Chapter 1 : SETS - ASSIGNMENT-3

(Based on KITE VICTERS Plus One Mathematics Class 03)

1. Which of the following sets are Finite or Infinite ?

   (i) Set of the Months of a year.

   Solution: Finite set

   {January,February,March,...,December}

   (ii) The set of prime numbers less than 100.

   Solution: Finite set

   {2,3,5,...,97}

   (iii) $B = \{y : y \in \mathbb{R} \text{ and } 1 \leq y < 3\}$

   Solution: Infinite set

   Between 1 and 3, there are infinitely many real numbers.

   Consider the Arithmetic Mean(A.M) of 1 and 3

   i.e., $\frac{1+3}{2} = 2 \in B$

   Using the same concept ,A.M of 1 and 2 is 1.5 $\in$ B and so on.

(iv) $C = \{(x, y) : x^2 + y^2 = 25, x, y \in \mathbb{Z}\}$

Solution: Finite set

Here each x and y can take a minimum value -5 and its maximum value is 5.But integer values are $\pm 5, \pm 4, \pm 3, \pm 2, \pm 1, 0$

and perfect square numbers ≤ 25 are 0,1,4,9,16,25

Given sum of the squares of two numbers =25

$\therefore$ possibilities are

$0 + 25 = 25 \implies 0^2 + (\pm 5)^2 = 25 \implies (0, -5), (0, 5) \in C$

and $25 + 0 = 25 \implies (-5, 0), (5, 0) \in C$

Also $9 + 16 = 25 \implies (\pm 3)^2 + (\pm 4)^2 = 25$

$\implies (3, 4)(-3, 4), (3, -4)(-3, -4) \in C$

and $16 + 9 = 25 \implies (\pm 4)^2 + (\pm 3)^2 = 25$

$\implies (4, 3)(4, -3), (-4, 3)(-4, -3) \in C$

$\therefore$C contains 12 elements.

2. From the following Sets , select Equal Sets.

(i) A={2,4,6,8}

(ii) B={1,3,5,...}

(iii) C={ -2,4,6,8 }

(iv) $D = \{x : x = 2n - 1, n \in \mathbb{N}\}$

(v) $E = \{2x : x \in \mathbb{N}, x < 5\}$

(vi) $F = \{x : x = 2n + 1, n \in \mathbb{N}\}$

Solution:

i) A={2,4,6,8}

ii) B={1,3,5,. . .}

iii)C={ -2,4,6,8 }

iv) Putting n= 1,2,3,... ,then D={1,3,5,...}

v) Here x takes the values 1,2,3,4 then 2x takes the values 2,4,6,8

$\therefore E = \{2, 4, 6, 8\}$

vi) Putting n=1,2,3,4,... $F = \{3, 5, 7, ...\}$

From the above sets $A = E \quad , \quad B = D$

Remark:-

i) A and C have same number of elements but the elements are not equal .i.e., the cardinality of A and C are equal.

ii) In above set F,if $n \in \mathbb{W}$ (set of Whole numbers),what will happen??

3. Check whether each of the following set is empty or non-empty. Also write the cardinality of each set.

Note : Cardinality of a set A is the "number of elements" of the set A, denoted by n(A).

(i) $A = \{x : x \in \mathbb{Z} \text{ and } x^2 - 1 = 0\}$

Solution:

$x^2 - 1 = 0 \implies x^2 = 1 \implies x = -1, 1$

$\therefore$ A={-1,1}, the set is non-empty

Then the cardinality(number of elements in a set), $n(A) = 2$

(ii) $B = \{x : x \in \mathbb{N} \text{ and } 0 < x < 1\}$

Solution:

There is no natural numbers between 0 and 1 .

i.e.,$B = \{\ \}$ *or* ϕ

so $n(B) = n(\phi) = 0$

(iii) $C = \{x : x \in Z \text{ and } 4x^2 - 1 = 0\}$

Solution:

$\mathbb{Z}$ is set of integers .

$4x^2 = 1 \implies x^2 = \frac{1}{4} \implies x = \frac{1}{2}\ and\ -\frac{1}{2}$

But $\frac{1}{2}$ and $-\frac{1}{2} \notin \mathbb{Z}$

$\therefore C = \{\ \}$ so $n(C) = 0$

(iv) $D = \{x : x \in \mathbb{Q} \text{ and } 4x^2 - 1 = 0\}$

Solution:

Here instead of $\mathbb{Z}$ the set is $\mathbb{Q}$ (set of rational numbers) .

$\therefore 4x^2 = 1 \implies x^2 = \frac{1}{4} \implies x = \pm\frac{1}{2} \in \mathbb{Q}$

i.e., $D = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$, D is non empty and $n(D) = 2$

(v) $E = \{x : x \text{ is an irrational number and } x^2 - 1 = 0\}$

Solution:

$x^2 = 1 \implies x = \pm 1 \notin \mathbb{Q}'$ (set of irrational numbers)

i.e., $E = \{\ \}$, an empty set and $n(E) = 0$

(vi) $F = \left\{\frac{n}{n+1}, n \in \mathbb{N}\right\}$

Solution:

$F = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right\}$ is an infinite set .As the cardinality is the number of elements in a set ,here we can't say the

cardinality of F.

