 - 2\,a\,b \cos C$

Pythagorean Identities

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a. $\sin^2 X + \cos^2 X = 1$
a. $1 + \tan^2 X = \sec^2 X$
b. $1 + \cot^2 X = \csc^2 X$

Addition Formulas:-

a. cos(X + Y) = cosXcosY - sinXsinY
b. cos(X - Y) = cosXcosY + sinXsinY
c. sin(X + Y) = sinXcosY + cosXsinY
d. sin(X - Y) = sinXcosY - cosXsinY
e. tan(X + Y) = [tanX + tanY] / [1 - tanXtanY]
f. tan(X - Y) = [tanX - tanY] / [1 + tanXtanY]
g. cot(X + Y) = [cotXcotY - 1] / [cotX + cotY]
h. cot(X - Y) = [cotXcotY + 1] / [cotX - cotY]

Sum to Product Formulas:-

a. cosX + cosY = 2cos[(X + Y) / 2] cos[(X - Y) / 2]
b. sinX + sinY = 2sin[(X + Y) / 2] cos[(X - Y) / 2]

Difference to Product Formulas

a. cosX - cosY = - 2sin[(X + Y) / 2] sin[(X - Y) / 2]
b. sinX - sinY = 2cos[(X + Y) / 2] sin[(X - Y) / 2]

Product to Sum/Difference Formulas

a. cosXcosY = (1/2) [cos (X - Y) + cos (X + Y)]
b. sinXcosY = (1/2) [sin (X + Y) + sin (X - Y)]
c. cosXsinY = (1/2) [sin (X + Y) - sin[(X - Y)]
d. sinXsinY = (1/2) [cos (X - Y) - cos (X + Y)]

Difference of Squares Formulas

a. $\sin^2 X - \sin^2 Y = \sin(X + Y)\sin(X - Y)$
b. $\cos^2 X - \cos^2 Y = -\sin(X + Y)\sin(X - Y)$
c. $\cos^2 X - \sin^2 Y = \cos(X + Y)\cos(X - Y)$

Double Angle Formulas

a. sin(2X) = 2 sinXcosX
b. $\cos(2X) = 1 - 2\sin^2 X = 2\cos^2 X - 1$
c. $\tan(2X) = 2\tan X / [\ 1 - \tan^2 X\ ]$

Multiple Angle Formulas

a. $\sin(3X) = 3\sin X - 4\sin^3 X$
b. $\cos(3X) = 4\cos^3 X - 3\cos X$
c. $\tan(3X) = \frac{3\tan X - \tan^3 X}{1 - 3\tan^2 X}$

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More half-angle formulas

$$\sin t = \frac{2\tan t/2}{1+\tan^2 t/2} \qquad \cos t = \frac{1-\tan^2 t/2}{1+\tan^2 t/2} \qquad \tan t = \frac{2\tan t/2}{1-\tan^2 t/2}$$

Identities expressing trig functions in terms of their complements

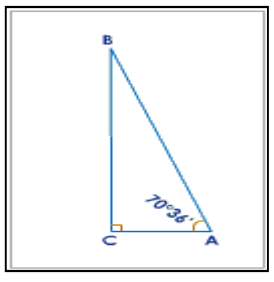
a. $\cos t = \sin(\pi/2 - t)$ $\sin t = \cos(\pi/2 - t)$
b. $\cot t = \tan(\pi/2 - t)$ $\tan t = \cot(\pi/2 - t)$
c. $\csc t = \sec(\pi/2 - t)$ $\sec t = \csc(\pi/2 - t)$

Height & Distance:-

Angle of elevation is the angle between a horizontal line and the line joining the observer's eye to some object above the horizontal line.

Angle of depression is the angle between a horizontal line and the line joining the observer's eye to some object beneath the horizontal line.

Eg- At a point 50 m from the foot of a building, the angle of elevation of the top of the building is $70^o\ 36'$. Find the height of the building.



BC is the building.

A is the observer.

AC is its distance of the building from the observer.

$\angle A$ is the angle of elevation.

$$\tan A = \frac{BC}{AC}$$

$$\therefore \quad \frac{BC}{50} = \tan 70^o\ 36'$$

$\therefore$ BC = 50 x tan $70^o\ 36'$ = 50 x 2.840 = 142.0 m

$\therefore$ BC = 142.0 m