# PRE- CLASS NOTES ON ANALYTICAL REASONING

**Concepts on Clock Problems:-**

Clock angle problems relate two different measurements: angles and time. To answer the problem the relationship between the time shown (or an elapsed time) and the position of the hands (as given by an angle) has to be found.

A general approach to such problems is to consider the rate of change of the angle in degrees per minute. The hour hand of a normal 12-hour analogue clock turns 360°in 12 hours (720 minutes) or 0.5°per minute. The minute hand rotates through 360°in 60 minutes or 6°per minute.

**Equation for the angle of the hour hand $\acute{\mathbf{O}}_{\mathbf{hr}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{M}_Z = \frac{1}{2}(60H + M)$**, where:

a. $\theta$ is the angle in degrees of the hand measured clockwise from the 12 o'clock position.
b. $H$ is the hour.
c. $M$ is the minutes past the hour.
d. $M_Z$ is the minutes past 12 o'clock.

**Equation for the degrees on the minute hand**
$\theta_{min.} = 6M$**,** where:

**a.** $\theta$ is the angle in degrees of the hand measured clockwise from the 12 o'clock position.
**b.** $M$ is the minute.

**Example**
The time is 5:24. The angle in degrees of the hour hand is:
$\acute{\mathbf{O}}_{\mathbf{hr}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{M}_Z = \frac{1}{2}(60H + M)$, = ½ (60 × 5 + 24) = 162
The angle in degrees of the minute hand is: $\theta_{min.} = 6M$**, =** 6× 24 = 144

**Equation for the angle between the hands**
The angle between the hands can also be found using the formula:

$$=> \Delta\acute{\mathbf{O}} = \left|\acute{\mathbf{O}}\mathbf{hr} - \theta\mathrm{min}\right| = \left|\frac{1}{2}(60\mathrm{H} + \mathrm{M}) - 6M\right|$$

**When are hour and minute hands of a clock superimposed?**

Hour and minute hands are superimposed only when their angle is the same.

$$\acute{\mathbf{O}}_{\mathbf{hr}=\theta\mathbf{min}} = \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{60H} + \mathbf{M}) = \mathbf{6}\boldsymbol{M}$$

⇨ **M = 5.45 h**

# PRE- CLASS NOTES ON ANALYTICAL REASONING

**Concepts on Calendars' Problems:-**

The year consists of 365 days, 5 hours, 48 minutes (52 weeks and 1 odd day). An extra day is added once in every fourth year which was called the leap year, which has 366 days (52 weeks and 2 odd days).

To find the day of any given date of the year, one need to understand the calendar calculations:

1. First thing to remember, first January 1 AD was Monday therefore, we must count days from Sunday. This means the 0th day was Sunday, so the 7th day was Sunday again and so on and so forth.

2. The day gets repeated after every seventh day (concept of a week), if today is Monday, then 28th day from now will also be Monday as it a multiple of 7 (28/7 = 4, so four weeks). Here the 30 day will be calculated by 30/7, which is 4 weeks and 2 days, these two days are called odd days. With starting day as Monday and two odd days, the day will be Wednesday; this point is the most critical in calendars. The other of looking at it is since the 28th day is Monday, so the 30th day will be Wednesday. But you have to understand and use the concept of odd days as the question may be about thousands of years.

3. In a normal year there are 365 days so 52 weeks and 1 odd day, in a leap year there are 366 days so 52 weeks and 2 odd days.

4. In 100 years there are 24 leap years and 76 normal years, so the number of odd days are 24(2) + 76 = 124, which is 17 weeks + 5 odd days, so 100 years have 5 odd days.

5. In 200 years the number of odd days is twice the number in 100 years which is 10, which is one week and 3 odd days, so 200 years have 3 odd days. In 300 years, the number of odd days is 15, which is two weeks and 1 odd day, so 300 years have one odd day.

6. 400 year is a leap year; similarly the multiples of 400 are also leap years.

7. In 400 years, the number of odd days become 20 + 1(from the leap year), so total days are 21, which is three weeks and 0 odd days. In 400 years there are 0 odd days

**Example1:** What was the day on 25th January, 1975?

Counting the years 1600 + 300 + 74
In 1600 years, there are zero odd days
In 300 years, there is one odd day
In 74 years, there are 18 leap years and 56 normal years, so the odd days are:
18(2) + 56(1) = 36 + 56 = 92,
Which is 13 weeks and 1 odd day

# PRE- CLASS NOTES ON ANALYTICAL REASONING

In 25 days of January, 1975, there are 3 weeks and 4 odd days
Total odd days = 0 + 1 + 1 + 4, six odd days, so it was a Saturday.