4. Which of the following pair of sets are Equal ? Justify your answer.

(i) $A = \{x : x$ is a letter in the word "MARS"$\}$

$B = \{y : y$ is a letter in the word "ARMS"$\}$

Solution:

$A = \{M, A, R, S\}$ and $B = \{A, R, M, S\}$

so $A = B$

(ii) $A = \{x : x \in \mathbb{R}$ and $x^2 < 1\ \}$

$B = \{x : x \in \mathbb{R}$ and $x < 1\ \}$

Solution:

A is the set of all real numbers between -1 and $+1$ but B is the set of all real number less than 1, $\implies A \neq B$.

For example Consider a real number less than 1 .

Let us take x= -2 .

Clearly $-2 \in$ B But $x^2 = (-2)^2 = 4 > 1$ so $-2 \notin A$

$\therefore A \neq B$

Or

$A = \{x : -1 < x < 1\}$ and $B = \{x : -\infty < x < 1\}$

so $A \neq B$

(iii) $A = \{x : x \in \mathbb{Z} \ \ and\ x^2 \leq 8\}$

$B = \{x : x \in \mathbb{R}\ and\ x^3 = 4x\}$

Solution:

$$A = \{x : x \in \mathbb{Z}\ and\ x^2 \leq 8\} \implies A = \{-2, -1, 0, 1, 2\}$$

$$B = \{x : x \in \mathbb{R}\ and\ x^3 = 4x\} \implies x^3 - 4x = 0$$

$$\implies x(x^2 - 4) = 0$$

$$\implies x = 0\ or\ (x^2 - 4) = 0$$

$$\implies x = 0, -2, 2$$

$$\therefore B = \{-2, 0, 2\} \implies A \neq B$$

(iv) $A = \{x : x = 2n - 1, n \in \mathbb{Z}\}$

$B = \{x : x = 2n + 1, n \in \mathbb{Z}\}$

Solution:

Putting $n = \ldots -2, -1, 0, 1, 2, \ldots$

$A = \{..., -3, -1, 1, 3, ...\}$

$B = \{..., -3, -1, 1, 3, ...\}$

$\therefore A = B$ i.e., both represents set of odd integers.

(v) $A = \{x : x = 2n, n \in \mathbb{R}\}$

$B = \{x : x = 4n, n \in \mathbb{R}\}$

Solution:

Here $n \in \mathbb{R}$ i.e., n is a real number so x is also a real number in both cases so $A = B = \mathbb{R}$

5. Which of the following sets are Singleton sets ?

(i) $A = \{x : x \in \mathbb{Z} \text{ and } x - 2 = 0\}$

Solution: $A = \{2\}$ so A is a singleton set.

(ii) $B = \{x : x \in \mathbb{Z} \text{ and } -2 \leq x \leq 2\}$

Solution: $B = \{-2, -1, 0, 1, 2\}$, contains 5 elements.

(iii) $C = \{x : x \in \mathbb{R}^+ \text{ and } x^2 - 2 = 0\}$

Solution:

$x^2 = 2 \implies x = \pm\sqrt{2}$

Here R^+ is the set of positive real numbers, $-\sqrt{2} \notin R^+$

$\therefore C = \{\sqrt{2}\}$,a singleton set.

(iv) $D = \{x : x \in \mathbb{Q} \text{ and } 3x - 2 = 0\}$

Solution: $D = \left\{\frac{2}{3}\right\}$, a singleton set.

(v) $E = \{x : |x| = 5, x \in \mathbb{Z}\}$

Solution:

Here $|x|$ is absolute value of x

so $|x| = 5 \implies x = -5, 5$

$\therefore E = \{-5, 5\}$, contains 2 elements.

(vi) $F = \{x : |x| = 5, x \in \mathbb{N}\}$

Solution:

$|x| = 5 \implies x = -5, 5$

Here $x \in \mathbb{N} \implies x = 5$ only.

$\therefore F = \{5\}$,a singleton set.

6. Which of the following sets are Equal ?

(i) $A = \{x \in \mathbb{Z} : -1 < x < 2\}$

(ii) $B = \{x \in \mathbb{Z} : 0 \leq x \leq 1\}$

(iii) $C = \{x \in \mathbb{R} : x^2 = x\}$

(iv) $D = \{n \in \mathbb{N} : \frac{1}{2}(1 + (-1)^n)\}$

Solution:

(i) $A = \{0, 1\}$

(ii) $B = \{0, 1\}$

(iii) For C ,$x^2 - x = 0 \implies x(x-1) = 0$ so $C = \{0, 1\}$

(iv) In D by putting n=1,2,3,... we are getting $D = \{0, 1\}$

So $A = B = C = D$

7. If $A = \{x : x$ is a vowel of English alphabets$\}$, and

   B=$\{$y : y$\in$ N and y $\leq$5$\}$. Is

   (i) $A = B$?

   (ii) $A \approx B$?

Note : Two sets A and B are said to be **Equivalent** (denoted by $A \approx B$), if the number of elements of A and B are equal. i.e., $n(A) = n(B)$

Solution:

$A = \{a, e, i, o, u\}$ and $B = \{1, 2, 3, 4, 5\}$ Here the elements are not equal but the number of elements are equal

$\therefore i) A \neq B$

ii) $A \approx B$

8. $A = \{x : x \in N$ and sum of digits of x is same as the number of digits of x$\}$. Is A finite or infinite set ?

Solution: Infinite set.

Let us consider numbers 1,11,111,1111,11111,.....all these are elements of our required set and there are infinite numbers like this.So the required set is infinite.

Note:-

i.e., stands for id est, which means "that is"