Time, Speed, Distance and Work, Pre-Class Notes

Time, speed, and distance are related by the formula: distance = speed x time. Therefore, if any two of the three quantities are known, the third can be found

a. If the ratio of speed is a:b:c, then the ratio of time taken is $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$
b. Since S ά $\frac{1}{T}$, speed is doubled, time taken to cover a distance would be half
c. Also S ά D, If speed is doubled, distance covered in given time will also doubled.

Relative Speed:

If Two bodies are moving (in the same direction or in opposite direction), then the speed of one body with respect to another is called its relative speed.

In Trains:

a. If two trains are moving in the same direction with speeds U & V respectively then relative speed = $|U - V|$
b. If two trains are moving in the opposite direction with speeds U & V respectively then relative speed = $|U + V|$
c. If two trains start from same A & B with speed U & V respectively, & after crossing each other take a & b hours to reach B & A respectively, then V = $\sqrt{\frac{b}{a}}$

In Boats & Streams:-

Downstream motion of a boat is its motion in the same direction as the flow of the river.
Upstream motion of a boat is its motion apposite to the direction of flow of the river.

If
b = speed of boat in still water
w= Speed of stream or water
u = speed of boat upstream
d = speed of boat downstream
Then

a. u = b-w
b. d= b + w
c. b= ½ (u +d)
d. w= ½ (d-u)

Results:

a. If a certain distance D, from point X to point Y is traveled at 'a' km/hr and the same distance is covered i.e. from Y to X at 'b' km/hr, then the average speed during the whole journey is $\frac{2ab}{a+b}$
b. While a certain distance 'D' is covered, if A man changes his speed in the ratio X : Y, then the ratio of time taken becomes Y : X.

c. If an object travels a distance D from X to Y with Speed 'a' in time T1 and travels back from Y to X with $\frac{m}{n}$ of the usual speed 'a', then the change in time taken to travel the same distance can be calculated as:
   Change in time = $(\frac{n}{m} - 1) \times T1$; for n > m
   Also Change in Time = $(1 - \frac{n}{m}) \times T1$; for m > n

d. IF two objects, say A and B, start at the same time in opposite directions from two different points (P and Q) and arrive at the opposite points in 'a' and 'b' hours respectively after having met
   Then the ratio of their speed is: $\frac{\text{A's speed}}{B's\ Speed} = \frac{\sqrt{b}}{\sqrt{a}}$

e. If one travel at x times his speed & is T min Late or early, then the usual time taken will be $\frac{x \times T}{X-1}$

f. Two person moving at speed U & V, then the time taken, for them to meet, if distance separating them is D will be $\frac{D}{u-v}$

g. Time taken By a train to cross a pole or a man or another train or bridge will be = $\frac{Sum\ Lengths}{u \pm v}$ (+ if opposite direction, - if same direction)

h. If T1 and T2 are the times taken to travel from X to Y and from Y to X respectively, the distance D from X to Y can be calculated as:
   i. D = (T1 + T2)$[\frac{ab}{a+b}]$km
   ii. D= (T1 – T2)$[\frac{ab}{a-b}]$km
   iii. (a - b)$[\frac{T1 \times T2}{T1 - T2}]$km

Work:

Work" problems involve situations such as two people working together to paint a house. You are usually told how long each person takes to paint a similarly-sized house, and you are asked how long it will take the two of them to paint the house when they work together

Eg- Suppose one painter can paint the entire house in twelve hours, and the second painter takes eight hours. How long would it take the two painters together to paint the house?

Sol. If the first painter can do the entire job in twelve hours and the second painter can do it in eight hours, then the first guy can do $^1/_{12}$ of the job *per hour*, and the second guy can do $^1/_8$*per hour*.

To find out how much they can do together *per hour*, add together what they can do individually *per hour*: $^1/_{12} + {}^1/_8 = {}^5/_{24}$. They can do $^5/_{24}$ of the job *per hour*. Now let "*t*" stand for how long they take to do the job together. Then they can do $^1/_t$ *per hour*, so $^5/_{24} = {}^1/_t$. Flip the equation, and get that $t = {}^{24}/_5$ = 4.8 hours.

Concepts:

a. If A can do a piece of work in 10 days, then in 1 day A will do 1/ 10th of the total work.
b. If A is thrice as good as B, then
   i. In a given amount of time, A will be able to do 3 times the work B will do
      Ratio of work done by A &B(In the same time) = 3:1
   ii. For the same amount of work, B will take thrice the time as A will.

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Ratio of time taken by A & B (same work done)= 1:3

c. Efficiency is directly proportional to work done & inversely proportional to time taken.

Pipes & Cisterns:

This consist of problems of like how long it will take for different pipes of different diameter to fill a cistern, The time taken to fill a cistern when one pipe is filling it while the other empties it etc.

Concepts:

a. If a pipe can fill a tank in x hr& another pipe can fill it in y hr, then the fraction of tank filled by both pipes in 1 hr= $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$

Or , the number of hours required to fill the tank by both pipes = $\frac{xy}{x+y}$

b. If a pipe can fill a tank in x hr& another pipe can empty it in y hr, then the fraction of tank filled by both pipes together in 1 hr = $\frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$

Results:

a. If one Person is M times a good worker & takes 1 day less than the other person, then Both working together can finish the work in T= $\frac{M}{M^2-1}$Days

b. One pipe can fill in a cistern in T_1 min & another pipe in T_2 min. If both pipes are open together , time taken to fill in the cistern T= $\frac{T1 \times T2}{T1+T2}$, Where T1 & T2 are time taken by pipes alone

c. One pipe can fill in a cistern in T1 min & another pipe can empty it in T2 min. If both pipes are opened together ,then time taken will be T= $\frac{T1 \times T2}{T1-T2}$, Where T1 & T2 are time taken by pipes alone

d. A cistern takes T1 min to fill by the filling pipes, but takes 'x'extra min to fill in due to a leak in the cistern. Time which leak takes too empty the cistern is T. then$\frac{1}{T1} - \frac{1}{T} = \frac{1}{T1-X}$