SI, CI, EQUATIONS AND ROOTS Pre Class Notes

Simple Interest:-

a. The money which has been lent or borrowed is called principal.
b. Interest is the additional amount, accrued on the principal of money borrowed
c. The Interest rate is the percentage growth on the principal for a given time period, usually calculated on the year bases.
d. The Interest payable on the principal is known as simple interest.
e. Formula for simple interest = $\frac{P \times R \times T}{100}$
   Here, p= principal; R= rate of interest; T= time period for which amount is borrowed
   Amount = Principal + simple Interest
   If T is not whole number, then the period is represented as a fraction of year, i.e 1 month = 1/ 12^{th} of the year.

Compound Interest:-

a. Compound interest is paid on the original principal **and** on the accumulated past interest
b. The amount received at the end of 1^{st} year becomes principal for 2^{nd} year, & so on
c. The interest is calculated on the new principal at the end of every time period
d. Formula for compound interest (C.I) = $P\{1+\frac{r}{100}\}^n - P$
   Where P= Principal; r = rate of interest; n= time period
   Amount = $P\{1+\frac{r}{100}\}^n$

Note:

a. When C.I is calculated annually , C.I = $P\{1+\frac{r}{100}\}^n - P$
b. When C.I is calculated semi-annually , C.I = $P\{1+\frac{r/2}{100}\}^{2n} - P$
c. When C.I is calculated Quarterly., C.I = $P\{1+\frac{r/4}{100}\}^{4n} - P$
d. If word interest is given & nothing is specified, the interest is considered as S.I
e. If the interest is given by bank & nothing is specified, it is always C.I
f. Population growth is always taken as compounding bases.

Results:

a. A amount becomes X times in 'n' year at C.I. The rate per annum will be
   r = $[\left(\frac{A}{P}\right)^{1/n} - 1] \times 100$; Here A= amount, P= principal, r= rate of interest
b. The C.I in the nth tear is Rs 'X'& C.I in (n+1) th year is Rs 'Y'. The ratio of interest will be;
   R= $\frac{Y-X}{X} \times 100$
c. A principal amount to 'X 'times in 'T 'years as S.I, The number of years taken to years become 'Y' times = $\frac{Y-1}{X-1} \times T$
d. A principal amount to 'X 'times in 'T 'years as C.I, The number of years taken to become 'Y' times = $T \times n$; where n= $X^n = Y$

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Equations & Roots:-

a. Equations can be classified as Linear equation(those with degree 1), Quadratic equation(those with degree 2), Cubic equation(those with degree 3), & higher equation.
b. Degree of an equation is same as the maximum value of the sum of power of the variables in any equation. Eg- x^3 -3x + 5; x+y+z +xyz; have degree 3

Quadratic equation:-

Quadratic equation is a polynomial equation of the second degree. The general form is $ax^2 + bx + c=0$

a. Where *x* represents a variable, and *a*, *b*, and *c*, constants, with *a* ≠ 0. (If *a* = 0, the equation becomes a linear equation.)
b. The constants *a*, *b*, and *c*, are called respectively, the quadratic coefficient, the linear coefficient and the constant term or free term
c. Roots of the quadratic equation $ax^2 + bx + c=0$; are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$; α & β areroots
d. $b^2 - 4ac$ is called the determinant, denoted by D. It determines nature of root.
   i. If D < 0; => roots are imaginary & of the form p + iq& p – iq,
   ii. If D= 0; => roots are rational & equal to each other
   iii. If D > 0; & D is a perfect square; => the roots are rational but equal
   iv. If D > 0 & D is not perfect square; => the roots are irrational & of the form $p + \sqrt{q}$& $p - \sqrt{q}$

Results:

a. Sum & product of the roots:
Sum = $\alpha + \beta = -\frac{b}{a}$ i.e $\{-\frac{coeff.of\ x}{coeff.of\ x^2}\}$
Product= $\alpha\beta = \frac{c}{a}$ i.e $\{\frac{constant\ term}{coeff.of\ x^2}\}$

b. equation whose roots are α & β:
= x^{2} – (sum of roots) x + (products of roots) = 0

c. Transformation of Equations:
Let α & β are roots of the equation $ax^2 + bx + c=0$ …..(1)

i. **Equation whose roots are negative** of eq-1 above
Replace roots of above equation by – α & - β,
This is effected by putting y = - α = -x; or x =-y in eq-1
Final equation will be $ax^2 - bx + c = 0$

ii. **Reciprocal of the roots of** eq-1
The required roots are $\frac{1}{\alpha}, \frac{1}{\beta}$
This is effected by putting y= $\frac{1}{\alpha} = \frac{1}{x}$or x = $\frac{1}{y}$ in eq -1
The required equation is $cy^2 + by + a = 0$

iii. **Square of the roots** of eq-1
The required roots are α^2& β^2
This is effected by putting $y = \alpha^2 = x^2$ or $x = \sqrt{y}$ in eq -1
The required equation will be $a^2y^2+(2ac - b^2)y - c^2$

iv. **Cube of the roo**t of eq-1
The required roots are α^3& β^3
This is effected by putting $y = \alpha^3 = x^3$ or $x = y^{1/3}$ in eq-1
The required equation will be $a^3 y^3 + y (b^3 - 3ab) + c^3 = 0$

Inequalities:-
Let a & b be real number, if a-b is negative, we say that a is less than b & write $a < b$. If $a - b$ is positive then a is greater than b, i.e $a > b$

Elementary properties of Inequalities:

a. For any two real number a & b, we have $a > b$ or $a = b$ or $a < b$
b. If $a > b$, $b > c$, then $a>c$
c. If $a> b$ then $a+m>b+m$, for any real number m.
d. If $a > b$, then $am >bm$ for $m>0$, & $am <bm$ for $m<0$, that is when we multiply both side of the inequality by a –ve quantity, the sign of inequality is reversed
e. If $a \neq 0$, $b \neq 0$ & $a > b$, then $\frac{1}{a}<\frac{1}{b}$
f. If $a_1> b_1, a_2 >b_2$ ….. $a_n>b_n$ then $a_1 + a_2 +\ldots a_n > b_1 + b_2 + \ldots bn$ and $a_1\ a_2 \ldots a_n> b_1\ b_2 +\ldots bn(a_i \geq 0$ & $b_i \geq 0$, $i = 1, 2, \ldots n)$

Some Important properties:-

a. If $x> 0$ & $a > b > 0$; then $a^x>b^x$
b. If $a > 1$ & $x > y > 0$, then $a^x> a^y$
c. If $0 < a < 1$ & $x > y > 0$, then $a^x< a^y$
d. If $a > 1$ & $x > y$, then $\log_a x >\log_a y$
e. If $0 < a < 1$ & $x > y$, then $\log_a x <\log_a y$.