SEQUENCE, SERIES & LOGARITHM-Pre class Notes

Arithmetic progression

An **arithmetic progression** (AP) or **arithmetic sequence** is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence 3, 5, 7, 9, 11, 13, … is an arithmetic progression with common difference 2.

If the initial term of an arithmetic progression is a_1 and the common difference of successive members is d, then the nth term of the sequence is given by:

a. The last term + First term = a + (n - 1) d
b. The sum **S** of the first **n** values of a finite sequence is given by the formula:
 $S = {}^1/_2(a_1 + a_n)n$, where $\mathbf{a_1}$ is the first term and $\mathbf{a_n}$ the last. Or $S = {}^1/_2(2a_1 + d(n-1))n$
 Eg- Find the sum of the first 10 numbers from this arithmetic progression 1, 11, 21, 31...

 Solution: we can use this formula $S = {}^1/_2(2a_1 + d(n-1))n$
 $S = {}^1/_2(2.1 + 10(10-1))10 = 5(2 + 90) = 5.92 = 460$

Arithmetic mean:

a. If a & b are any two number, m is called arithmetic mean of 'a' & 'b' & is given by $m = \frac{a+b}{2}$
b. 'n' Arithmetic mean between two quantities 'a' & 'b'
 If between two given quantities 'a' & 'b' we insert 'n' A.M, $x_1, x_2, x_3 \dots x_n$, then a, $x_1, x_2, x_3 \dots x_n$, b will be in A.P
 In order to find the values of these means, we require the common difference.
 The above series consist of(n+2) terms, & last term is b & first term is 'a'

 ⇨ $b = T_{n+2} = a+(n-1)d$; => $d=\frac{b-a}{n+1}$
 ⇨ $x_1= T_2= a+d$; $x_2 = T_3 = a + 2d$….. ; $x_n= T_{n+1}= a+nd$
 ⇨ On putting value of d in $x_1= \frac{na+b}{n+1}$
 ⇨ $X_n = \frac{nb+a}{n+1}$
c. Sum of 'n' A.M = n{single A.M}= $n\{\frac{a+b}{2}\}$

Results:

a. If each term of a given arithmetical progression be increased, decreased, multiplied or divided by the same non zero quantity, then the resulting series thus obtained will also be in A.P
b. Any three number in A.P be taken as a-d, a. a+d; Any four number in A.P are taken as
 a-3d, a-d, a+d, a+3d.
c. In an A.P, the sum of terms equidistant from the beginning & end is constant & equal to the sum of first & last term.

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d. $T_n = S_n - S_{n-1}$ (n≥2)
e. Sum & difference of corresponding term of two A.P will also form a series in A.P
f. m^{th} term of A.P from end is T_{n-m-1} term from the beginning.

Geometric Progression:-

Geometric progression, also known as a geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the *common ratio*.
For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2.

⇨ The general term form of G.P with 'n' terms ia a, ar, ar^2, ar^3,….ar^{n-1}.
⇨ If a = the first term, r = the common ratio, $T_n = n^{th}$ term & S_n = sum of n terms; then

a. $T_n = ar^{n-1}$
b. Sun of n terms of G.P (S_n) = a $\frac{(1-r^n)}{(1-r)}$, where (r <1)
& S_n = a $\frac{(r^n-1)}{(r-1)}$ where r > 1
c. Sum of Infinite term of G.P when $|r| < 1$
S∞ = - $\frac{a}{1-r}$

Geometric Mean:

⇨ Are any two terms, 'G' is called the geometric mean of 'a' & 'b' & is given by
G = $\sqrt{ab}$
⇨ Product of ' n' geometric mean $G^n = (\sqrt{ab})^n$

Results:

a. Three number in an G.P should be taken as $\frac{a}{r}$, a, ar
b. Four number in an G.P should be taken as $\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3
c. If a_1, a_2, a_3…. & b_1, b_2, b_3….be two G.P of common ratio of r_1 & r_2 respectively, then a_1b_1, a_2b_2, a_3b_3…. & $\frac{a1}{b1}$, $\frac{b1}{b2}$, $\frac{a3}{b3}$….. will also form G.P, whose common ratio will be r_1r_2 & $\frac{r1}{r2}$ respectively.
d. If a_1, a_2, a_3…be a G.P of +ve terms, then log a_1 , log a_2 , log a_3 will be an A.P & conversely

Harmonic Progression:-

A series of quantities are said to be in harmonic progression when their reciprocals are in A.P

Eg- $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$…. Are H.P as their reciprocals 3, 5, 7…. Are in A.P

⇨ n^{th} term of H.P (T_n) = $\frac{1}{a+(n-1)d}$

Harmonic Mean:-

The H.M between two quantities 'a' & 'b' will be 'x' if a, x, b be in harmonica progression

⇨ x = $\frac{2ab}{a+b}$

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Important Note: For any set of 'n' positive numbers, the following relationship always hold.

- ⇨ A.M ≥ G.M ≥ H.M

Logarithms:-

The power to which a base, such as 10, must be raised to produce a given number. If $n^x = a$, the logarithm of *a,* with *n* as the base, is *x;* symbolically, $\log_n a = x$.

- ⇨ For example, $10^3 = 1{,}000$; therefore, $\log_{10} 1{,}000 = 3$.
- ⇨ The kinds most often used are the common logarithm (base 10), the natural logarithm (base *e*), and the binary logarithm (base 2).
- ⇨ If $a^y = x$, then $Y = \log_a x$

Properties and Formulas of Logarithms:-

a. $\text{Log}_a 1 = 0$ {Property}

b. $\text{Log}_a a = 1$ {Property}

c. Loga $a^x = x$ {Property}

d. $\log_a a^x = x$ {Property}

e. $a^{\log_a x} = x$ {Property}

f. $\log_a mn = \log_a m + \log_a n$ {Product Rule}

g. $\log_a \frac{m}{n} = \log_a m - \log_a n$ {Quotient rule}

h. $\log_a m^p = p \log_a m$

i. $\log_a m^{\frac{1}{r}} = \frac{1}{r} \log_a m$ {The root rule}

j. $\log_a m = \log_a n \longleftrightarrow m = n, \text{ for } a > 0 \text{ and } a \neq 1$ {property of logarithmic equality}

k. **Changing the base**

$$\log_b a = \frac{\log_d a}{\log_d b}$$