PERMUTATION & COMBINATION & PROBABILITY, Pre Class Notes

Fundamental principal of counting:-

1. **Product Rule:**
   If there are ‘m’ ways of doing a first thing & for each of them there is ‘n’ ways of doing a second thing, then total number of ways of doing both the thing together is **m × n**
   Eg: - In how many different ways can 3 travelers stay in 4 hotels, when each one should stay
   in different hotel?
   Solution: For first traveler there are 4 choices, for second traveler there are 3 choices, for third traveler there are only 2 choices
   Therefore total ways = 4 ×3 × 2 = 24 ways
2. **Addition Rule:**
   Eg- If there are 4 different ways from Surat to Ahmedabad, & 3 different ways from Surat to Mumbai, then in how many ways can a person can go to Ahmedabad or Mumbai from Surat?
   As From Surat a person can go either Mumbai or Ahmedabad. When we have OR it is two different cases hence the number of ways is 4 + 3 = 7
   Note: Addition Rule & Product Rule signifies the cases of “OR” & “AND”

Permutation:

Each of the different arrangements which can be made by taking some or all of a number of things is called permutation.
Eg-Suppose Amy, Brian, and Charles are to sit side by side. Then there are 6 different orders in which they can arrange themselves.
ABC, ACB, BAC, BCA, CAB, CBA – each of these permutation is different from the others.

Results:

a. Number of permutation of ‘n’ dissimilar things taken r at time = ${}^{n}P_{r} = \frac{n!}{(n-r)!}$.
   - ⇨ In above case repetition is not allowed, ie if you fill the first place in ‘n’ ways, then second place will be filled by (n-1) ways, third by (n-2) ways…..&rth by (n-r +1) ways
   - ⇨ Now if repetition is allowed, in this case each place is filled by n ways. Hence by fundamental theorem of counting all r places can be filled in n^r ways.

b. Number of permutation of ‘n’ dissimilar things taken all at a time= ${}^{n}P_{n} = n!$

c. If out of ‘n’ things ‘p’ are exactly alike of one kind, ‘q’ exactly alike of second kind, & ‘r’ exactly alike of third kind & rest are different, then the number of permutation of ‘n’ things taken all at a time
   - ⇨ $\frac{n!}{p!+q!+r!}$

d. Number of circular permutation of ‘n’ different things taken all at a times
   - ⇨ (n-1)!.
   - ⇨ Particular case: Necklace
     Number of arrangement of ‘n’ beads all different to form a necklace or on a circular wire will be ½ (n-1)!

e. Number of permutation of ‘n’ dissimilar things taken ‘r’ at a time when ‘p’ particulat things always occur
   ⇨ ${}^{n-p}C_{r-p}$ .r!
f. Number of permutation of ‘n’ dissimilar things taken ‘r’ at a time when ‘p’ particulat things never occur
   ⇨ ${}^{n-p}C_r$ .r!

Combinations:

Each of different groups or selection which can be made by taking some or all of a numbers of things (irrespective of order) is called combinations.

Eg-Suppose we wish to know how many color combinations can be made from four different colored marbles if we use only three marbles at a time. The marbles are colored red, green, white, and yellow

Sol. We list the possible combinations as follows:RGW RGY RWY GWY

i. If we rearrange the groups, for example RGW, to form GWR or RWG, we still have the same color combination within each group; therefore, order is not important.

Results:

a. Number of combination of ‘n’ dissimilar things taken ‘r’ at a time
   ⇨ ${}^{n}C_r = \frac{n!}{(n-r)!r!}$
b. Number of combination of ‘n’ dissimilar things taken ‘r’ at a time
   ⇨ ${}^{n}C_n = 1$
c. Number of combinations of ‘n’ dissimilar things taken ‘r’at a time, when ‘p’ particular things always occur
   ⇨ ${}^{n-p}C_{r-p}$
d. Number of combinations of ‘n’ dissimilar things taken ‘r’at a time, when ‘p’ particular things always occur
   ⇨ ${}^{n-p}C_r$

Special cases:

a. Division Into Groups:-

i. The number of way in which (m + n) things can be divided into two groups containing ‘m’ & ‘n’ things respectively

⇨ $\frac{(\boldsymbol{m}+\boldsymbol{n})!}{\boldsymbol{m}!\,\boldsymbol{n}!}$

ii. If n = m, the group are equal, & in this case the number of different ways of subdivision

⇨ $\frac{2m!}{m!\,m!\,2!}$

iii. If 2m things are to be divided equally between two persons, then the number of division

⇨ $\frac{2\ m!}{m!\,m!}$

iv. The number of division of m+n+p things into group of m, n, & p things respectively

⇨ $\frac{(m+n+p)!}{m!\,n!\,p!}$

b. Selection out of identical things:-

i. If there be 'n' identical things out of which we have to draw r things, then the number of combinations or selection will be 1

⇨ But if there be 'n' different things, then the number of selection of 'r' things will be ^nC_r

ii. If there be 'n' identical things & we have to make a selection of any number, it would mean that we may select none, one, two, or three…..or n things. Thus the number of selection will be n+1.

⇨ But for **non- empty selection** it will be 'n' only, and the case of none will be excluded

⇨ But if there be 'n' different things, then the number of selection of any number r(r may be 0, 1, 2, ….n) will be = 2^n.

⇨ But for non-empty selection = (2^n – 1)

Points to remember:

a. $^nC_r = {}^nC_{n-r}$
b. $^nCr_1 = {}^nCr_2 \Rightarrow r_1 = r_2$ or $r_1 + r_2 = n$
c. $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
d. Selection of **at least one lady,** from the committee out of gents & ladies= total – (no lady) = selection of committees with at least one.
   ⇨ Similarly at least two ladies = Total – (No lady + one lady)

Probability:-

Probability is the measure of the likelihood of occurrence of an event.

Probability of an event =$\frac{number\ of\ favourable\ outcome}{Number\ of\ all\ possible\ outcome}$

a. If an event E is sure to occur, we say that the probability of the event E is equal to 1 & we write P(E) =1,Such events are known as certain events.
b. If an event E is sure not to occur, we say that the probability of the event E is equal to 0 & we write P(E) =0,Such events are known as impossible events.

Therefore, for any event E, $0 \le P(E) \le 1$

For example, if we toss a coin, it is more likely for a 'head' or a 'tail' too come up? If the coin is unbiased, we find that there is an equal chance for a 'head' or a 'tail' to come up. Thus the chance for a 'head' (or a 'tail') to come up is ½.

Notes:

a. If the outcome of an operation can occur in 'n' equally likely ways, & if 'm' of these ways are favorable to an event E, then the probability of E, denoted by P(E), is given by P(E) = $\frac{m}{n}$

b. As $0 \le m \le n$, therefore for any event E, we have $0 \le P(E) \le 1$
c. The probability of E not occurring, denoted by P(not E), is given by P(not E)
   Or $P(\vec{E}) = 1 - P(E)$
d. Odds in Favour = $\frac{Number\ of\ favourable\ cases}{Number\ of\ unfavourable\ cases}$
e. Odds against = $\frac{Number\ of\ unfavourable\ cases}{Number\ of\ favourable\ cases}$

Mutually exclusive events:-

If two events are said to be mutually exclusive then if one happens, the other cannot happen & vice versa. In other words events have no simultaneous occurrence.
Eg- In rolling a die;

a. E : The event that the number is odd
b. F: The event that the number is even
c. G: The number that the number is a multiple of three

In drawing a card from a pack of 52 cards:

a. E: The event that it is a spade.
b. F: The event that it is a club
c. G: The event that it is a King

In the above 2 cases event E & F are mutually exclusive but the events E& G mutually exclusive or disjoint since they may have common outcomes.
Additional law of Probability:-
If E & F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by: **P(E or F) = p(E) + P(F)**
If the events are not mutually exclusive, then **P(E or F) = p(E) + P(F) – P(E & F together)**

Independent Events:
Two events are independent if the occurrence of one has no effect on the occurrence ot the other.
Eg-

1. On rolling a die & tossing a coin together:
   E: The event that number 6 turn up
   F: The event that head turns up
2. In shooting a target:
   E: event that the first trial is missed
   F: Event that the second trail is missed

In both these cases events E & F are independent.

Multiplication Law of Probability:-
If the event E & F are independent, then **P(E & F) = P(E) ×P(F)**

Conditional Probability:-

Let A & B are two dependent events, then probability of occurrence of events A when B has already occurred is given by P$(\frac{A}{B})$ = P$\frac{P(A \cap B)}{P(A)}$

Bayes' theorem.

Let A_1, A_2, ... , A_n be a set of mutually exclusive events that together form the sample space S. Let B be any event from the same sample space, such that P(B) > 0. Then,

P(A_k| B) = $\frac{P(\text{Ak} \cap \text{B})}{\text{P(A1}\cap \text{B) + P(A2}\cap \text{B) + ...+ P(An}\cap \text{B)}}$